Chapter 4 – Past Test and Quiz Problems

You are given that $F_{50}(t) = 0.0004t^2 \quad \text{for} \;\; 0 \le t \le 50$ and $\delta = 0.05$.

Calculate $1000\bar{A}_{50}$.

Solutions:

$$S_{50}(t) = {}_t p_x = 1 - 0.0004t^2$$

$$1000\bar{A}_{50} = 1000\int_0^{50} v^t \cdot {}_t p_{50} \cdot \mu_{50+t} \cdot dt$$

$$\mu_{50+t} = \frac{-\frac{d}{dt}\,{}_t p_x}{{}_t p_x} = \frac{0.0008t}{{}_t p_x}$$

$$1000\int_0^{50} v^t \cdot {}_t p_{50} \cdot \mu_{50+t} \cdot dt = 1000\int_0^{50} v^t (0.0008t)dt = 0.8\int_0^{50} te^{-0.05t}dt$$

$$u = t \implies du = dt \quad \text{and} \quad dv = e^{-0.05t} \implies v = -\frac{e^{-0.05t}}{0.05}$$

$$0.8\int_0^{50} te^{-0.05t}dt = 0.8\left(\left[-\frac{e^{-0.05t}}{0.05}\right]_0^{50} + \int_0^{50} \frac{e^{-0.05t}}{0.05}dt\right) = 0.8\left(-82.0850 + \left[-\frac{e^{-0.05t}}{(0.05)^2}\right]_0^{50}\right)$$

$$0.8(-82.0850 - 32.8340 + 400) = 228.0648$$

Let Z_x be the present value random variable for a whole life insurance policy on (x) with a death benefit of 1 payable at the end of the year of death.

You are given:

a. $A_{50} = 0.300$

b. $Var[Z_{50}] = 0.110$

c. $q_{50} = 0.01$

d. $v = 0.93$

Calculate the $Var[Z_{51}]$.

Solution:

$$Var[Z_{50}] = {}^2A_{50} - (A_{50})^2 \Longrightarrow 0.11 = {}^2A_{50} - (0.3)^2 \Longrightarrow {}^2A_{50} = 0.2$$

$$A_{50} = vq_{50} + vp_{50}A_{51}$$

$$0.3 = (0.93)(0.01) + (0.93)(0.99)(A_{51}) \Longrightarrow A_{51} = \frac{0.3 - 0.0093}{(0.93)(0.99)} = 0.31574$$

$${}^2A_{50} = v^2q_{50} + v^2p_{50} \cdot {}^2A_{51}$$

$$0.2 = (0.93)^2(0.01) + (0.93)^2(0.99)({}^2A_{51}) \Longrightarrow {}^2A_{51} = \frac{0.2 - (0.93)^2(0.01)}{(0.93)^2(0.99)} = 0.22348$$

$$Var[Z_{51}] = 0.22348 - (0.31574)^2 = 0.1238$$

You are given that mortality follows the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
101	0.10	0.30	0.50	103
102	0.20	0.40	0.80	104
103	0.25	0.50	1.00	105

Further, you are given that $d = 0.10$.

Let Z be the present value random variable for a whole life insurance on (102) that pays a death benefit of 2000 at the end of the year of death.

Calculate the $E[Z] + \sqrt{Var[Z]}$.

$l_{[102]} = 1000$

$l_{[102]+1} = 1000(0.8) = 800$

$l_{[102]+2} = 800(0.6) = 480$

$l_{105} = 480(0.2) = 96$

$l_{106} = 0$

$$2000 \cdot A_x = E[Z] = \frac{200v + 320v^2 + 384v^3 + 96v^4}{1000} \cdot 2000 = 1,564.2432$$

$$2000^2 \cdot {}^2A_x = \frac{200v^2 + 320v^4 + 384v^6 + 96v^8}{1000} \cdot 2000^2 = 2,469,400.784$$

$$Var[Z] = {}^2A_x - (A_x)^2 = 2,469,400.784 - (1,564.2432)^2 = 22,544$$

$$E[Z] + \sqrt{Var[Z]} = 1,564.2432 + \sqrt{22,544} = 1,714.39$$

You are given:

i. $A_{60} = 0.500$

ii. ${}^2A_{60} = 0.350$

iii. $p_{59} = 0.97$

iv. $p_{60} = 0.96$

v. $i = 0.10$

Let Z be the present value random variable for a whole life to (59) with a death benefit of 1 paid at the end of the year of death.

Calculate the $Var[Z]$.

$$A_x = vq_x + vp_x[A_{x+1}]$$

$${}^2A_x = v^2q_x + v^2p_x[{}^2A_{x+1}]$$

$$A_{59} = \frac{0.03}{1.1} + \frac{0.97}{1.1}(0.5) = 0.468181$$

$${}^2A_{59} = \frac{0.03}{1.21} + \frac{0.97}{1.21}(0.350) = 0.30537$$

$$Var[Z] = 0.30537 - 0.468181^2 = 0.08618$$

You are given the following mortality table:

x	q_x
106	0.10
107	0.20
108	0.30
109	0.40
110	0.50
111	0.60
112	0.70
113	0.80
114	0.90
115	1.00

You are also given that $i = 0.10$.

Calculate 12,000 $A^{1}_{106:\overline{4}|}$.

$$l_{106} = 1000$$
$$l_{107} = 1000(0.9) = 900$$
$$l_{108} = 900(0.8) = 720$$
$$l_{109} = 720(0.7) = 504$$
$$l_{110} = 504(0.6) = 302.4$$

$$1000 \cdot A^{1}_{106:\overline{4}|} = (1000-900)v + (900-720)v^2 + (720-504)v^3 + (504-302.4)v^4$$

$$1000 \cdot A^{1}_{106:\overline{4}|} = 100v + 180v^2 + 216v^3 + 201.6v^4$$

$$A^{1}_{106:\overline{4}|} = 0.539648931$$

$$\therefore 12000 \cdot A^{1}_{106:\overline{4}|} = 6475.787173$$

Let Z be the present value random variable for a discrete whole life policy with a death benefit of 100 sold to (101) who has just been underwritten.

You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
100	0.05	0.15	0.35	102
101	0.10	0.30	0.50	103
102	0.20	0.40	0.80	104
103	0.25	0.50	1.00	105

You are also given that $d = 0.07$.

Calculate $Var[Z]$.

Solution:

$$v = 1 - 0.07 = 0.93$$

$$Var[Z] = 100^2\left({}^2A_{[101]} - (A_{[101]})^2\right)$$

$$l_{[101]} = 1000; l_{[101]+1} = 1000(0.9) = 900; l_{[101]+2} = 900(0.7) = 630$$
$$l_{[101]+3} = 630(0.5) = 315; l_{[101]+4} = 315(0.2) = 63; l_{[101]+5} = 63(0) = 0$$

$$A_{[101]} = \frac{(1000-900)(.93) + (900-630)(.93)^2 + (630-315)(.93)^3 + (315-63)(.93)^4 + (63-0)(.93)^5}{1000}$$

$$= 0.8122$$

$${}^2A_{[101]} = \frac{(1000-900)(.93)^2 + (900-630)(.93)^4 + (630-315)(.93)^6 + (315-63)(.93)^8 + (63-0)(.93)^{10}}{1000}$$

$$= 0.6638$$

$$Var[Z] = 100^2(0.6638 - 0.8122^2) = 40.4912$$

You are given:

vi. $A_{60} = 0.500$

vii. ${}^2A_{60} = 0.350$

viii. $p_{60} = 0.96$

ix. $p_{61} = 0.95$

x. $i = 0.06$

Let Z be the present value random variable for a whole life to (61) with a death benefit of 1 paid at the end of the year of death.

Calculate the $Var(Z)$.

Solution:

$$A_{60} = vq_{60} + vp_{60}A_{61}$$

$$0.5 = (1.06)^{-1}(1-.96) + (1.06)^{-1}A_{61}$$

$$A_{61} = 0.51042$$

$$Var[Z] = {}^2A_{61} - (A_{61})^2$$

$${}^2A_{60} = v^2q_{60} + v^2p_{60}\,{}^2A_{61}$$

$$0.35 = (1.06)^{-2}(.04) + (1.06)^{-2} \cdot {}^2A_{61}$$

$${}^2A_{61} = 0.36798$$

$$Var[Z] = 0.36798 - (0.51042)^2 = 0.10745$$

Jeff is (63). He wants to buy a life insurance policy from Wu Life Insurance Company. However, he is not sure which policy to buy. All calculations assume:

i. Mortality follows the Standard Ultimate Life Table

ii. $i = 0.05$

iii. Deaths are uniformly distributed between integral ages.

First, he decides to consider a whole life insurance policy that pays a death benefit of 100,000 at the end of the year of death. Jeff asks Shina who is the chief actuary at Wu Life to do several things for him.

a. (4 points) Write the present value random variable Z for this policy.

$$Z = 100,000 v^{k_x+1} = 100,000\left(\frac{1}{1.05}\right)^{k_x+1}$$

b. (4 points) Jeff estimates that the expected present value of this whole life policy is 33,000 to the nearest 1000. Calculate it to the nearest 1.

$$100,000 A_{63} = 100,000(0.32785) = 32,785$$

c. (4 points) What would be the expected present value of the whole life policy if it paid a death benefit at the moment of death instead of at the end of the year of death.

$$100,000\bar{A}_{63} = 100,000 A_{63}\left(\frac{i}{\delta}\right) = 32,785\left(\frac{0.05}{\ln(1.05)}\right) = 33,597.96$$

d. (12 points) Jeff decides to purchase the whole life insurance policy in Part b. With this sale, Wu Life now has 400 identical whole life policies sold to 400 independent lives. Wu decides to hold 13,500,000 to cover the future death benefit payments on these policies. Using the normal distribution, calculate the probability that the present value of the death benefits will be greater than the 13,500,000.

Solution:

$$E[Port] = 400(32,785) = 13,114,000$$

$$Var[Z] = 100,000^2[{}^2A_{63} - (A_{63})^2] = 100,000^2[0.13421 - (0.32785)^2] = 267,243,775$$

$$Var[Port] = 400(267,243,775)$$

$$13,114,000 + Z\left(\sqrt{400(267,243,775)}\right) = 13,500,000$$

$$Z = 1.18$$

$$P(Z > 1.18) = 1 - 0.8810 = 0.1190$$

e. Jeff asks Shina to calculate his median future life time. The median future life time is the point at which ${}_{n+s}p_{63} = 0.5$ where $0 \le s \le 1$. Jeff knows that $n = 25$.

Determine s accurate to three decimal places.

Solution:

$${}_{25+s}p_{63} = 0.5 = \frac{l_{88+s}}{l_{63}} = \frac{l_{88+s}}{95,534.40} ==> l_{88+s} = 47,767.20$$

$$l_{88}(1-s) + l_{89}(s) = 47,767.20$$

$$50,038.60(1-s) + 45,995.6(s) = 47,767.20$$

$$50,038.6 - 4043s = 47,767,20$$

$$s = 0.562$$

f. (10 points) Jeff decides to also purchase a term insurance policy that pays a death benefit of 100,000 payable at the moment of death if he dies in the next 26 years. Determine the expected present value of this term insurance.

Solution:

$$100,000\bar{A}^{1}_{63:\overline{26}|} = 100,000\left[A_{63}\left(\frac{i}{\delta}\right) - {}_{26}E_{63}A_{89}\left(\frac{i}{\delta}\right)\right]$$

$$\frac{i}{\delta} = \frac{0.05}{\ln(1.05)} = 1.0248$$

$${}_{26}E_{63} = (1.05)^{-26}\left(\frac{l_{89}}{l_{63}}\right)$$

$$100,000\left[0.32785(1.0248) - (1.05)^{-26}\left(\frac{45,995.6}{95,534.4}\right)(0.73853)(1.0248)\right] = 23,349.93$$

g. (5 points) Explain why the expected present value of the term insurance is less than the expected present value of the whole life insurance.

Solution:

The expected present value of term insurance is lower because the death benefit only gets paid to the beneficiary if Jeff dies within the 26-year term; with whole life insurance the death benefit is paid no matter when Jeff dies therefore the present value of whole life is greater than that of term.

You are given the following mortality table:

x	q_x
90	0.2
91	0.4
92	0.6
93	0.8
94	1.0

Let Z be the present value random variable for a discrete three year term insurance policy issued to (90) with a death benefit of 5,000 paid at the end of the year of death.

You are also given that $d = 0.10$.

Calculate $Var[Z]$.

Solution:

$$v = 1 - 0.10 = 0.90$$

$$Var[Z] = {}^2A^{1}_{90:\overline{3}|} - (A^{1}_{90:\overline{3}|})^2$$

$$l_{90} = 1000; l_{91} = 1000(0.8) = 800; l_{92} = 900(0.6) = 480$$
$$l_{93} = 630(0.4) = 192; l_{94} = 315(0.2) = 38.4; l_{95} = 63(0) = 0$$

$$A^{1}_{90:\overline{3}|} = \frac{(1000-800)(.9) + (800-480)(.9)^2 + (480-192)(.9)^3}{1000} = 0.649152$$

$${}^2A^{1}_{90:\overline{3}|} = \frac{(1000-800)(.9)^2 + (800-480)(.9)^4 + (480-192)(.9)^6}{1000} = 0.525007$$

$$Var[Z] = 5000^2(0.525007 - 0.649152^2) = 2590217.22$$

Jeff is (70). He wants to buy a life insurance policy from Lai Life Insurance Company. However, he is not sure which policy to buy. All calculations assume:

i. Mortality follows the Standard Ultimate Life Table

ii. $i = 0.05$

iii. Deaths are uniformly distributed between integral ages.

First, he decides to consider a whole life insurance policy that pays a death benefit of 100,000 at the moment of death. Jeff asks Jake who is the chief actuary at Lai Life to do several things for him.

a. (4 points) Write the present value random variable Z for this policy.

$$Z = 100{,}000 v^{T_{70}} = 100{,}000(1.05)^{-T_{70}}$$

b. (5 points) Jeff estimates that the expected present value of this whole life policy is 44,000 to the nearest 1000. Calculate it to the nearest 1.

$$EPV = 100{,}000\bar{A}_{70} = 100{,}000\left(\frac{i}{\delta}\right)A_{70}$$

$$= 100{,}000(1.02480)(0.42818) = 43{,}880$$

c. (10 points) Calculate the probability that Z is less than 51,000. The probability needs to be accurate to 3 decimal places.

Solution:

$$\Pr(Z < 51{,}000) = \Pr(100{,}000 v^{T_{70}} < 51{,}000)$$

$$v^{T_{70}} < 0.51 \Rightarrow (1.05)^{-T_{70}} < 0.51 ==> T_{70} > \frac{-\ln(0.51)}{\ln(1.05)} \Rightarrow T_{70} > 13.8008$$

$$P(T_{70} > 13.8008) = {}_{13.8008}p_{70}$$

$$\frac{l_{83.8008}}{l_{70}} = \frac{0.1992 l_{83} + 0.8008 l_{84}}{l_{70}}$$

$$= \frac{0.1992(67{,}614.6) + 0.8008(64{,}506.5)}{91{,}082.40} = 0.71502$$

d. (10 points) Jeff decides to purchase a 13 year endowment insurance policy with a death benefit of 100,000 payable at the end of the year of death.

Determine the expected present value of this endowment insurance.

Solution:

$$EPV = 100,000 A_{70:\overline{13}|}$$

$$= 100,000\left(A^{1}_{70:\overline{13}|} + {}_{13}E_{70}\right) = 100,000[{}_{70} - A_{83}({}_{13}E_{70}) + {}_{13}E_{70}]$$

$$= 100,000(0.42818 - 0.64336({}_{13}p_{70})v^{13} + ({}_{13}p_{70})v^{13}$$

$$= 100,000\left[0.42818 - 0.64336\left(\frac{67614.6}{91082.4}\right)(1.05)^{-13} + \left(\frac{67614.6}{91082.4}\right)(1.05)^{-13}\right]$$

$$= 56,858.26$$

e. (5 Points) Explain why the expected present value of the endowment insurance is greater than the expected present value of the whole life insurance.

Solution:

The expected present value of endowment insurance is greater because the longest you will have to wait to get paid is the 13 year term which is given; if you die before then the death benefit will be paid when you die but if you die after 13 years you still receive it as a pure endowment at the end of the set term (in this case 13 years). For the whole life, if you survive 13 years, the death benefit will not be paid until you die. Since this death benefit would be paid after the time of the pure endowment, it has a smaller present value.

You are given:

i. Z is the present value for a whole life policy sold to (x) with a death benefit of 1 payable at the end of the year of death.

ii. ${}^2A_x = 0.41$

iii. $Var(Z) = 0.05$

iv. $q_x = 0.035$

v. $q_{x+1} = 0.037$

vi. $i = 0.06$

Calculate A_{x+2} accurate to 4 decimal places.

Solution:

$$A_{x+1} = vq_{x+1} + vp_{x+1}A_{x+2}$$

$$A_x = vq_x + vp_x A_{x+1}$$

$$Var[Z] = {}^2A_x - (A_x)^2$$

$$0.05 = 0.41 - \left(A_x\right)^2 \Rightarrow A_x = 0.6$$

$$0.6 = (1.06)^{-1}(0.035) + (1.06)^{-1}(1-0.035)A_{x+1} \Rightarrow A_{x+1} = 0.62280$$

$$0.62280 = (1.06)^{-1}(0.037) + (1.06)^{-1}(1-0.037)A_{x+2} \Rightarrow A_{x+2} = 0.64711$$

You are given that mortality follows the following mortality table:

Age x	q_x
100	0.20
101	0.30
102	0.50
103	0.75
104	1.00

You are also given that $d = 0.10$ which means that $v = 0.90$. Further, you are given that deaths are uniformly distributed between integral ages for ages 100 and 101 and between ages 101 and 102. For ages over 102, mortality follows a constant force of mortality between integral ages.

a. (6 points) Calculate ${}_{0.8}p_{101.6}$.

Solution:

$${}_{0.8}p_{101.6} = \frac{l_{102.4}}{l_{101.6}}$$

$$l_{100} = 1000; l_{101} = (1000)(1-0.2) = 800; l_{102} = (800)(1-0.3) = 560; l_{103} = 560(0.5) = 280$$

$$l_{104} = (280)(1-0.75) = 70; l_{105} = 0$$

$${}_{0.8}p_{101.6} = \frac{l_{102.4}}{l_{101.6}} = \frac{(560)^{0.6}(280)^{0.4}}{(800)(0.4)+(560)(0.6)} = 0.64695$$

b. (6 points) Calculate μ_{102}.

Solution:

For a constant force of mortality, $\mu_x = -\ln[p_x]$

$$\mu_{102} = -\ln[p_{102}] = -\ln[0.5] = 0.69315$$

c. Let Z^{CONT} be the present value random variable for a whole life insurance policy to (100) with a death benefit of 10,000 paid at the moment of death.

i. (4 points) Write an expression of Z^{CONT}.

Solution:

$$Z^{CONT} = 10,000v^{T_{100}} = (10,000)(0.9)^{T_{100}}$$

d. Let $Z^{Discrete}$ be the present value random variable for a 3 year endowment insurance to (100) with a death benefit of 10,000 paid at the end of the year of death.

i. (8 points) The expected present value which is $10,000A_{100:\overline{3}|}$ is 7800 to the nearest 100. Calculate $10,000A_{100:\overline{3}|}$ to the nearest 1.

Solution:

Using the ls from Part a:

$$1000A_{100:\overline{3}|} = 200v + 240v^2 + 560v^3 = 782.64 \implies A_{100:\overline{3}|} = 0.78264$$

$$10,000A_{100:\overline{3}|} = 7826.4$$

ii. (8 points) Calculate $Var[Z^{Discrete}]$

Solution:

Using the ls from Part a:

$$1000[{}^2A_{100:\overline{3}|}] = 200v^2 + 240v^4 + 560v^6 = 617.07096 \implies {}^2A_{100:\overline{3}|} = 0.61707096$$

$$Var[Z^{Discrete}] = (10,000)^2[{}^2A_{100:\overline{3}|} - (A_{100:\overline{3}|})^2]$$

$$= (10,000)^2[0.61707096 - (0.78264)^2] = 454,559.04$$

Let Z be the present value for a whole life insurance to (50) with a death benefit of 1 paid at the end of the year of death.

You are given:

1. $A_{50} = 0.3$
2. $i = 0.05$
3. $q_{50} = 0.0018 \quad \text{and} \quad q_{51} = 0.0020 \quad \text{and} \quad q_{52} = 0.0022$

Determine $1000A_{52}$

Solution:

$$A_{50} = vq_{50} + vp_{50} \cdot A_{51} \text{ ==> } 0.3 = (1.05)^{-1}(0.018) + (1.05)^{-1}(1-0.018)(A_{51}) \text{ ==> } A_{51} = 0.31376$$

$$A_{51} = vq_{51} + vp_{51} \cdot A_{52} \text{ ==> } 0.31376 = (1.05)^{-1}(0.002) + (1.05)^{-1}(1-0.002)(A_{52}) \text{ ==> } A_{52} = 0.32810$$

$$1000A_{52} = 328.10$$

You are given that mortality follows the following mortality table:

Age x	q_x
100	0.20
101	0.30
102	0.50
103	0.75
104	1.00

You are also given that $d = 0.10$ which means that $v = 0.90$. Further, you are given that deaths are uniformly distributed between integral ages for ages 100 and 101 and between ages 101 and 102. For ages over 102, mortality follows a constant force of mortality between integral ages.

a. (6 points) Calculate ${}_{0.6}p_{101.8}$.

Solution:

$${}_{0.6}p_{101.8} = \frac{l_{102.4}}{l_{101.8}}$$

$$l_{100} = 1000; l_{101} = (1000)(1-0.2) = 800; l_{102} = (800)(1-0.3) = 560; l_{103} = 560(0.5) = 280$$

$${}_{0.6}p_{101.8} = \frac{l_{102.4}}{l_{101.8}} = \frac{(560)^{0.6}(280)^{0.4}}{(800)(0.2)+(560)(0.8)} = 0.69803$$

b. (6 points) Calculate μ_{102} .

Solution:

For a constant force of mortality, $\mu_x = -\ln[p_x]$

$$\mu_{102} = -\ln[p_{102}] = -\ln[0.5] = 0.69315$$

c. Let Z^{CONT} be the present value random variable for a whole life insurance policy to (100) with a death benefit of 10,000 paid at the moment of death.

iii. (4 points) Write an expression of Z^{CONT}.

Solution:

$$Z^{CONT} = 10,000v^{T_{100}} = (10,000)(0.9)^{T_{100}}$$

You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
75	0.10	0.20	0.4	77
76	0.15	0.30	0.6	78
77	0.20	0.40	0.8	79
78	0.25	0.55	0.9	80
79	0.30	0.70	1.00	81

You are also given that $d = 0.08$.

a. (8 points) If $l_{[75]} = 100,000$, calculate $l_{[76]}$.

Solution:

$$l_{[75]+1} = (100,000)(1-0.1) = 90,000$$

$$l_{77} = (90,000)(1-0.2) = 72,000$$

$$l_{78} = (72,000)(1-0.4) = 43,200$$

$$l_{78} = l_{[76]+1}(1-0.3) \implies l_{[76]+1} = \frac{l_{78}}{0.7} = \frac{43,200}{0.7} = 61,714.28571$$

$$l_{[76]+1} = l_{[76]}(1-0.15) \implies l_{[76]} = \frac{61,714.28571}{0.85} = 72,605.04202$$

b. (8 points) Calculate $100,000A_{[77]}$ which is a whole life insurance to a newly underwritten life age 77 with a death benefit of 100,000 paid at the end of the year of death.

Solution:

$$l_{[77]} = 100,000; l_{[77]+1} = (100,000)(0.8) = 80,000; l_{79} = (80,000)(0.6) = 48,000$$
$$l_{80} = 48,000(0.2) = 9600; l_{81} = 9600(0.1) = 96; l_{82} = 0$$

$$100,000A_{[77]} = (100,000-80,000)(0.92) + (80,000-48,000)(0.92)^2 +$$
$$(48,000-9600)(0.92)^3 + (9600-96)v^4 + 96v^5 = 82,208.77$$

Let Z be the present value for a whole life insurance to (50) with a death benefit of 1 paid at the end of the year of death.

You are given:

1. $A_{52} = 0.32$
2. $i = 0.05$
3. $q_{50} = 0.0018 \quad \text{and} \quad q_{51} = 0.0020 \quad \text{and} \quad q_{52} = 0.0022$

Determine $1000A_{50}$.

Solution:

$$A_{51} = vq_{51} + vp_{51} \cdot A_{52} = (1.05)^{-1}(0.002) + (1.05)^{-1}(1-0.002)(0.32) = 0.30606$$

$$A_{50} = vq_{50} + vp_{50} \cdot A_{51} = (1.05)^{-1}(0.018) + (1.05)^{-1}(1-0.018)(0.30606) = 0.29267$$

$$1000A_{50} = 292.67$$

You are given that mortality follows Gompertz Law with:

$$B = 0.00001 \text{ and } c = 1.11$$

You are also given that $i = 0.06$.

Calculate $1000A^{\ 1}_{80:\overline{2}|}$.

Solution:

$$p_{80} = \exp\left[\frac{-0.00001}{\ln(1.11)}(1.11)^{80}(1.11-1)\right] = 0.95644$$

$$p_{81} = \exp\left[\frac{-0.00001}{\ln(1.11)}(1.11)^{81}(1.11-1)\right] = 0.95177$$

$$l_{80} = 1000; l_{81} = (1000)(0.95644) = 956.44; l_{82} = (956.44)(0.95177) = 910.31$$

$$1000A^{\ 1}_{80:\overline{2}|} = (1000-956.44)(1.06)^{-1} + (956.44-910.31)(1.06)^{-2} = 82.15$$

You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
75	0.10	0.20	0.4	77
76	0.15	0.30	0.6	78
77	0.20	0.40	0.8	79
78	0.25	0.55	0.9	80
79	0.30	0.70	1.00	81

You are also given that $d = 0.1$.

Calculate $100,000A_{[77]}$ which is a whole life issued to a newly underwritten life age 77 with a death benefit of 100,000 paid at the end of the year of death.

Solution:

$$l_{[77]} = 100,000; l_{[77]+1} = (100,000)(0.8) = 80,000; l_{79} = (80,000)(0.6) = 48,000$$
$$l_{80} = 48,000(0.2) = 9600; l_{81} = 9600(0.1) = 96; l_{82} = 0$$

$$100,000A_{[77]} = (100,000 - 80,000)(0.9) + (80,000 - 48,000)(0.9)^2 +$$
$$(48,000 - 9600)(0.9)^3 + (9600 - 96)(0.9)^4 + 96(0.9)^5 = 78,149.17$$

Let Z be the present value for a whole life insurance to (50) with a death benefit of 1 paid at the end of the year of death.

You are given:

1. $A_{50} = 0.3$
2. $Var[Z] = 0.04$
3. $i = 0.05$
4. $q_{50} = 0.0018 \quad \text{and} \quad q_{51} = 0.0020 \quad \text{and} \quad q_{52} = 0.0022$

Determine ${}^2A_{52}$ to four decimal places.

Solution:

$$Var(Z) = {}^2A_{50} - (A_{50})^2 \implies 0.04 = {}^2A_{50} - (0.3)^2 \implies {}^2A_{50} = 0.13$$

$${}^2A_{50} = v^2 q_{50} + v^2 p_{50} \, {}^2A_{51}$$

$$0.13 = (1.05)^{-2}(0.0018) + (1.05)^{-2}(1-0.0018)^2 A_{51} \implies {}^2A_{51} = 0.141780204$$

$${}^2A_{51} = v^2 q_{51} + v^2 p_{51} \, {}^2A_{52}$$

$$0.141780204 = (1.05)^{-2}(0.002) + (1.05)^{-2}(1-0.002)^2 A_{52} \implies {}^2A_{52} = 0.15462$$

You are given that mortality follows Makeham's Law with:

$$A = 0.004,\ B = 0.00001, \text{ and } c = 1.11$$

a. (6 points) Calculate probability that a person age 90 is still alive at age 95.

Solution:

$${}_5p_{90} = \exp\left[-(0.004(5) - \frac{0.00001}{\ln(1.11)}(1.11)^{90}(1.11^5 - 1)\right] = 0.44596284$$

Let L_5 be the random variable representing the number of people alive at age 95 if there were 50,000 people alive at age 90.

b. (4 points) Using the probability from Part (a), calculate $Var[L_5]$. If you are not able to calculate the probability in Part (a), assume that it is 0.45. If you get an answer for Part (a), please use that answer.

Solution:

$$Var[L_5] = npq = (50,000)(0.44596284)(1 - 0.44596284) = 12,353.99927$$

c. (6 points) You are also given that $i = 0.06$. Calculate $1000A^{\ 1}_{90:\overline{2}|}$.

Solution:

$$p_{90} = \exp\left[-0.004 - \frac{0.00001}{\ln(1.11)}(1.11)^{90}(1.11 - 1)\right] = 0.877698382$$

$$p_{91} = \exp\left[-0.004 - \frac{0.00001}{\ln(1.11)}(1.11)^{91}(1.11 - 1)\right] = 0.865574333$$

$$l_{90} = 1000; l_{91} = (1000)(0.877698382) = 877.698382;$$

$$l_{92} = (877.698382)(0.865574333) = 759.7131916$$

$$1000A^{\ 1}_{90:\overline{2}|} = (1000 - 877.698382)(1.06)^{-1} + (877.698382 - 759.7131916)(1.06)^{-2} = 220.39$$

You are given:

i. $A_{70} = 0.6435$

ii. $A_{71} = 0.7000$

iii. $v = 0.90$

a. Calculate q_{70} .

Solution:

$$A_{70} = vq_{70} + vp_{70}A_{71}$$

$$0.6435 = (0.9)(q_{70}) + (0.9)(1 - q_{70})(0.700)$$

$$0.6435 = 0.9q_{70} + 0.63 - 0.63q_{70} ==> q_{70} = \frac{0.6435 - 0.63}{0.9 - 0.63} = 0.05$$

b. Calculate ${}_{0.5}q_{70.2}$ assuming that there is a constant force of mortality between ages 70 and 71.

Solution:

$${}_{0.5}q_{70.2} = \frac{l_{70.2} - l_{70.7}}{l_{70.2}}$$

$$l_{70} = 1000; l_{71} = (1000)(1 - 0.05) = 950$$

$${}_{0.5}q_{70.2} = \frac{l_{70.2} - l_{70.7}}{l_{70.2}} = \frac{(1000)^{1-0.2}(950)^{0.2} - (1000)^{1-0.7}(950)^{0.7}}{(1000)^{1-0.2}(950)^{0.2}} = 0.02532$$

Filza is 95 and buys a whole life policy with a death benefit of 100,000 paid at the end of the year of death.

You are given the following mortality table and $i = 0.08$.

x	q_x
95	0.2
96	0.5
97	0.8
98	1.0

Let Z be the present value random variable of Filza's whole life insurance.

Calculate the $\sqrt{Var[Z]}$.

Solution:

$$l_{95} = 100; l_{96} = (100)(1-0.2) = 80; l_{97} = (80)(1-0.5) = 40; l_{98} = (40)(1-0.8) = 8; l_{99} = 0$$

$$A_{95} = \frac{20(1.08)^{-1} + 40(1.08)^{-2} + 32(1.08)^{-3} + 8(1.08)^{-4}}{100} = 0.840949$$

$$^2A_{95} = \frac{20(1.08)^{-2} + 40(1.08)^{-4} + 32(1.08)^{-6} + 8(1.08)^{-8}}{100} = 0.710355$$

$$Var[Z] = (100,000)^2(0.710355 - 0.840949^2) = 31,597,793.99$$

$$\sqrt{Var[Z]} = 5621$$

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%. You are also given that deaths are uniformly distributed between integral ages.

a. (10 points) Calculate the Expected Present Value for a 30 year term insurance issued to (50) with a death benefit of 10,000 paid **at the end of the year of death**.

Solution:

$$10,000A^{\,1}_{50:\overline{30}|} = 10,000(A_{50} - {}_{30}E_{50} \cdot A_{80})$$

$$= (10,000)(0.18931 - (0.34824)(0.50994)(0.59293)) = 840.1645$$

b. (10 points) Calculate the $Var[Z]$ where Z is the present value random variable for a 30 year term insurance issued to (50) with a death benefit of 10,000 paid **at the end of the year of death.**

Solution:

$$Var = (10,000)^2\left[{}^2A^{\,1}_{50:\overline{30}|} - \{A^{\,1}_{50:\overline{30}|}\}^2\right]$$

$${}^2A^{\,1}_{50:\overline{30}|} = {}^2A_{50} - {}^2_{30}E_{50} \cdot {}^2A_{80} = 0.05108 - (1.05)^{-30}(0.34824)(0.50994)(0.38134)$$

$$= 0.03541$$

$$Var = (10,000)^2\left[{}^2A^{\,1}_{50:\overline{30}|} - \{A^{\,1}_{50:\overline{30}|}\}^2\right] = (10,000)^2\left[0.03541 - (0.08401645)^2\right]$$

$$= 2,835,258.1$$

c. (10 points) Calculate the Expected Present Value for a 30 year endowment insurance issued to (50) with a death benefit of 10,000 **paid at the moment of death**.

Solution:

$$10,000\bar{A}_{50:\overline{30}|} = (10,000)\left[\left(\frac{i}{\delta}\right)A^{\ 1}_{50:\overline{30}|} + {}_{30}E_{50}\right]$$

$$= (10,000)[(1.02480)(0.08401645) + (0.50994)(0.59293)] = 2636.8181$$

d. (4 points) Explain why the Expected Present Value in c. is greater than the Expected Present Value in a. Provide two reasons.

Solution:

Endowment insurance will pay during the first 30 years if someone dies and will also pay at the end of 30 years to anyone that is alive. Term insurance only pays to those that die during the first 30 years.

Part c pays at the moment of death while part a pays at the end of the year of death. This means that part c pays earlier that part a so the present value is larger.

(10 points) Alisa (20) buys a special whole life policy with a non-level death benefit. The death benefits are paid at the end of the year of death and are listed in the following table:

Years	Death Benefit
1-30	125,000
31-50	300,000
51+	50,000

Using the Standard Ultimate Life Table with $i = 5\%$, calculate the expected present value of this insurance.

Solution:

$$125,000A_{20} + 175,000 {}_{30}E_{20}A_{50} - 250,000 {}_{50}E_{20}A_{70}$$

$${}_{30}E_{20} {}_{10}E_{20} \cdot {}_{20}E_{30}$$

$${}_{50}E_{20} \Rightarrow (1.05)^{-50}\left(\frac{l_{70}}{l_{20}}\right)$$

$$125,000(0.04922) + 175,000(0.61224)(0.37254)(0.18931)$$
$$- 250,000(1.05)^{-50}\left(\frac{91,082.4}{100,000}\right)(0.42818)$$

$$= 6152.5 + 7556.2482 - 8502.2897 = 5206.46$$

(9 points) Under the Standard Ultimate Life Table:

a. $e_{60:\overline{10}|} = 9.733$

b. $e_{70:\overline{10}|} = 9.201$

c. $e_{80} = 10.606$

Determine e_{60}.

Solution:

$$e_{60} = e_{60:\overline{10}|} + e_{70:\overline{10}|}\left({}_{10}p_{60}\right) + e_{80}\left({}_{20}p_{60}\right)$$

$$= 9.733 + 9.201\left(\frac{l_{70}}{l_{60}}\right) + 10.606\left(\frac{l_{80}}{l_{60}}\right)$$

$$= 9.733 + 9.201\left(\frac{91082.4}{96634.1}\right) + 10.606\left(\frac{75657.2}{96634.1}\right)$$

$$= 26.7091$$

(10 points) Jimmy (30) buys a special 45 year term insurance policy with a non-level death benefit. The death benefits are paid at the end of the year of death and are listed in the following table:

Years	Death Benefit
1-20	100,000
21-35	50,000
36-45	25,000

Using the Standard Ultimate Life Table with $i = 5\%$, calculate the expected present value of this insurance.

Solution:

$$EPV = 100,000A_{30} - 50,000A_{50}({}_{20}E_{30}) - 25,000A_{65}({}_{35}E_{30}) - 25,000A_{75}({}_{45}E_{30})$$

$$= 100,000(0.07698) - 50,000(0.18931)(0.37254)$$
$$- 25,000(0.35477)(1.05)^{-35}\left(\frac{94,579.7}{99,727.3}\right) - 25,000(0.50868)(1.05)^{-45}\left(\frac{85,203.5}{99,727.3}\right)$$

$$= 1,437.577$$

(10 points) Abishek is (25) and purchases a special term policy with a non-level death benefit. The death benefit paid at the end of the year of death is 50,000 if he dies between ages 25 and 40. The death benefit is 100,000 if he dies between ages 40 and 60. For death between ages 60 and 75, the death benefit is 25,000. No death benefit is payable after age 75 is Abishek lives 50 years.

You are given that the mortality follows the Standard Ultimate Life Table and the annual effective interest rate is 5%.

Calculate the present value of this life insurance policy.

Solution:

$$EPV = 50,000A_{25} + 50,000\,{}_{15}E_{25} \cdot A_{40} - 75,000\,{}_{35}E_{25} \cdot A_{60} - 25,000\,{}_{50}E_{25} \cdot A_{75}$$

$$= 50,000A_{25} + 50,000\,{}_{5}E_{25} \cdot {}_{10}E_{30} \cdot A_{40}$$
$$- 70,000\,{}_{20}E_{25} \cdot {}_{10}E_{45} \cdot {}_{5}E_{55} \cdot A_{60} - 25,000\,{}_{20}E_{25} \cdot {}_{20}E_{45} \cdot {}_{10}E_{65} \cdot A_{70}$$

$$= (50,000)(0.06147) + (50,000)(0.78240)(0.61152)(0.12106)$$
$$- (75,000)(0.37373)(0.60655)(0.77382)(0.29028)$$
$$- (25,000)(0.37373)(0.35994)(0.55305)(0.50868)$$

$$= 1204.54$$

(8 points) Megan (25) buys a 20 year term policy with a death benefit of 1,000,000 payable at the end of the year of death.

You are given that mortality follows the Standard Ultimate Life Table with an interest rate of 5%.

Calculate the expected present value which is $1,000,000 A^{1}_{25:\overline{20|}}$.

Solution:

$$A^{1}_{25:\overline{20|}} = A_{25:\overline{20|}} - {}_{20}E_{25} = 0.37854 - 0.37373 = 0.00481$$

$$1,000,000 A^{1}_{25:\overline{20|}} = (1,000,000)(0.00481) = 4810$$

(8 points) You are given that mortality follows Gompertz Law with:

$$B = 0.00001 \text{ and } c = 1.11$$

You are also given that $i = 0.06$.

Calculate $1000 \, {}_{10}E_{70}$.

Solution:

$$_{10}p_{70} = \exp\left[\frac{-0.00001}{\ln(1.11)}(1.11)^{70}(1.11^{10} - 1)\right] = 0.76930$$

$$1000 \, {}_{10}E_{70} = v^{10} \cdot {}_{10}p_{70} = (1000)(1.06)^{-10}(0.76930) = 429.57$$

Megan is (45). She purchases a whole life policy with death benefit of 250,000 payable at the moment of death.

You are given that mortality follows the Standard Ultimate Life Table and $i = 0.05$. You are also given that deaths are uniformly distributed between integral ages.

a. (4 points) The actuarial present value of Megan's death benefit is 38,800 to the nearest 100. Calculate it to the nearest 1.

Solution:

$$APV = 250{,}000\bar{A}_{45} = (250{,}000)\left(\frac{i}{\delta}\right)A_{45} = (250{,}000)\left(\frac{0.05}{\ln(1.05)}\right)(0.15161) = 38{,}842$$

(6 points) A special whole life insurance policy to (70) provides a death benefit payable at the moment of death. The death benefit for the first 10 years is 100,000. The death benefit for death between ages 80 and 90 is 75,000. The death benefit after age 90 is 25,000.

You are given that mortality follows the Standard Ultimate Life Table and $i = 0.05$. You are also given that deaths are uniformly distributed between integral ages.

Calculate the Expected Present Value of this special whole life.

Solution:

$$APV = 100,000\bar{A}_{70} - 25,000\,{}_{10}E_{70} \cdot \bar{A}_{80} - 50,000\,{}_{20}E_{70} \cdot \bar{A}_{90}$$

$$= 100,000\left(\frac{i}{\delta}\right)A_{70} - 25,000\,{}_{10}E_{70}\left(\frac{i}{\delta}\right)A_{80} - 50,000\,{}_{20}E_{70}\left(\frac{i}{\delta}\right)A_{90}$$

$$= (100,000)(1.0248)(0.42818) - (25,000)(0.50994)(1.0248)(0.59293) - (50,000)(0.17313)(1.0248)(0.75317)$$

$$= 29,452$$

Valerie buys a special whole life insurance with the death benefit paid at the end of the year of death. Valerie is age 25. The death benefit is 50,000 for the first 20 years. The death benefit is 200,000 for the second 20 years (ages 45 to 65). The death benefit after age 65 is 100,000.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

Calculate the Actuarial Present Value of Valerie's policy.

Solution:

$$APV = 50,000 A_{25} + 150,000 \, {}_{20}E_{25} A_{45} - 100,000 \, {}_{40}E_{25} A_{65}$$

$$= 50,000 A_{25} + 150,000 \, {}_{20}E_{25} A_{45} - 100,000 \, {}_{20}E_{25} \cdot {}_{20}E_{45} A_{65}$$

$$= (50,000)(0.06147) + (150,000)(0.37373)(0.15161) - (100,000)(0.37373)(0.35994)(0.35477)$$

$$= 6800.30$$

Filza who is (45) buys a 25 year term insurance with a death benefit of 1,000,000 paid at the moment of death.

You are given that:

a. Mortality follows the Standard Ultimate Life Table

b. $i = 0.05$

c. Deaths are uniformly distributed between integral ages.

Calculate the Actuarial Present Value of Filza's policy.

Solution:

$$APV = 1,000,000\bar{A}^{1}_{45:\overline{25|}} = 1,000,000(\bar{A}_{45} - {}_{25}E_{45}\bar{A}_{70})$$

$$1,000,000(i/\delta)(A_{45} - {}_{20}E_{45} \cdot {}_{5}E_{65}A_{70})$$

$$= (1,000,000)(1.0248)[0.15161 - (0.35994)(0.75455)(0.42818)]$$

$$= 36,195.35$$

Tara buys a special 40 year term life insurance with the death benefit paid at the end of the year of death. Tara is age 35. The death benefit is 100,000 for the first 20 years. The death benefit is 50,000 for the last 20 years (ages 55 to75). There is no death benefit after age 75.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

Calculate the Actuarial Present Value of Tara's policy.

Solution:

$$APV = 100,000 A_{35} - 50,000 \, {}_{20}E_{35} A_{55} - 50,000 \, {}_{40}E_{35} A_{75}$$

$$= 100,000 A_{35} - 50,000 \, {}_{20}E_{35} A_{55} - 50,000 \, {}_{20}E_{35} \cdot {}_{20}E_{55} A_{75}$$

$$= (100,000)(0.09653) - (50,000)(0.37041)(0.23524) - (50,000)(0.37041)(0.32819)(0.50868)$$

$$= 2204.36$$

Arthur is (80) and buys a three year endowment insurance with a death benefit of 10,000 paid at the end of the year of death.

a. Write the random variable for Arthur's endowment insurance.

Solution:

$$Z = \begin{cases} 10,000v^{K_{80}+1} & \text{for } K_x \le 2 \\ 10,000v^3 & \text{for } K_x \ge 3 \end{cases}$$

You are given that:

i. $q_{80} = 0.08$

ii. $q_{81} = 0.10$

iii. $v = 0.93$

b. Calculate the Actuarial Present Value of Arthur's endowment insurance.

Solution:

$$l_{80} = 1000; l_{81} = (1000)(1-0.08) = 920; l_{82} = (920)(1-0.1) = 828$$

$$A_{80:\overline{3}|} = \frac{(1000-920)(0.93)+(920-828)(0.93)^2+828(0.93)^3}{1000} = 0.81997840$$

Note that for the 828 people alive at the beginning of the third year, it does not matter if they live or die because they will get paid at the end of the third year.

$$Answer = (10,000)(0.81997840) = 8199.78$$

Dylan is (35) and buys a 10 year term policy with a death benefit of 500,000 paid at the end of the year of death.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

b. Calculate the Expected Present Value of this term policy.

Solution:

$$500,000A^{1}_{35:\overline{10|}} = 500,000(A_{35:\overline{10|}} - {}_{10}E_{35}) = (500,000)(0.61464 - 0.061069) = 1975$$

c. Explain why a 10 year term insurance has an actuarial present value that is less than a 20 year term insurance.

Solution:

A ten year term provides a death benefit if a person dies in the first 10 years while a 20 year term provides a death benefit if the person dies in the first 20 years. Therefore, the 20 year term provides the benefits of the 10 year term plus additional benefits if you die during the last ten years. Therefore, the APV of the 20 year term must be greater.

(8 points) You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
75	0.10	0.20	0.4	77
76	0.15	0.30	0.6	78
77	0.20	0.40	0.8	79
78	0.25	0.55	0.9	80
79	0.30	0.70	1.00	81

You are also given that $d = 0.1$.

Calculate $100,000A_{[77]:\overline{4}|}$ which is a four year endowment insurance to a newly underwritten life age 77 with a death benefit of 100,000 paid at the end of the year of death.

Solution:

$$l_{[77]} = 100,000; l_{[77]+1} = (100,000)(0.8) = 80,000; l_{79} = (80,000)(0.6) = 48,000$$
$$l_{80} = 48,000(0.2) = 9600; l_{81} = 9600(0.1) = 96; l_{82} = 0$$

$$100,000A_{[77]:\overline{4}|} = (100,000-80,000)(0.9) + (80,000-48,000)(0.9)^2 +$$
$$(48,000-9600)(0.9)^3 + 9600v^4 = 78,212.34$$

(8 points) Madeline (25) buys a 32 year pure endowment contract that will pay her 1,000,000 at the end of the term provided she is alive.

You are given that mortality follows the Standard Ultimate Life Table with an interest rate of 5%.

Calculate the expected present value which is $1,000,000 A_{25:\overline{32|}}^{\;\;\;\;1}$.

Solution:

$$1,000,000 A_{25:\overline{32|}}^{\;\;\;\;1} = (1,000,000) v^{32} \frac{l_{57}}{l_{25}}$$

$$= (1,000,000)(1.05)^{-32} \frac{97,435.2}{99,871.1} = 204,747.44$$

You are given that mortality follows the following mortality table:

Age x	q_x
100	0.20
101	0.30
102	0.50
103	0.75
104	1.00

You are also given that $d = 0.09$ which means that $v = 0.91$. Further, you are given that between integral ages for ages 100 and 101 and between ages 101 and 102, mortality follows a constant force of mortality. For ages over 102, deaths are uniformly distributed between integral ages.

a. (6 points) Calculate ${}_{1.2}p_{101.6}$.

Solution:

$${}_{1.2}p_{101.6} = \frac{l_{102.8}}{l_{101.6}}$$

$$l_{100} = 1000; l_{101} = (1000)(1-0.2) = 800; l_{102} = (800)(1-0.3) = 560; l_{103} = 560(0.5) = 280$$

$$l_{104} = (280)(1-0.75) = 70; l_{105} = 0$$

$${}_{1.2}p_{101.6} = \frac{l_{102.8}}{l_{101.6}} = \frac{(560)(0.2)+(280)(0.8)}{(800)^{0.4}(560)^{0.6}} = 0.520224$$

b. (4 points) Calculate $\mu_{102.2}$.

Solution:

$$\mu_{x+s} = \frac{q_x}{1 - s \cdot q_x} \quad \text{under UDD.}$$

$$\mu_{102.2} = \frac{q_{102}}{1-(0.2)q_{102}} = \frac{0.5}{1-(0.2)(0.5)} = 0.555556$$

c. (6 points) The expected value of $E[K_{102}]$ is 0.6 to the nearest 0.1. Calculate it to the nearest 0.001.

Solution:

$$E[K_{102}] = \sum_{k=1}^{3} {}_t p_{102} = (0.5) + (0.5)(0.25) + (0.5)(0.25)(0) = 0.625$$

d. (6 points) Calculate the $Var[K_{102}]$.

Solution:

From Part e, $E[K_{102}] = 0.625$

$$Var[K_{102}] = E[K_{102}{}^2] - \left(E[K_{102}]\right)^2$$

$$E[K_{102}{}^2] = 2\sum_{k=1}^{3} t \cdot {}_t\, p_{102} - e_{102} = 2\left[(1)(0.5) + (2)(0.5)(0.25) + (3)(0.5)(0.25)(0)\right] - 0.625 = 0.875$$

$$Var[K_{102}] = E[K_{102}{}^2] - \left(E[K_{102}]\right)^2 = 0.875 - (0.625)^2 = 0.484375$$

e. (3 points) Calculate the complete expectation of life for a life age 102 which is $\mathring{e}_{102}$.

Solution:

$$\mathring{e}_{102} = e_{102} + \frac{1}{2} \text{ under UDD and we have UDD for ages 102 and above.}$$

$$\mathring{e}_{102} = 0.625 + 0.5 = 1.125$$

f. Let Z be the present value random variable for a 3 year endowment insurance to (100) with a death benefit of 10,000 paid at the end of the year of death.

iv. (3 points) Write an expression of Z.

Solution:

$$Z = \begin{cases} 10,000v^{K_x+1} & \text{for } K_x < n \\ 10,000v^n & \text{for } K_x \geq n \end{cases}$$

v. (6 points) The expected present value which is $10,000A_{100:\overline{3}|}$ is 7800 to the nearest 100. Calculate $10,000A_{100:\overline{3}|}$ to the nearest 1.

Solution:

Using the ls from Part a:

$$1000A_{100:\overline{3}|} = 200v + 240v^2 + 560v^3 = 802.74376 ==> A_{100:\overline{3}|} = 0.80274376$$

$$10,000A_{100:\overline{3}|} = 8027.44$$

vi. (6 points) Calculate $Var[Z]$.

Solution:

Using the ls from Part a:

$$1000[{}^2A_{100:\overline{3}|}] = 200v^2 + 240v^4 + 560v^6 = 648.2066875 ==> {}^2A_{100:\overline{3}|} = 0.6482066875$$

$$Var[Z] = (10,000)^2[{}^2A_{100:\overline{3}|} - (A_{100:\overline{3}|})^2]$$

$$= (10,000)^2[0.6482066875 - (0.80274376)^2] = 380,914$$

(6 points) Varun (30) buys a 10 year term policy with a death benefit of 1,000,000 payable at the end of the year of death.

You are given that mortality follows the Standard Ultimate Life Table with an interest rate of 5%.

Calculate the expected present value which is $1,000,000 A^{\,1}_{30:\overline{10|}}$.

Solution:

$$A^{\,1}_{30:\overline{10|}} = A_{30:\overline{10|}} - {}_{10}E_{30} = 0.61447 - 0.61152 = 0.00295$$

$$1,000,000 A^{\,1}_{30:\overline{10|}} = (1,000,000)(0.00295) = 2950$$

(4 points) Explain why an n-year endowment insurance is equivalent to purchasing an n-year term policy and an n-year pure endowment.

Solution:

An n-year endowment pays a benefit when the insured dies during the n-year period or pays a benefit at the end of n-years if the insured is still alive.

An n-year term pays a benefit when the insured dies during the n-year period but makes no payment if the insured is alive at the end of n years.

An n-year pure endowment pays a benefit at the end of n-years if the insured is still alive but does not pay a benefit if the insured dies.

If you add the benefits paid by the term insurance and the pure endowment, you will get the benefits paid the n-year endowment insurance.

Jiaying is (60) and buys a pure endowment of 25,000 to be paid at the end of 10 years if she is alive at that time.

You are given that $\mu_{60+t} = 0.01t$. You are also given that $\delta = 0.06$.

Calculate the Actuarial Present Value of Jiaying's pure endowment.

Solution:

$$APV = 25,000\,{}_{10}E_{60} = (25,000)v^{10}\,{}_{10}p_{60}$$

$$v^{10} = e^{-0.06(10)}$$

$$ {}_{10}p_{60} = e^{-\int_0^{10} \mu_{60+t}dt} = e^{-\int_0^{10} 0.01t \cdot dt} = e^{-\left[0.005t^2\right]_0^{10}} = e^{-0.5}$$

$$APV = (25,000)e^{-0.6}e^{-0.5} = 8321.78$$

(10 points) Valerie is 70 and purchases a whole life policy with a death benefit of 1000 paid at the end of the year of death.

You are given:

i. Mortality follows the Standard Ultimate Life Table.

ii. $i = 0.07$

Calculate the $\Pr(Z < 500)$

Solution:

$$Z = 1000v^{K_{70}+1}$$

$$1000v^{K_{70}+1} = 500 \text{ ==> } v^{K_{70}+1} = 0.5$$

$$K_{70} + 1 = \frac{\ln(0.5)}{\ln[(1.07)^{-1}]} = 10.24$$

$$K_{70} = 9.24 \text{ ==> Round Up ==> } 10$$

$$\Pr[Z < 500] = {}_{10}p_{70} = \frac{l_{80}}{l_{70}} = \frac{75,657.2}{91,082.4} = 0.830646$$