Chapter 3 – Past Test and Quiz Problems – Select and Ultimate Mortality

(4 points) You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
90	0.1	0.2	0.3	92
91	0.12	0.25	0.5	93

If $l_{[90]} = 100,000$, calculate $l_{[91]}$.

Solution:

$$l_{93} = l_{[90]} \cdot p_{[90]} \cdot p_{[91]} \cdot p_{92} \quad \text{and} \quad l_{93} = l_{[91]} \cdot p_{[91]} \cdot p_{[92]}$$

so

$$l_{[90]} \cdot p_{[90]} \cdot p_{[91]} \cdot p_{92} = l_{[91]} \cdot p_{[91]} \cdot p_{[92]}$$

$$(100,000)(1-0.1)(1-0.2)(1-0.3) = l_{[91]}(1-0.25)(1-0.12)$$

$$l_{[91]} = \frac{(100,000)(1-0.1)(1-0.2)(1-0.3)}{(1-0.25)(1-0.12)} = 76,363.64$$

(6 points) You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
90	0.1	0.2	0.3	92
91	0.12	0.25	0.5	93

Deaths between integral ages follows a constant force of mortality.

Calculate ${}_{1.7}q_{[90]+0.4}$.

Solution:

$$ {}_{1.7}q_{[90]+0.4} = \frac{l_{[90]+0.4} - l_{[90]+2.1}}{l_{[90]+0.4}} $$

We need to create $l\text{'}s$.

$$ l_{[90]} = 1000; l_{[90]+1} = (1000)(1-0.1) = 900; l_{[90]+2} = (900)(1-0.2) = 720; l_{[90]+3} = (720)(1-0.3) = 504 $$

$$ l_{[90]+0.4} = (1000)^{1-0.4}(900)^{0.4} = 958.73315155 $$

$$ l_{[90]+2.1} = (720)^{0.9}(504)^{0.1} = 694.7719885 $$

$$ {}_{1.7}q_{[90]+0.4} = \frac{l_{[90]+0.4} - l_{[90]+2.1}}{l_{[90]+0.4}} = \frac{958.73315155 - 694.7719885}{958.73315155} = 0.27532 $$

(4 points) You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
90	0.1	0.2	0.3	92
91	0.12	0.25	0.5	93

If $e_{92} = 2.00$, calculate $e_{[90]}$.

Solution:

$$e_{[90]} = p_{[90]}(1+e_{[90]+1})$$

$$e_{[90]+1} = p_{[91]}(1+e_{92})$$

$$e_{[90]} = p_{[90]}[1+p_{[90]+1}(1+e_{92})] = (0.9)[1+(0.8)(1+2.00)] = 3.06$$

(5 points) You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
100	0.20	0.40	0.70	102
101	0.30	0.60	1.00	103

Calculate $Var[K_{[100]}]$

Solution:

$$Var[K_{[100]}] = 2\sum k \cdot {}_k p_x - e_x - (e_x)^2$$

$$e_{[100]} = p_{[100]} + {}_2 p_{[100]} + {}_3 p_{[100]} = (0.8) + (0.8)(0.6) + (0.8)(0.6)(0.3) = 1.424$$

$$\sum k \cdot {}_k p_x = (1)(0.8) + (2)(0.8)(0.6) + (3)(0.8)(0.6)(0.3) = 2.192$$

$$Var = (2)(2.192) - 1.424 - (1.424)^2 = 0.9322$$

(6 points) You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
100	0.05	0.15	0.25	102
101	0.10	0.30	0.50	103
102	0.20	0.40	0.80	104
103	0.25	0.50	1.00	105

During the first two years, deaths are assumed to be distributed uniformly between integral ages. After two years, it is assumed that we have a constant force of mortality between integral ages.

Calculate ${}_{0.7|0.9}q_{[101]+0.6}$.

Solution:

$$ {}_{0.7|0.9}q_{[101]+0.6} = \frac{l_{[101]+1.3} - l_{103.2}}{l_{[101]+0.6}} \Rightarrow \frac{0.7l_{[101]+1} + 0.3l_{[101]+2} - (l_{103})^{0.8}(l_{104})^{0.2}}{0.4l_{[101]} + 0.6l_{[101]+1}} $$

$$ l_{[101]} = 1000 $$
$$ l_{[101]+1} = 1000(1-0.1) = 900 $$
$$ l_{103} = 900(1-0.3) = 630 $$
$$ l_{104} = 630(1-0.5) = 315 $$

$$ {}_{0.7|0.9}q_{[101]+0.6} = \frac{0.7(900) + .3(630) - (630)^{0.8}(315)^{0.2}}{0.4(1000) + 0.6(900)} = 0.2878 $$

(6 points) You are given that mortality follows the following select and ultimate mortality table.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	x
93	0.14	0.24	0.36	0.50	96
94	0.21	0.32	0.45	0.70	97
98	0.28	0.40	0.63	0.90	98
99	0.36	0.56	0.81	1.00	99

Further, you are given that deaths are uniformly distributed during the first two years following underwriting. Mortality after the first two years following underwriting follows a constant force of mortality.

Calculate ${}_{1.5\,|\,2.3}q_{[93]+0.4}$

Solution:

$${}_{1.5\,|\,2.3}q_{[93]+0.4} = \frac{l_{[93]+0.4+1.5} - l_{[93]+0.4+1.5+2.3}}{l_{[93]+0.4}} = \frac{l_{[93]+1.9} - l_{97.2}}{l_{[93]+0.4}}$$

$$l_{[93]} = 1000 \Longrightarrow l_{[93]+1} = (1000)(1-0.14) = 860 \Longrightarrow l_{[93]+2} = 860(1-0.24) = 653.6$$

$$\Longrightarrow l_{96} = (653.6)(1-0.36) = 418.304 \Longrightarrow l_{97} = (418.304)(1-0.5) = 209.152$$

$$\Longrightarrow l_{98} = (209.152)(1-0.7) = 62.7456$$

$$l_{[93]+0.4} = (0.6)(1000) + (0.4)(860) = 944$$
$$l_{[93]+1.9} = (0.1)(860) + (0.9)(653.6) = 674.24$$
$$l_{97.2} = (209.152)^{0.8}(62.7456)^{0.2} = 164.3941174$$

$${}_{1.5\,|\,2.3}q_{[93]+0.4} = \frac{674.24 - 164.3941174}{944} = 0.54009$$

(4 points) You are given the following one year select and ultimate mortality table:

$[x]$	$q_{[x]}$	q_{x+1}	$x+1$
80	0.05	0.10	81
81	0.07	0.12	82
82	0.10	0.15	83
83	0.13	0.19	84

If $l_{[81]} = 100,000$, calculate $l_{[80]}$.

Solution:

$$l_{[82]} = l_{[80]} \cdot p_{[80]} \cdot p_{81}$$

$$l_{[82]} = l_{[81]} \cdot p_{[81]}$$

$$==> l_{[80]} \cdot p_{[80]} \cdot p_{81} = l_{[81]} \cdot p_{[81]}$$

$$==> l_{[80]} = \frac{l_{[81]} \cdot p_{[81]}}{p_{[80]} \cdot p_{81}} = \frac{(100,000)(1-0.07)}{(1-0.05)(1-0.10)} = 108,771.93$$

(4 points) You are given the following one year select and ultimate mortality table:

$[x]$	$q_{[x]}$	q_{x+1}	$x+1$
80	0.05	0.10	81
81	0.07	0.12	82
82	0.10	0.15	83
83	0.13	0.19	84

Calculate $e_{[81]:\overline{3}|}$.

Solution:

$$e_{[81]:\overline{3}|} = {}_1 p_{[81]} + {}_2 p_{[81]} + {}_3 p_{[81]}$$

$$= (1-0.07) + (1-1.07)(1-0.12) + (1-1.07)(1-0.12)(1-0.15)$$

$$= 2.44404$$