**Chapter 4 – Past Test and Quiz Problems – Whole Life, EOY of Death**

(6 points) You are given that mortality follows the following select and ultimate mortality table:

| $[x]$ | $q_{[x]}$ | $q_{[x]+1}$ | $q_{x+2}$ | $x+2$ |
|---|---|---|---|---|
| 101 | 0.10 | 0.30 | 0.50 | 103 |
| 102 | 0.20 | 0.40 | 0.80 | 104 |
| 103 | 0.25 | 0.50 | 1.00 | 105 |

Further, you are given that $d = 0.10$.

Let $Z$ be the present value random variable for a whole life insurance on (102) that pays a death benefit of 2000 at the end of the year of death.

Calculate the $E[Z] + \sqrt{Var[Z]}$ .

$l_{[102]} = 1000$

$l_{[102]+1} = 1000(0.8) = 800$

$l_{[102]+2} = 800(0.6) = 480$

$l_{105} = 480(0.2) = 96$

$l_{106} = 0$

$$2000 \cdot A_x = E[Z] = \frac{200v + 320v^2 + 384v^3 + 96v^4}{1000} \cdot 2000 = 1,564.2432$$

$$2000^2 \cdot {}^2A_x = \frac{200v^2 + 320v^4 + 384v^6 + 96v^8}{1000} \cdot 2000^2 = 2,469,400.784$$

$$Var[Z] = {}^2A_x - (A_x)^2 = 2,469,400.784 - (1,564.2432)^2 = 22,544$$

$$E[Z] + \sqrt{Var[Z]} = 1,564.2432 + \sqrt{22,544} = 1,714.39$$

(6 points) Let $Z$ be the present value random variable for a discrete whole life policy with a death benefit of 100 sold to (101) who has just been underwritten.

You are given the following select and ultimate mortality table:

| $[x]$ | $q_{[x]}$ | $q_{[x]+1}$ | $q_{x+2}$ | $x+2$ |
|---|---|---|---|---|
| 100 | 0.05 | 0.15 | 0.35 | 102 |
| 101 | 0.10 | 0.30 | 0.50 | 103 |
| 102 | 0.20 | 0.40 | 0.80 | 104 |
| 103 | 0.25 | 0.50 | 1.00 | 105 |

You are also given that $d = 0.07$ .

Calculate $Var[Z]$ .

**Solution:**

$$v = 1 - 0.07 = 0.93$$

$$Var[Z] = 100^2 \left( {}^2A_{[101]} - (A_{[101]})^2 \right)$$

$$l_{[101]} = 1000; l_{[101]+1} = 1000(0.9) = 900; l_{[101]+2} = 900(0.7) = 630$$
$$l_{[101]+3} = 630(0.5) = 315; l_{[101]+4} = 315(0.2) = 63; l_{[101]+5} = 63(0) = 0$$

$$A_{[101]} = \frac{(1000-900)(.93) + (900-630)(.93)^2 + (630-315)(.93)^3 + (315-63)(.93)^4 + (63-0)(.93)^5}{1000}$$

$$= 0.8122$$

$${}^2A_{[101]} = \frac{(1000-900)(.93)^2 + (900-630)(.93)^4 + (630-315)(.93)^6 + (315-63)(.93)^8 + (63-0)(.93)^{10}}{1000}$$

$$= 0.6638$$

$$Var[Z] = 100^2 (0.6638 - 0.8122^2) = 40.4912$$

Jeff is (63). He wants to buy a life insurance policy from Wu Life Insurance Company. However, he is not sure which policy to buy. All calculations assume:

i. Mortality follows the Standard Ultimate Life Table

ii. $i = 0.05$

iii. Deaths are uniformly distributed between integral ages.

First, he decides to consider a whole life insurance policy that pays a death benefit of 100,000 at the end of the year of death. Jeff asks Shina who is the chief actuary at Wu Life to do several things for him.

a. (2 points) Write the present value random variable $Z$ for this policy.

$$Z = 100,000 v^{k_x+1} = 100,000 \left( \frac{1}{1.05} \right)^{k_x+1}$$

b. (2 points) Jeff estimates that the expected present value of this whole life policy is 33,000 to the nearest 1000. Calculate it to the nearest 1.

$$100,000 A_{63} = 100,000(0.32785) = 32,785$$

(4 points) You are given the following select and ultimate mortality table:

| $[x]$ | $q_{[x]}$ | $q_{[x]+1}$ | $q_{x+2}$ | $x+2$ |
|---|---|---|---|---|
| 75 | 0.10 | 0.20 | 0.4 | 77 |
| 76 | 0.15 | 0.30 | 0.6 | 78 |
| 77 | 0.20 | 0.40 | 0.8 | 79 |
| 78 | 0.25 | 0.55 | 0.9 | 80 |
| 79 | 0.30 | 0.70 | 1.00 | 81 |

You are also given that $d = 0.1$.

Calculate $100,000A_{[77]}$ which is a whole life issued to a newly underwritten life age 77 with a death benefit of 100,000 paid at the end of the year of death.

**Solution:**

$$l_{[77]} = 100,000; l_{[77]+1} = (100,000)(0.8) = 80,000; l_{79} = (80,000)(0.6) = 48,000$$
$$l_{80} = 48,000(0.2) = 9600; l_{81} = 9600(0.1) = 96; l_{82} = 0$$

$$100,000A_{[77]} = (100,000 - 80,000)(0.9) + (80,000 - 48,000)(0.9)^2 + (48,000 - 9600)(0.9)^3 + (9600 - 96)(0.9)^4 + 96(0.9)^5 = 78,149.17$$

(6 points) Filza is 95 and buys a whole life policy with a death benefit of 100,000 paid at the end of the year of death.

You are given the following mortality table and $i = 0.08$ .

| $x$ | $q_x$ |
|---|---|
| 95 | 0.2 |
| 96 | 0.5 |
| 97 | 0.8 |
| 98 | 1.0 |

Let $Z$ be the present value random variable of Filza's whole life insurance.

Calculate the $\sqrt{Var[Z]}$ .

**Solution:**

$$l_{95} = 100; l_{96} = (100)(1-0.2) = 80; l_{97} = (80)(1-0.5) = 40; l_{98} = (40)(1-0.8) = 8; l_{99} = 0$$

$$A_{95} = \frac{20(1.08)^{-1} + 40(1.08)^{-2} + 32(1.08)^{-3} + 8(1.08)^{-4}}{100} = 0.840949$$

$$^{2}A_{95} = \frac{20(1.08)^{-2} + 40(1.08)^{-4} + 32(1.08)^{-6} + 8(1.08)^{-8}}{100} = 0.710355$$

$$Var[Z] = (100,000)^2(0.710355 - 0.840949^2) = 31,597,793.99$$

$$\sqrt{Var[Z]} = 5621$$