Chapter 4 – Past Test and Quiz Problems – Endowment and Pure Endowment

(6 points) Jeff is (70). He wants to buy a life insurance policy from Lai Life Insurance Company. However, he is not sure which policy to buy. All calculations assume:

i. Mortality follows the Standard Ultimate Life Table

ii. $i = 0.05$

iii. Deaths are uniformly distributed between integral ages.

Jeff decides to purchase a 13-year endowment insurance policy with a death benefit of 100,000 payable at the end of the year of death.

Determine the expected present value of this endowment insurance.

Solution:

$$EPV = 100,000 A_{70:\overline{13}|}$$

$$= 100,000\left(A^{1}_{70:\overline{13}|} + {}_{13}E_{70}\right) = 100,000[{}_{70} - A_{83}({}_{13}E_{70}) + {}_{13}E_{70}]$$

$$= 100,000(0.42818 - 0.64336({}_{13}p_{70})v^{13} + ({}_{13}p_{70})v^{13}$$

$$= 100,000\left[0.42818 - 0.64336\left(\frac{67614.6}{91082.4}\right)(1.05)^{-13} + \left(\frac{67614.6}{91082.4}\right)(1.05)^{-13}\right]$$

$$= 56,858.26$$

You are given that mortality follows the following mortality table:

Age x	q_x
100	0.20
101	0.30
102	0.50
103	0.75
104	1.00

You are also given that $d = 0.10$ which means that $v = 0.90$. Further, you are given that deaths are uniformly distributed between integral ages for ages 100 and 101 and between ages 101 and 102. For ages over 102, mortality follows a constant force of mortality between integral ages.

a. (2 points) Let Z^{CONT} be the present value random variable for a whole life insurance policy to (100) with a death benefit of 10,000 paid at the moment of death.

   i. (4 points) Write an expression of Z^{CONT}.

   Solution:

   $$Z^{CONT} = 10{,}000v^{T_{100}} = (10{,}000)(0.9)^{T_{100}}$$

b. (4 points) Let $Z^{Discrete}$ be the present value random variable for a 3 year endowment insurance to (100) with a death benefit of 10,000 paid at the end of the year of death.

   i. (8 points) The expected present value which is $10{,}000A_{100:\overline{3}|}$ is 7800 to the nearest 100. Calculate $10{,}000A_{100:\overline{3}|}$ to the nearest 1.

   Solution:

   $$l_{100} = 1000; l_{101} = (1000)(1-0.2) = 800; l_{102} = (800)(1-0.3) = 560; l_{103} = 560(0.5) = 280$$

   $$l_{104} = (280)(1-0.75) = 70; l_{105} = 0$$

   $$1000A_{100:\overline{3}|} = 200v + 240v^2 + 560v^3 = 782.64 ==> A_{100:\overline{3}|} = 0.78264$$

   $$10{,}000A_{100:\overline{3}|} = 7826.4$$

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ii. (4 points) Calculate $Var[Z^{Discrete}]$

Solution:

Using the ls from Part a:

$$1000[{}^2A_{100:\overline{3}|}] = 200v^2 + 240v^4 + 560v^6 = 617.07096 \Longrightarrow {}^2A_{100:\overline{3}|} = 0.61707096$$

$$Var[Z^{Discrete}] = (10,000)^2[{}^2A_{100:\overline{3}|} - (A_{100:\overline{3}|})^2]$$

$$= (10,000)^2[0.61707096 - (0.78264)^2] = 454,559.04$$

(4 points) You are given that mortality follows the Standard Ultimate Life Table with interest at 5%. You are also given that deaths are uniformly distributed between integral ages.

Calculate the Expected Present Value for a 30 year endowment insurance issued to (50) with a death benefit of 10,000 **paid at the moment of death**.

Solution:

$$10,000\bar{A}_{50:\overline{30}|} = (10,000)\left[\left(\frac{i}{\delta}\right)(A_{50} - {}_{30}E_{50} \cdot A_{80}) + {}_{30}E_{50}\right]$$

$$= (10,000)[(1.02480)(0.18931 - (0.34824)(0.50994)(0.59293)) + (0.50994)(0.59293)]$$
$$= 2636.8181$$

Arthur is (80) and buys a 3-year endowment insurance with a death benefit of 10,000 paid at the end of the year of death.

a. (2 points) Write the random variable for Arthur's endowment insurance.

Solution:

$$Z = \begin{cases} 10,000v^{K_{80}+1} & \text{for } K_x \le 2 \\ 10,000v^3 & \text{for } K_x \ge 3 \end{cases}$$

You are given that:

i. $q_{80} = 0.08$

ii. $q_{81} = 0.10$

iii. $v = 0.93$

b. (4 points) Calculate the Actuarial Present Value of Arthur's endowment insurance.

Solution:

$$l_{80} = 1000; l_{81} = (1000)(1-0.08) = 920; l_{82} = (920)(1-0.1) = 828$$

$$A_{80:\overline{3}|} = \frac{(1000-920)(0.93)+(920-828)(0.93)^2+828(0.93)^3}{1000} = 0.81997840$$

Note that for the 828 people alive at the beginning of the third year, it does not matter if they live or die because they will get paid at the end of the third year.

$$Answer = (10,000)(0.81997840) = 8199.78$$

(4 points) You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
75	0.10	0.20	0.4	77
76	0.15	0.30	0.6	78
77	0.20	0.40	0.8	79
78	0.25	0.55	0.9	80
79	0.30	0.70	1.00	81

You are also given that $d = 0.1$.

Calculate $100,000A_{[77]:\overline{4}|}$ which is a 4-year endowment insurance to a newly underwritten life age 77 with a death benefit of 100,000 paid at the end of the year of death.

Solution:

$$l_{[77]} = 100,000; l_{[77]+1} = (100,000)(0.8) = 80,000; l_{79} = (80,000)(0.6) = 48,000$$
$$l_{80} = 48,000(0.2) = 9600; l_{81} = 9600(0.1) = 96; l_{82} = 0$$

$$100,000A_{[77]:\overline{4}|} = (100,000 - 80,000)(0.9) + (80,000 - 48,000)(0.9)^2 +$$
$$(48,000 - 9600)(0.9)^3 + 9600v^4 = 78,212.34$$

(4 points) Madeline (25) buys a 32-year pure endowment contract that will pay her 1,000,000 at the end of the term provided she is alive.

You are given that mortality follows the Standard Ultimate Life Table with an interest rate of 5%.

Calculate the expected present value which is $1,000,000 A_{25:\overline{32|}}^{\;\;\;\;1}$.

Solution:

$$1,000,000 A_{25:\overline{32|}}^{\;\;\;\;1} = (1,000,000) v^{32} \frac{l_{57}}{l_{25}}$$

$$= (1,000,000)(1.05)^{-32} \frac{97,435.2}{99,871.1} = 204,747.44$$

(4 points) Jiaying is (60) and buys a pure endowment of 25,000 to be paid at the end of 10 years if she is alive at that time.

You are given that $\mu_{60+t} = 0.01t$. You are also given that $\delta = 0.06$.

Calculate the Actuarial Present Value of Jiaying's pure endowment.

Solution:

$$APV = 25,000\,{}_{10}E_{60} = (25,000)v^{10}\,{}_{10}p_{60}$$

$$v^{10} = e^{-0.06(10)}$$

$${}_{10}p_{60} = e^{-\int_0^{10} \mu_{60+t}dt} = e^{-\int_0^{10} 0.01t\cdot dt} = e^{-\left[0.005t^2\right]_0^{10}} = e^{-0.5}$$

$$APV = (25,000)e^{-0.6}e^{-0.5} = 8321.78$$