

Chapter 4 – Past Test and Quiz Problems – Single Policy Probability (Percentile)

Jeff is (70). He wants to buy a life insurance policy from Lai Life Insurance Company. However, he is not sure which policy to buy. All calculations assume:

- i. Mortality follows the Standard Ultimate Life Table
- ii. $i = 0.05$
- iii. Deaths are uniformly distributed between integral ages.

First, he decides to consider a whole life insurance policy that pays a death benefit of 100,000 at the moment of death. Jeff asks Jake who is the chief actuary at Lai Life to do several things for him.

- a. (2 points) Write the present value random variable Z for this policy.

$$Z = 100,000v^{T_{70}} = 100,000(1.05)^{-T_{70}}$$

- b. (1 points) Jeff estimates that the expected present value of this whole life policy is 44,000 to the nearest 1000. Calculate it to the nearest 1.

$$\begin{aligned} EPV &= 100,000\bar{A}_{70} = 100,000\left(\frac{i}{\delta}\right)A_{70} \\ &= 100,000(1.02480)(0.42818) = 43,880 \end{aligned}$$

(Continued from Prior Page)

- c. (5 points) Calculate the probability that Z is less than 51,000. The probability needs to be accurate to 3 decimal places.

Solution:

$$\Pr(Z < 51,000) = \Pr(100,000v^{T_{70}} < 51,000)$$

$$v^{T_{70}} < 0.51 \Rightarrow (1.05)^{-T_{70}} < 0.51 \Rightarrow T_{70} > \frac{-\ln(0.51)}{\ln(1.05)} \Rightarrow T_{70} > 13.8008$$

$$P(T_{70} > 13.8008) = {}_{13.8008}P_{70}$$

$$\frac{l_{83.8008}}{l_{70}} = \frac{0.1992l_{83} + 0.8008l_{84}}{l_{70}}$$

$$= \frac{0.1992(67,614.6) + 0.8008(64,506.5)}{91,082.40} = 0.71502$$

(5 points) Valerie is 70 and purchases a whole life policy with a death benefit of 1000 paid at the end of the year of death.

You are given:

- i. Mortality follows the Standard Ultimate Life Table.
- ii. $i = 0.07$

Calculate the $\Pr(Z < 500)$

Solution:

$$Z = 1000v^{K_{70}+1}$$

$$1000v^{K_{70}+1} = 500 \implies v^{K_{70}+1} = 0.5$$

$$K_{70} + 1 = \frac{\ln(0.5)}{\ln[(1.07)^{-1}]} = 10.24$$

$$K_{70} = 9.24 \implies \text{Round Up} \implies 10$$

$$\Pr[Z < 500] = {}_{10}p_{70} = \frac{l_{80}}{l_{70}} = \frac{75,657.2}{91,082.4} = 0.830646$$