Chapter 4 – Past Test and Quiz Problems – Whole Life, moment of death

(6 points) You are given that $F_{50}(t) = 0.0004t^2$ for $0 \le t \le 50$ and $\delta = 0.05$.

Calculate $1000\overline{A}_{50}$.

Solutions:

$$S_{50}(t) =_t p_x = 1 - 0.0004t^2$$

$$1000\overline{A}_{50} = 1000\int_0^{50} v^t \cdot_t p_{50} \cdot \mu_{50+t} \cdot dt$$

$$\mu_{50+t} = \frac{-\dfrac{d}{dt} \,_t p_x}{_t p_x} = \frac{0.0008t}{_t p_x}$$

$$1000\int_0^{50} v^t \cdot_t p_{50} \cdot \mu_{50+t} \cdot dt = 1000\int_0^{50} v^t (0.0008t)dt = 0.8\int_0^{50} te^{-0.05t}dt$$

$$u = t \Longrightarrow du = dt \quad \text{and} \quad dv = e^{-0.05t} \Longrightarrow v = -\frac{e^{-0.05t}}{0.05}$$

$$0.8\int_0^{50} te^{-0.05t}dt = 0.8\left(-\frac{e^{-0.05t}}{0.05}\Big]_0^{50} + \int_0^{50} \frac{e^{-0.05t}}{0.05}dt\right) = 0.8\left(-82.0850 + \left[-\frac{e^{-0.05t}}{(0.05)^2}\right]_0^{50}\right)$$

$$0.8(-82.0850 - 32.8340 + 400) = 228.0648$$

(2 points) Jeff is (70). He wants to buy a life insurance policy from Lai Life Insurance Company. However, he is not sure which policy to buy. All calculations assume:

     i.   Mortality follows the Standard Ultimate Life Table

     ii. $i = 0.05$

     iii.   Deaths are uniformly distributed between integral ages.

First, he decides to consider a whole life insurance policy that pays a death benefit of 100,000 at the moment of death. Jeff asks Jake who is the chief actuary at Lai Life to do several things for him.

Write the present value random variable Z for this policy.

Solution:

$$Z = 100,000 v^{T_{70}} = 100,000(1.05)^{-T_{70}}$$

(2 points) You are given that mortality follows the following mortality table:

Age x	q_x
100	0.20
101	0.30
102	0.50
103	0.75
104	1.00

You are also given that $d = 0.10$ which means that $v = 0.90$. Further, you are given that deaths are uniformly distributed between integral ages for ages 100 and 101 and between ages 101 and 102. For ages over 102, mortality follows a constant force of mortality between integral ages.

Let Z^{CONT} be the present value random variable for a whole life insurance policy to (100) with a death benefit of 10,000 paid at the moment of death.

Write an expression of Z^{CONT}.

Solution:

$$Z^{CONT} = 10,000 v^{T_{100}} = (10,000)(0.9)^{T_{100}}$$