Chapter 4 – Past Test and Quiz Problems – Whole Life, recursive

(6 points) Let Z_x be the present value random variable for a whole life insurance policy on (x) with a death benefit of 1 payable at the end of the year of death.

You are given:

a. $A_{50} = 0.300$

b. $Var[Z_{50}] = 0.110$

c. $q_{50} = 0.01$

d. $v = 0.93$

Calculate the $Var[Z_{51}]$.

Solution:

$$Var[Z_{50}] = {}^2A_{50} - (A_{50})^2 \Longrightarrow 0.11 = {}^2A_{50} - (0.3)^2 \Longrightarrow {}^2A_{50} = 0.2$$

$$A_{50} = vq_{50} + vp_{50}A_{51}$$

$$0.3 = (0.93)(0.01) + (0.93)(0.99)(A_{51}) \Longrightarrow A_{51} = \frac{0.3 - 0.0093}{(0.93)(0.99)} = 0.31574$$

$${}^2A_{50} = v^2q_{50} + v^2p_{50} \cdot {}^2A_{51}$$

$$0.2 = (0.93)^2(0.01) + (0.93)^2(0.99)({}^2A_{51}) \Longrightarrow {}^2A_{51} = \frac{0.2 - (0.93)^2(0.01)}{(0.93)^2(0.99)} = 0.22348$$

$$Var[Z_{51}] = 0.22348 - (0.31574)^2 = 0.1238$$

(6 points) You are given:

i. $A_{60} = 0.500$

ii. ${}^{2}A_{60} = 0.350$

iii. $p_{60} = 0.96$

iv. $p_{61} = 0.95$

v. $i = 0.06$

Let Z be the present value random variable for a whole life to (61) with a death benefit of 1 paid at the end of the year of death.

Calculate the $Var(Z)$.

Solution:

$$A_{60} = vq_{60} + vp_{60}A_{61}$$

$$0.5 = (1.06)^{-1}(1-.96) + (1.06)^{-1}A_{61}$$

$$A_{61} = 0.51042$$

$$Var[Z] = {}^{2}A_{61} - (A_{61})^{2}$$

$${}^{2}A_{60} = v^{2}q_{60} + v^{2}p_{60}{}^{2}A_{61}$$

$$0.35 = (1.06)^{-2}(.04) + (1.06)^{-2} \cdot {}^{2}A_{61}$$

$${}^{2}A_{61} = 0.36798$$

$$Var[Z] = 0.36798 - (0.51042)^{2} = 0.10745$$

(6 points) You are given:

i. Z is the present value for a whole life policy sold to (x) with a death benefit of 1 payable at the end of the year of death.

ii. ${}^{2}A_x = 0.41$

iii. $Var(Z) = 0.05$

iv. $q_x = 0.035$

v. $q_{x+1} = 0.037$

vi. $i = 0.06$

Calculate A_{x+2} accurate to 4 decimal places.

Solution:

$$A_{x+1} = vq_{x+1} + vp_{x+1}A_{x+2}$$

$$A_x = vq_x + vp_x A_{x+1}$$

$$Var[Z] = {}^{2}A_x - (A_x)^2$$

$$0.05 = 0.41 - (A_x)^2 \Rightarrow A_x = 0.6$$

$$0.6 = (1.06)^{-1}(0.035) + (1.06)^{-1}(1-0.035)A_{x+1} \Rightarrow A_{x+1} = 0.62280$$

$$0.62280 = (1.06)^{-1}(0.037) + (1.06)^{-1}(1-0.037)A_{x+2} \Rightarrow A_{x+2} = 0.64711$$

(6 points) Let Z be the present value for a whole life insurance to (50) with a death benefit of 1 paid at the end of the year of death.

You are given:

1. $A_{50} = 0.3$
2. $i = 0.05$
3. $q_{50} = 0.0018 \quad \text{and} \quad q_{51} = 0.0020 \quad \text{and} \quad q_{52} = 0.0022$

Determine $1000A_{52}$

Solution:

$$A_{50} = vq_{50} + vp_{50} \cdot A_{51} \Longrightarrow 0.3 = (1.05)^{-1}(0.018) + (1.05)^{-1}(1-0.018)(A_{51}) \Longrightarrow A_{51} = 0.31376$$

$$A_{51} = vq_{51} + vp_{51} \cdot A_{52} \Longrightarrow 0.31376 = (1.05)^{-1}(0.002) + (1.05)^{-1}(1-0.002)(A_{52}) \Longrightarrow A_{52} = 0.32810$$

$$1000A_{52} = 328.10$$