Chapter 3 – Past Test and Quiz Problems

You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
90	0.1	0.2	0.3	92
91	0.12	0.25	0.5	93

If $l_{[90]} = 100,000$, calculate $l_{[91]}$.

Solution:

$$l_{93} = l_{[90]} \cdot p_{[90]} \cdot p_{[91]} \cdot p_{92} \quad \text{and} \quad l_{93} = l_{[91]} \cdot p_{[91]} \cdot p_{[92]}$$

so

$$l_{[90]} \cdot p_{[90]} \cdot p_{[91]} \cdot p_{92} = l_{[91]} \cdot p_{[91]} \cdot p_{[92]}$$

$$(100,000)(1-0.1)(1-0.2)(1-0.3) = l_{[91]}(1-0.25)(1-0.12)$$

$$l_{[91]} = \frac{(100,000)(1-0.1)(1-0.2)(1-0.3)}{(1-0.25)(1-0.12)} = 76,363.64$$

You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
90	0.1	0.2	0.3	92
91	0.12	0.25	0.5	93

Deaths between integral ages follows a constant force of mortality.

Calculate ${}_{1.7}q_{[90]+0.4}$.

Solution:

$$ {}_{1.7}q_{[90]+0.4} = \frac{l_{[90]+0.4} - l_{[90]+2.1}}{l_{[90]+0.4}} $$

We need to create $l's$.

$$ l_{[90]} = 1000; l_{[90]+1} = (1000)(1-0.1) = 900; l_{[90]+2} = (900)(1-0.2) = 720; l_{[90]+3} = (720)(1-0.3) = 504 $$

$$ l_{[90]+0.4} = (1000)^{1-0.4}(900)^{0.4} = 958.73315155 $$

$$ l_{[90]+2.1} = (720)^{0.9}(504)^{0.1} = 694.7719885 $$

$$ {}_{1.7}q_{[90]+0.4} = \frac{l_{[90]+0.4} - l_{[90]+2.1}}{l_{[90]+0.4}} = \frac{958.73315155 - 694.7719885}{958.73315155} = 0.27532 $$

You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
90	0.1	0.2	0.3	92
91	0.12	0.25	0.5	93

If $e_{92} = 2.00$, calculate $e_{[90]}$.

Solution:

$$e_{[90]} = p_{[90]}(1 + e_{[90]+1})$$

$$e_{[90]+1} = p_{[91]}(1 + e_{92})$$

$$e_{[90]} = p_{[90]}[1 + p_{[90]+1}(1 + e_{92})] = (0.9)[1 + (0.8)(1 + 2.00)] = 3.06$$

You are given the following mortality table:

x	q_x
90	0.1
91	0.3
92	0.5
93	0.7
94	1.0

For ages 90 to 91 and 91 to 92, deaths are uniformly distributed between integral ages.

For ages 92 to 93 and 93 to 94, there is a constant force of mortality between integral ages.

Calculate ${}_{0.8|1.5}q_{90.6}$.

Solution:

$$ {}_{0.8|1.5}q_{90.6} = \frac{l_{91.4} - l_{92.9}}{l_{90.6}} $$

$$ l_{90} = 1000; l_{91} = (1000)(1-0.1) = 900; l_{92} = (900)(1-0.3) = 630; l_{93} = (630)(1-0.5) = 315 $$

$$ l_{90.6} = (1-0.6)(1000) + (0.6)(900) = 940 $$
$$ l_{91.4} = (1-0.4)(900) + (0.4)(630) = 792 $$
$$ l_{92.9} = (630)^{(1-0.9)}(315)^{0.9} = 337.60864 $$

$$ {}_{0.8|1.5}q_{90.6} = \frac{l_{91.4} - l_{92.9}}{l_{90.6}} = \frac{792 - 337.60864}{940} = 0.4834 $$

You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
100	0.20	0.40	0.70	102
101	0.30	0.60	1.00	103

Calculate $Var[K_{[100]}]$

Solution:

$$Var[K_{[100]}] = 2\sum k \cdot {}_k p_x - e_x - (e_x)^2$$

$$e_{[100]} = p_{[100]} + {}_2 p_{[100]} + {}_3 p_{[100]} = (0.8) + (0.8)(0.6) + (0.8)(0.6)(0.3) = 1.424$$

$$\sum k \cdot {}_k p_x = (1)(0.8) + (2)(0.8)(0.6) + (3)(0.8)(0.6)(0.3) = 2.192$$

$$Var = (2)(2.192) - 1.424 - (1.424)^2 = 0.9322$$

The curtate expectation of life, which is e_{80}, is 10.606 based on the Standard Ultimate Life Table.

If $q_{80} = 0.015$ instead of 0.032658, but the other mortality rates are unchanged, determine e_{80} accurate to three decimal places.

Solution:

$$e_{80} = p_x(1 + e_{81})$$

$$10.606 = (1 - 0.032658)(1 + e_{81}) \Rightarrow e_{81} = 9.9641$$

$$e_{80} = (1 - 0.015)(1 + 9.9641) = 10.800$$

You are given the following mortality table:

x	q_x
90	0.2
91	0.4

You are also given that deaths are uniformly distributed between ages 90 and 91 and the deaths follow a constant force of mortality between ages 91 and 92.

Calculate ${}_{0.2|0.9}q_{90.2}$.

Solution:

$$l_{90} = 1000$$
$$l_{91} = 1000(1-0.2) = 800$$
$$l_{92} = 800(1-0.4) = 480$$

$${}_{0.2|0.9}q_{90.2} = \frac{l_{90.4} - l_{91.3}}{l_{90.2}}$$

$$= \frac{[1000(0.6)+800(0.4)] - 800^{0.7}(480)^{0.3}}{1000(0.8)+800(0.2)}$$

$$= \frac{233.6662}{960} = 0.2434$$

Under the Standard Ultimate Life Table:

a. $e_{60:\overline{10}|} = 9.733$

b. $e_{70:\overline{10}|} = 9.201$

c. $e_{80} = 10.606$

Determine e_{60}.

Solution:

$$e_{60} = e_{60:\overline{10}|} + e_{70:\overline{10}|}\left({}_{10}p_{60}\right) + e_{80}\left({}_{20}p_{60}\right)$$

$$= 9.733 + 9.201\left(\frac{l_{70}}{l_{60}}\right) + 10.606\left(\frac{l_{80}}{l_{60}}\right)$$

$$= 9.733 + 9.201\left(\frac{91082.4}{96634.1}\right) + 10.606\left(\frac{75657.2}{96634.1}\right)$$

$$= 26.7091$$

You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
100	0.05	0.15	0.025	102
101	0.10	0.30	0.50	103
102	0.20	0.40	0.80	104
103	0.25	0.50	1.00	105

During the first two years, deaths are assumed to be distributed uniformly between integral ages. After two years, it is assumed that we have a constant force of mortality between integral ages.

Calculate ${}_{0.7|0.9}q_{[101]+0.6}$.

Solution:

$${}_{0.7|0.9}q_{[101]+0.6} = \frac{l_{[101]+1.3} - l_{103.2}}{l_{[101]+0.6}} \Rightarrow \frac{0.7l_{[101]+1} + 0.3l_{[101]+2} - (l_{103})^{0.8}(l_{104})^{0.2}}{0.4l_{[101]} + 0.6l_{[101]+1}}$$

$$l_{[101]} = 1000$$
$$l_{[101]+1} = 1000(1-0.1) = 900$$
$$l_{103} = 900(1-0.3) = 630$$
$$l_{104} = 630(1-0.5) = 315$$

$${}_{0.7|0.9}q_{[101]+0.6} = \frac{0.7(900) + .3(630) - (630)^{0.8}(315)^{0.2}}{0.4(1000) + 0.6(900)} = 0.2878$$

There are currently 1000 independent lives all age 80 who own life insurance policies at Maxwell Life Insurance Company.

You are given that mortality for these policies follows Gompertz Law with $B = 0.000005$ and $c = 1.10$.

Let L_{90} be the random variable representing the number of lives alive at the end of 10 years.

Calculate the $Var(L_{90})$.

Solutions:

$$L_{90} \sim Bin(1000, {}_{10}p_{80})$$

$${}_{10}p_{80} = e^{-\frac{-0.000005}{\ln(1.1)}(1.1)^{80}(1.1^{10}-1)} = 0.8426$$

$$Var[L_{90}] = npq = 1000(0.8426)(1-0.8426) = 132.6253$$

1. You are given that Mortality follows the Standard Ultimate Life Table.

   You are also given that mortality between 90 and 91 and between 91 and 92 follows a constant force of mortality while deaths are uniformly distributed between 92 and 93 and between 93 and 94.

   Calculate ${}_{1.6\,|\,1.2}q_{90.8}$.

   Solution:

$$ {}_{1.6\,|\,1.2}q_{90.8} = \frac{l_{92.4} - l_{93.6}}{l_{90.8}} $$

$$ l_{90.8} = (l_{90})^{0.2}(l_{91})^{0.8} = (41,841.1)^{0.2}(37,618.6)^{0.8} = 38,427.55 $$

$$ l_{92.4} = (0.6)(33,379.9) + (0.4)(29,183.8) = 31,701.46 $$

$$ l_{93.6} = (0.4)(29,183.8) + (0.6)(25,094.3) = 26,730.1 $$

$$ {}_{1.6\,|\,1.2}q_{90.8} = \frac{31,701.46 - 26,730.1}{38,427.55} = 0.12937 $$

You are given the following values from a mortality table:

$$q_{90} = 0.25 \qquad q_{91} = 0.40 \qquad q_{92} = 0.50$$

You are also given that deaths are uniformly distributed between 90 and 91 and between 91 and 92. Mortality between 92 and 93 follows a constant force of mortality.

Calculate ${}_{0.7\,|\,1.3}q_{90.6}$.

Solution:

$${}_{0.7\,|\,1.3}q_{90.6} = \frac{l_{91.3} - l_{92.6}}{l_{90.6}}$$

$$l_{90} = 1000 \implies l_{91} = (1000)(1-0.25) = 750 \implies l_{92} = 750(1-0.4) = 450 \implies l_{93} = (450)(1-0.5) = 225$$

$$l_{90.6} = (0.4)(1000) + (0.6)(750) = 850$$

$$l_{91.3} = (0.7)(750) + (0.3)(450) = 660$$

$$l_{92.6} = (450)^{0.4}(225)^{0.6} = 296.88928$$

$${}_{0.7\,|\,1.3}q_{90.6} = \frac{660 - 296.8828}{850} = 0.42719$$

You are given that mortality follows the following select and ultimate mortality table.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	x
93	0.14	0.24	0.36	0.50	96
94	0.21	0.32	0.45	0.70	97
98	0.28	0.40	0.63	0.90	98
99	0.36	0.56	0.81	1.00	99

Further, you are given that deaths are uniformly distributed during the first two years following underwriting. Mortality after the first two years following underwriting follows a constant force of mortality.

Calculate ${}_{1.5\,|\,2.3}q_{[93]+0.4}$

Solution:

$${}_{1.5\,|\,2.3}q_{[93]+0.4} = \frac{l_{[93]+0.4+1.5} - l_{[93]+0.4+1.5+2.3}}{l_{[93]+0.4}} = \frac{l_{[93]+1.9} - l_{97.2}}{l_{[93]+0.4}}$$

$$l_{[93]} = 1000 \implies l_{[93]+1} = (1000)(1-0.14) = 860 \implies l_{[93]+2} = 860(1-0.24) = 653.6$$

$$\implies l_{96} = (653.6)(1-0.36) = 418.304 \implies l_{97} = (418.304)(1-0.5) = 209.152$$

$$\implies l_{98} = (209.152)(1-0.7) = 62.7456$$

$$l_{[93]+0.4} = (0.6)(1000) + (0.4)(860) = 944$$
$$l_{[93]+1.9} = (0.1)(860) + (0.9)(653.6) = 674.24$$
$$l_{97.2} = (209.152)^{0.8}(62.7456)^{0.2} = 164.3941174$$

$${}_{1.5\,|\,2.3}q_{[93]+0.4} = \frac{674.24 - 164.3941174}{944} = 0.54009$$

You are given:

a. $q_{85} = 0.2$

b. $q_{86} = 0.4$

c. Deaths are uniformly distributed between age 85 and 86.

d. There is a constant force of mortality between age 86 and 87.

Calculate ${}_{0.4|0.7}q_{85.6}$.

Solution:

$${}_{0.4|0.7}q_{85.6} = \frac{l_{86} - l_{86.7}}{l_{85.6}} = \frac{800 - 559.49455}{880} = 0.27330$$

$$l_{85} = 1000$$

$$l_{86} = (1000)(1 - 0.2) = 800$$

$$l_{87} = 800(1 - 0.4) = 480$$

$$l_{85.6} = (1000)(1 - 0.6) + 800(0.6) = 880 \quad \text{since it is UDD}$$

$$l_{86.7} = (800)^{(1-0.7)} \cdot (480)^{0.3} = 559.49445$$

You are given the following one year select and ultimate mortality table:

$[x]$	$q_{[x]}$	q_{x+1}	$x+1$
80	0.05	0.10	81
81	0.07	0.12	82
82	0.10	0.15	83
83	0.13	0.19	84

If $l_{[81]} = 100,000$, calculate $l_{[80]}$.

Solution:

$$l_{[82]} = l_{[80]} \cdot p_{[80]} \cdot p_{81}$$

$$l_{[82]} = l_{[81]} \cdot p_{[81]}$$

$$==> l_{[80]} \cdot p_{[80]} \cdot p_{81} = l_{[81]} \cdot p_{[81]}$$

$$==> l_{[80]} = \frac{l_{[81]} \cdot p_{[81]}}{p_{[80]} \cdot p_{81}} = \frac{(100,000)(1-0.07)}{(1-0.05)(1-0.10)} = 108,771.93$$

You are given:

a. $\mu_{x+0.4} = 0.13$

b. $q_{x+1} = q_x + 0.02$

c. Deaths are uniformly distributed between integral ages.

Calculate ${}_{0.2|0.5}q_{x+0.7}$.

Solution:

$$\text{Under UDD} \Longrightarrow \mu_{x+s} = \frac{q_x}{1 - s \cdot q_x} \Longrightarrow 0.13 = \frac{q_x}{1-(0.4)q_x}$$

$$\Longrightarrow 0.13 - 0.052q_x = q_x \Longrightarrow q_x = \frac{0.13}{1.052} = 0.123574$$

$$q_{x+1} = q_x + 0.02 = 0.143574$$

$$l_x = 1000; l_{x+1} = (1000)(1-0.123574) = 876.426; l_{x+2} = (876.426)(1-0.143574) = 750.594$$

$${}_{0.2|0.5}q_{x+0.7} = \frac{l_{x+0.9} - l_{x+1.4}}{l_{x+0.7}}$$

$$= \frac{(1000)(1-0.9) + (876.426)(0.9) - \left[(876.426)(1-0.4) + (750.594)(0.4)\right]}{(1000)(1-0.7) + (876.426)(0.7)} = 0.06863$$

You are given the following one year select and ultimate mortality table:

$[x]$	$q_{[x]}$	q_{x+1}	$x+1$
80	0.05	0.10	81
81	0.07	0.12	82
82	0.10	0.15	83
83	0.13	0.19	84

Calculate $e_{[81]:\overline{3}|}$.

Solution:

$$e_{[81]:\overline{3}|} = {}_1 p_{[81]} + {}_2 p_{[81]} + {}_3 p_{[81]}$$

$$= (1-0.07) + (1-1.07)(1-0.12) + (1-1.07)(1-0.12)(1-0.15)$$

$$= 2.44404$$