Chapter 7 – Past Test and Quiz Problems – Gross Premium Policy Values

(8 points) Conley Life Insurance Company sells a whole life policy to Andrew who is (60). The policy pays a death benefit of 100,000 at the end of the year of death. The premiums for the policy are paid annually.

You are given that:

i. Mortality follows the Standard Ultimate Life Table.

ii. $i = 0.05$

iii. Commissions of 40% of premium in year 1 and 8% of premium thereafter.

iv. Issue expenses of 400 per policy at time 0.

v. Maintenance expenses of 30 at the beginning of each year including year 1.

vi. Expense of paying a death claim is 300 and will be incurred at the end of the year of death.

a. (4 points) Calculate the gross premium using the equivalence principle.

Solution:

$$PVP = PVB + PVE$$

$$P\ddot{a}_{60} = 100,000A_{60} + 0.32P + 0.08P\ddot{a}_{60} + 400 + 30\ddot{a}_{60} + 300A_{60}$$

$$P = \frac{100,300A_{60} + 400 + 30\ddot{a}_{60}}{0.92\ddot{a}_{60} - 0.32} = \frac{(100,300)(0.29028) + 400 + (30)(14.9041)}{(0.92)(14.9041) - 0.32} = 2,237.36$$

b. (4 points) Calculate the gross premium policy value at the end of the 10th year.

Solution:

$${}_{10}V^{g} = PVFB + PVFE - PVFP$$

$$= 100,000A_{70} + (0.08)(2237.36)\ddot{a}_{70} + 30\ddot{a}_{70} + 300A_{70} - 2237.36\ddot{a}_{70}$$

$$= (100,300)(0.42818) - [(0.92)(2237.36) - 30](12.0083) = 18,589.16$$

(4 points) A whole life insurance policy on (80) pays a death benefit of 150,000 at the end of the year of death.

You are given:

i. Mortality follows the Standard Ultimate Life Table.

ii. $i = 4\%$

iii. ${}_{15}V^{g} = 90,000$

iv. The gross premium is 13,000.00.

v. Commissions are 100% in the first year and 8% thereafter

vi. Issue Expenses are 1000 at the beginning of the first year.

vii. Maintenance expenses are 35 per policy at the beginning of every year including the first year.

Determine ${}_{16}V^{g}$.

Solution:

$$({}_{t}V + P_{t} - e_{t} - X_{t}^{BOY})(1+i) = (S_{t+1} + E_{t+1})(q_{+t}) + {}_{t+1}V(1-q_{+t})$$

$$(90,000 + 13,000 - (0.08)(13,000) - 35)(1.04) = (150,000 + 0)(0.173599) + {}_{16}V(1 - 0.173599)$$

$${}_{16}V = \frac{(90,000 + 13,000 - (0.08)(13,000) - 35)(1.04) - (150,000 + 0)(0.173599)}{(1 - 0.173599)}$$

$$= 96,759.50$$

A whole life insurance policy to (60) pays a death benefit of 100,000 at the end of the year of death. The gross annual premium is 2400 payable for the life of the insured. It was not calculated using the equivalence principle.

You are given the following policy value basis:

i. Mortality follows the Standard Ultimate Life Table

ii. $i = 0.05$

iii. Expenses as follows:

1. Issue Expense at time 0 of 925.
2. Maintenance expense of 37 at the start of every year including the first year.
3. Termination expense of 1500 paid at the end of the year of death.
4. Commissions of 40% in the first year and 6.5% thereafter

a. (9 points) Calculate the gross premium policy value at time 10.

Solution:

$$_{10}V = PVFB + PVFE - PVFP$$

$$= 100,000A_{70} + 37\ddot{a}_{70} + 1500A_{70} + 0.065(2400)\ddot{a}_{70} - (2400)\ddot{a}_{70}$$

$$= (101,500)(0.42818) - \left[(0.935)(2400) - 37\right](12.0083) = 16,957.95$$

Kayla who is (65) buys a whole life policy. The death benefit is 200,000 paid at the end of the year of death. Annual premiums are payable for the life of the policy.

The gross premium policy value at the end of the 10th year is 44,699.20. The gross premium policy value at the end of the 11th year is 49,973.11.

Policy values are based on the following assumptions:

i. Mortality follows the Standard Ultimate Life Table.
ii. $i = 0.05$
iii. Expenses:
    1. Commissions of 50% of premiums year 1 and 7% year 2+
    2. Issue Expense of 400 per policy at the start of year 1 only
    3. Maintenance expense of 52 per policy at the start of every year including the first year.

(6 points) Calculate the gross premium. The gross premium is not determined using the equivalence principle.

Solution:

$$({}_{10}V^g + P^g - 0.07P^g - 52)(1.05) = (200,000)q_{75} + {}_{11}V^g(1 - q_{75})$$

$$(44,699.20 + (0.93)P^g - 52)(1.05) = (200,000)(0.018433) + (49,973.11)(1 - 0.018433)$$

$$P^g = \frac{[(200,000)(0.018433) + (49,973.11)(1 - 0.018433)](1.05)^{-1} - 44,699.20 + 52}{0.93} = 6000.00$$

Ranya who is (21) purchases a whole life insurance policy with a death benefit of 100,000 payable at the end of the year of death. The policy has annual premiums. **The gross premium for this policy is 360.**

You are given:

i. Mortality follows that Standard Ultimate Life Table.

ii. $i = 0.05$

iii. Deaths are uniformly distributed between integral ages.

This policy has the following expenses:

i. First year expense of 400 per policy plus 53% of premium

ii. Expense of 50 per policy plus 5% of premium in years 2+

iii. Claim expense of 500 incurred at the end of the year of death

Per policy expenses are incurred at the beginning of the policy year.

a. (4 points) The gross premium policy value at the end of 20 years is 7370 to the nearest 10. Calculate the policy value to the nearest 0.1. Remember that the gross premium is 360.

Solution:

$$_{20}V^g = PVFB + PVFE - PVFP$$

$$= 100,000A_{41} + 50\ddot{a}_{41} + 0.05P\ddot{a}_{41} + 500A_{41} - P\ddot{a}_{41}$$

$$= (100,500)(0.12665) - [(0.95)(360) - 50](18.3403) = 7372.96$$

b. (4 points) Use the recursive formula to find the gross premium policy value at the end of 21 years.

Solution:

$$_{21}V^g = \frac{(_{20}V^g + 0.95P - 50)(1.05) - (100,500)q_{41}}{1 - q_{41}}$$

$$= \frac{(7372.96 + (0.95)(360) - 50)(1.05) - (100,500)(0.000565)}{1 - 0.000565} = 7995.94$$

(4 points) A whole life insurance policy is issued to (70) and pays a death benefit of 78,000 at the end of the year of death. The policy has level annual premiums for as long as the insured is alive.

You are given:

i. Mortality follows the Standard Ultimate Life Table
ii. $i = 0.05$
iii. The policy pays commissions of 50% for the first year and 5% thereafter.
iv. The per policy expenses is 200.
v. The maintenance expense for the policy is 40 at the beginning of every year including the first year.

The gross premium for this policy is 3200. Calculate the gross premium policy value at the end of 10 years.

Solution:

$$_{10}V^{g} = PVFB + PVFE - PVFP = (78,000)A_{80} + (0.05)(3200)\ddot{a}_{80} + 40\ddot{a}_{80} - 3200\ddot{a}_{80}$$

$$= (78,000)(0.59293) - (3200 - 160 - 40)(8.5484) = 20,603.34$$

(4 points) Richard buys a whole life policy when he is (70). The policy pays a death benefit of 200,000 at the end of the year of death. The premiums are paid annually as long as Richard is alive.

You are given that mortality follows the Standard Ultimate Mortality Table with interest at 5%.

The gross premium for this policy is 8308.22. The premium was determined using the equivalence principal.

The policy value basis for the gross premium policy values is:

- Mortality follows the Standard Ultimate Mortality Table
- $i = 0.05$
- Commissions are 65% of premiums in the first year and 8% thereafter
- Issue Expenses are 400 per policy and 1.00 per thousand
- Maintenance expenses are 50 at the beginning of each year including the first year.
- Termination expense of 500 paid at the end of the year of death

Determine the gross premium policy value at the end of the 10th year.

Solution:

$$_{10}V^{g} = PVFB + PVFE - PVFP =$$

$$(200,000 + 500)A_{80} - (8308.22 - 0.08(8308.22) - 50)\ddot{a}_{80}$$

$$= 200,500(0.59293) - 7593.5624(8.5484) = 53,969.66$$

(4 points) Jaden who is (25) buys a whole life policy with a death benefit of 75,000 paid at the end of the year of death. The policy has annual gross premiums.

The policy value basis for the gross premium policy values is:

- Mortality follows the Standard Ultimate Mortality Table
- $i = 0.06$ ← Note that this is not $i = 0.05$.
- Expenses are 10% of premium and 40 per policy at the start of each year.
- Termination expense of 250 paid at the end of the year of death

The gross premium policy values for the 9th, 10th, and 11th year are given in the following table:

Time	Gross Premium Policy value
9	1000.00
10	1271.51
11	1558.02

The gross premium is 300 to the nearest 25. Determine the gross premium to the nearest 1.

Solution:

$$ {}_{10}V = \frac{({}_9V + P - .1P - 40)(1.06) - (75000 + 250)q_{34}}{p_{34}} $$

$$ 1271.51 = \frac{(960 + 0.9P)(1.06) - (75,250)(0.000372)}{1 - 0.000372} $$

$$ .9P = 265.5 ==> P = 295 $$

A whole life policy is issued to (70) with a death benefit of 25,000 paid at the end of the year of death. The gross premium paid annually for the life of the policy is P. This premium was NOT determined by the equivalence principle.

The policy value basis is the Standard Ultimate Life Table with interest at 6%. (Note that the interest rate is NOT 5%.)

The expenses for this policy are:

- Commissions of 50% of premiums in the first year and 9% thereafter.
- Per policy expenses of 300 in the first year and 50 each year thereafter.
- Claim expense of 350 paid at the end of the year of death.

The gross premium policy value at the end of the 10th year is 6000.00. The gross premium policy value at the end of the 11th year is 6894.60.

Determine P.

Solution:

$$ {}_{11}V^{g} = \frac{({}_{10}V^{g} + P_{10} - e_{10} - X_{10}^{BOY})(1+i) - (S_{11} + E_{11})q_{80}}{p_{80}} $$

$$ 6894.60 = \frac{(6000 + 0.91P - 50)(1.06) - (25,000 + 350)(0.032658)}{(1 - 0.032658)} $$

$$ P = \frac{\left[\frac{6894.60(1-0.032658) + (25,350)(0.032658)}{1.06} - 5950\right]}{0.91} = 1234 $$