Chapter 4 – Past Test and Quiz Problems – Term Insurance

(4 points) You are given the following mortality table:

x	q_x
106	0.10
107	0.20
108	0.30
109	0.40
110	0.50
111	0.60
112	0.70
113	0.80
114	0.90
115	1.00

You are also given that $i = 0.10$.

Calculate 12,000 $A^{\,1}_{106:\overline{4}|}$.

$$\begin{aligned} l_{106} &= 1000 \\ l_{107} &= 1000(0.9) = 900 \\ l_{108} &= 900(0.8) = 720 \\ l_{109} &= 720(0.7) = 504 \\ l_{110} &= 504(0.6) = 302.4 \end{aligned}$$

$$1000 \cdot A^{\,1}_{106:\overline{4}|} = (1000-900)v + (900-720)v^2 + (720-504)v^3 + (504-302.4)v^4$$

$$1000 \cdot A^{\,1}_{106:\overline{4}|} = 100v + 180v^2 + 216v^3 + 201.6v^4$$

$$A^{\,1}_{106:\overline{4}|} = 0.539648931$$

$$\therefore 12000 \cdot A^{\,1}_{106:\overline{4}|} = 6475.787173$$

(5 points) Jeff is (63). He wants to buy a life insurance policy from Wu Life Insurance Company. However, he is not sure which policy to buy. All calculations assume:

i. Mortality follows the Standard Ultimate Life Table

ii. $i = 0.05$

iii. Deaths are uniformly distributed between integral ages.

Jeff decides to purchase a term insurance policy that pays a death benefit of 100,000 payable at the moment of death if he dies in the next 26 years. Determine the expected present value of this term insurance.

Solution:

$$100,000\bar{A}^{1}_{63:\overline{26}|} = 100,000\left[A_{63}\left(\frac{i}{\delta}\right) - {}_{26}E_{63}A_{89}\left(\frac{i}{\delta}\right)\right]$$

$$\frac{i}{\delta} = \frac{0.05}{\ln(1.05)} = 1.0248$$

$${}_{26}E_{63} = (1.05)^{-26}\left(\frac{l_{89}}{l_{63}}\right)$$

$$100,000\left[0.32785(1.0248) - (1.05)^{-26}\left(\frac{45,995.6}{95,534.4}\right)(0.73853)(1.0248)\right] = 23,349.93$$

(5 points) You are given the following mortality table:

x	q_x
90	0.2
91	0.4
92	0.6
93	0.8
94	1.0

Let Z be the present value random variable for a discrete 3-year term insurance policy issued to (90) with a death benefit of 5,000 paid at the end of the year of death.

You are also given that $d = 0.10$.

Calculate $Var[Z]$.

Solution:

$$v = 1 - 0.10 = 0.90$$

$$Var[Z] = {}^2 A^{1}_{90:\overline{3}|} - (A^{1}_{90:\overline{3}|})^2$$

$$l_{90} = 1000; l_{91} = 1000(0.8) = 800; l_{92} = 900(0.6) = 480$$
$$l_{93} = 630(0.4) = 192; l_{94} = 315(0.2) = 38.4; l_{95} = 63(0) = 0$$

$$A^{1}_{90:\overline{3}|} = \frac{(1000-800)(.9) + (800-480)(.9)^2 + (480-192)(.9)^3}{1000} = 0.649152$$

$${}^2 A^{1}_{90:\overline{3}|} = \frac{(1000-800)(.9)^2 + (800-480)(.9)^4 + (480-192)(.9)^6}{1000} = 0.525007$$

$$Var[Z] = 5000^2(0.525007 - 0.649152^2) = 2590217.22$$

You are given that mortality follows Makeham's Law with:

$$A = 0.004,\ B = 0.00001, \text{ and } c = 1.11$$

a. (3 points) Calculate probability that a person age 90 is still alive at age 95.

Solution:

$$_5p_{90} = \exp\left[-(0.004(5) - \frac{0.00001}{\ln(1.11)}(1.11)^{90}(1.11^5 - 1)\right] = 0.44596284$$

Let L_5 be the random variable representing the number of people alive at age 95 if there were 50,000 people alive at age 90.

b. (3 points) Using the probability from Part (a), calculate $Var[L_5]$. If you are not able to calculate the probability in Part (a), assume that it is 0.45. If you get an answer for Part (a), please use that answer.

Solution:

$$Var[L_5] = npq = (50,000)(0.44596284)(1 - 0.44596284) = 12,353.99927$$

c. (6 points) You are also given that $i = 0.06$. Calculate $1000A^{\ \,1}_{90:\overline{2}|}$.

Solution:

$$p_{90} = \exp\left[-0.004 - \frac{0.00001}{\ln(1.11)}(1.11)^{90}(1.11 - 1)\right] = 0.877698382$$

$$p_{91} = \exp\left[-0.004 - \frac{0.00001}{\ln(1.11)}(1.11)^{91}(1.11 - 1)\right] = 0.865574333$$

$$l_{90} = 1000; l_{91} = (1000)(0.877698382) = 877.698382;$$

$$l_{92} = (877.698382)(0.865574333) = 759.7131916$$

$$1000A^{\ \,1}_{90:\overline{2}|} = (1000 - 877.698382)(1.06)^{-1} + (877.698382 - 759.7131916)(1.06)^{-2} = 220.39$$

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%. You are also given that deaths are uniformly distributed between integral ages.

a. (3 points) Calculate the Expected Present Value for a 30 year term insurance issued to (50) with a death benefit of 10,000 paid **at the end of the year of death**.

Solution:

$$10,000 A^{\ 1}_{50:\overline{30|}} = 10,000(A_{50} - {}_{30}E_{50} \cdot A_{80})$$

$$= (10,000)(0.18931 - (0.34824)(0.50994)(0.59293)) = 840.1645$$

b. (4 points) Calculate the $Var[Z]$ where Z is the present value random variable for a 30 year term insurance issued to (50) with a death benefit of 10,000 paid **at the end of the year of death.**

Solution:

$$Var = (10,000)^2 \left[{}^2A^{\ 1}_{50:\overline{30|}} - \{A^{\ 1}_{50:\overline{30|}}\}^2 \right]$$

$${}^2A^{\ 1}_{50:\overline{30|}} = {}^2A_{50} - {}^2_{30}E_{50} \cdot {}^2A_{80} = 0.05108 - (1.05)^{-30}(0.34824)(0.50994)(0.38134)$$

$$= 0.03541$$

$$Var = (10,000)^2 \left[{}^2A^{\ 1}_{50:\overline{30|}} - \{A^{\ 1}_{50:\overline{30|}}\}^2 \right] = (10,000)^2 \left[0.03541 - (0.08401645)^2 \right]$$

$$= 2,835,258.1$$

(5 points) Filza who is (45) buys a 25 year term insurance with a death benefit of 1,000,000 paid at the moment of death.

You are given that:

a. Mortality follows the Standard Ultimate Life Table

b. $i = 0.05$

c. Deaths are uniformly distributed between integral ages.

Calculate the Actuarial Present Value of Filza's policy.

Solution:

$$APV = 1,000,000\bar{A}^{1}_{45:\overline{25}|} = 1,000,000(\bar{A}_{45} - {}_{25}E_{45}\bar{A}_{70})$$

$$1,000,000(i/\delta)(A_{45} - {}_{20}E_{45} \cdot {}_{5}E_{65}A_{70})$$

$$= (1,000,000)(1.0248)[0.15161 - (0.35994)(0.75455)(0.42818)]$$

$$= 36,195.35$$