Chapter 3 – Past Test and Quiz Problems – Life Tables, UDD

(4 points) The curtate expectation of life, which is e_{80}, is 10.606 based on the Standard Ultimate Life Table.

If $q_{80} = 0.015$ instead of 0.032658, but the other mortality rates are unchanged, determine e_{80} accurate to three decimal places.

Solution:

$$e_{80} = p_x(1 + e_{81})$$

$$10.606 = (1 - 0.032658)(1 + e_{81}) \Rightarrow e_{81} = 9.9641$$

$$e_{80} = (1 - 0.015)(1 + 9.9641) = 10.800$$

(5 points) Under the Standard Ultimate Life Table:

a. $e_{60:\overline{10}|} = 9.733$

b. $e_{70:\overline{10}|} = 9.201$

c. $e_{80} = 10.606$

Determine e_{60} .

Solution:

$$e_{60} = e_{60:\overline{10}|} + e_{70:\overline{10}|}({}_{10}p_{60}) + e_{80}({}_{20}p_{60})$$

$$= 9.733 + 9.201\left(\frac{l_{70}}{l_{60}}\right) + 10.606\left(\frac{l_{80}}{l_{60}}\right)$$

$$= 9.733 + 9.201\left(\frac{91082.4}{96634.1}\right) + 10.606\left(\frac{75657.2}{96634.1}\right)$$

$$= 26.7091$$

(8 points) You are given:

a. $\mu_{x+0.4} = 0.13$

b. $q_{x+1} = q_x + 0.02$

c. Deaths are uniformly distributed between integral ages.

Calculate ${}_{0.2|0.5}q_{x+0.7}$.

Solution:

$$\text{Under UDD} ==> \mu_{x+s} = \frac{q_x}{1 - s \cdot q_x} ==> 0.13 = \frac{q_x}{1-(0.4)q_x}$$

$$==> 0.13 - 0.052 q_x = q_x ==> q_x = \frac{0.13}{1.052} = 0.123574$$

$$q_{x+1} = q_x + 0.02 = 0.143574$$

$$l_x = 1000; l_{x+1} = (1000)(1-0.123574) = 876.426; l_{x+2} = (876.426)(1-0.143574) = 750.594$$

$${}_{0.2|0.5}q_{x+0.7} = \frac{l_{x+0.9} - l_{x+1.4}}{l_{x+0.7}}$$

$$= \frac{(1000)(1-0.9) + (876.426)(0.9) - \left[(876.426)(1-0.4) + (750.594)(0.4)\right]}{(1000)(1-0.7) + (876.426)(0.7)} = 0.06863$$