

Chapter 7 – Past Test and Quiz Problems – Net Premium Policy Value

(5 points) Conley Life Insurance Company sells a whole life policy to Andrew who is (60). The policy pays a death benefit of 100,000 at the end of the year of death. The premiums for the policy are paid annually.

You are given that:

- i. Mortality follows the Standard Ultimate Life Table.
- ii. $i = 0.05$
- iii. Commissions of 40% of premium in year 1 and 8% of premium thereafter.
- iv. Issue expenses of 400 per policy at time 0.
- v. Maintenance expenses of 30 at the beginning of each year including year 1.
- vi. Expense of paying a death claim is 300 and will be incurred at the end of the year of death.

Calculate the net premium policy value at the end of the 10th year.

Solution:

$$P = \frac{100,000 A_{60}}{\ddot{a}_{60}} = \frac{(100,000)(0.29028)}{14.9041} = 1947.6520$$

$${}_{10}V^n = 100,000 A_{70} - 1947.6520 \ddot{a}_{70}$$

$$= (100,000)(0.42818) - (1947.6520)(12.0083) = 19,430.01$$

Or

$${}_{10}V^n = 100,000 \left(1 - \frac{\ddot{a}_{70}}{\ddot{a}_{60}} \right) = (100,000) \left(1 - \frac{12.0083}{14.9041} \right) = 19,429.55$$

***2nd method is a shortcut that we may not have yet covered in class**

(6 points) A 20-year endowment insurance to (50) pays a death benefit of 100,000 at the end of the year of death.

You are given:

- i. Mortality follows the Standard Ultimate Life Table.
- ii. $i = 5\%$
- iii. Net premiums are determined using the equivalence principle.

Determine the net premium policy value at the end of ten years.

Solution:

$$PVP = PVB$$

$$P\ddot{a}_{50:\overline{20}|} = 100,000A_{50:\overline{20}|}$$

$$P(12.8424) = (100,000)(0.38844)$$

$$P = \frac{38,844.00}{12,8424} = 3024.57$$

$${}_{10}V^n = PVFB - PVFP = (100,000)A_{60:\overline{10}|} - 3024.57\ddot{a}_{60:\overline{10}|}$$

$$= (100,000)(0.62116) - (3024.57)(7.9555) = 38,054.03$$

Kayla who is (65) buys a whole life policy. The death benefit is 200,000 paid at the end of the year of death. Annual premiums are payable for the life of the policy.

The gross premium policy value at the end of the 10th year is 44,699.20. The gross premium policy value at the end of the 11th year is 49,973.11.

Policy values are based on the following assumptions:

- i. Mortality follows the Standard Ultimate Life Table.
- ii. $i = 0.05$
- iii. Expenses:
 - 1. Commissions of 50% of premiums year 1 and 7% year 2+
 - 2. Issue Expense of 400 per policy at the start of year 1 only
 - 3. Maintenance expense of 52 per policy at the start of every year including the first year.

- a. (2 points) Calculate the net premium.

Solution:

$$P^n = \frac{(200,000)A_{65}}{\ddot{a}_{65}} = \frac{(200,000)(0.35477)}{13.5498} = 5236.53$$

- b. (2 points) Calculate the net premium policy value at the end of 10 years.

Solution:

$$\begin{aligned} {}_{10}V &= PVFB - PVFP = 200,000A_{75} - 5236.53\ddot{a}_{75} \\ &= (200,000)(0.50868) - (5236.53)(10.3178) = 47,706.48 \end{aligned}$$

or

$${}_{10}V = (200,000)\left(1 - \frac{\ddot{a}_{75}}{\ddot{a}_{65}}\right) = (200,000)\left(1 - \frac{10.3178}{13.5498}\right) = 47,705.50$$

(6 points) Ranya who is (21) purchases a whole life insurance policy with a death benefit of 100,000 payable at the end of the year of death. The policy has annual premiums. **The gross premium for this policy is 360.**

You are given:

- i. Mortality follows that Standard Ultimate Life Table.
 - ii. $i = 0.05$
 - iii. Deaths are uniformly distributed between integral ages.
- a. (2 points) The net premium is 260 to the nearest 10. Calculate the net premium to the nearest 0.01.

Solution:

$$PVP = PVB$$

$$P^n \ddot{a}_{21} = 100,000 A_{21} \implies P^n = \frac{(100,000)(0.051441)}{19.9197} = 258.24$$

- b. (4 points) Calculated the net premium policy value at the end of 20 years.

Solution:

$${}_{20}V^n = PVFB - PVFP^n = 100,000 A_{41} - 258.24 \ddot{a}_{41}$$

$$= (100,000)(0.12665) - 258.24(18.3403) = 7928.80$$

Or

$${}_{20}V^n = (100,000) \left(1 - \frac{\ddot{a}_{41}}{\ddot{a}_{21}} \right) = (100,000) \left(1 - \frac{18.3403}{19.9197} \right) = 7928.83$$

(14 points) Jake who is (70) purchases a 30-year term insurance policy with a death benefit of 500,000 paid at the moment of death. For this policy, premiums are paid quarterly for 20 years.

You are given:

- i. Mortality follows that Standard Ultimate Life Table.
 - ii. $i = 0.05$
 - iii. Deaths are uniformly distributed between integral ages.
- a. (5 points) The quarterly net premium is 4920 to the nearest 10. Calculate the quarterly net premium to the nearest 0.01 .

Solution:

$$PVP = PVB \implies 4P\ddot{a}_{70:\overline{20}|}^{(4)} = 500,000\bar{A}_{70:\overline{30}|}^1$$

$$\ddot{a}_{70:\overline{20}|}^{(4)} = \ddot{a}_{70}^{(4)} - {}_{20}E_{70} \cdot \ddot{a}_{90}^{(4)} = [\alpha(4) \cdot \ddot{a}_{70} - \beta(4)] - {}_{20}E_{70}[\alpha(4) \cdot \ddot{a}_{90} - \beta(4)]$$

$$= [(1.00019)(12.0083) - 0.38272] - (0.38272)[(1.00019)(5.1835) - 0.38272]$$

$$= 10.79653$$

$$\bar{A}_{70:\overline{30}|}^1 = \left(\frac{i}{\delta}\right)({}_{A_{70}} - {}_{30}E_{70} \cdot A_{100}) = \left(\frac{0.05}{\ln(1.05)}\right)(0.42818 - (0.17313)(0.09168)(0.87068))$$

$$= 0.42463$$

$$P = \frac{500,000(0.42463)}{4(10.79653)} = 4916.33$$

- b. (5 points) Calculate the net premium policy value at the end of 10 years.

Solution:

$${}_{10}V = PVB - PVP \implies 500,000 \bar{A}_{80:\overline{20}|}^1 - 4P\ddot{a}_{80:\overline{10}|}^{(4)}$$

$$\ddot{a}_{80:\overline{10}|}^{(4)} = \ddot{a}_{80}^{(4)} - {}_{10}E_{80} \cdot \ddot{a}_{90}^{(4)} = [\alpha(4) \cdot \ddot{a}_{80} - \beta(4)] - {}_{10}E_{80}[\alpha(4) \cdot \ddot{a}_{90} - \beta(4)]$$

$$= [(1.00019)(8.5484) - 0.38272] - (0.33952)[(1.00019)(5.1838) - 0.38272]$$

$$= 6.53701$$

$$\bar{A}_{80:\overline{20}|}^1 = \left(\frac{i}{\delta} \right) (A_{80} - {}_{20}E_{80} \cdot A_{100}) = \left(\frac{0.05}{\ln(1.05)} \right) (0.59293 - (0.03113)(0.87068))$$

$$= 0.57986$$

$${}_{10}V = (500,000)(0.57986) - (4)(4916.33)(6.53701) = 161,377.61$$

- c. (4 points) Calculate the net premium policy value at the end of 20 years.

Solution:

$${}_{20}V = PVB - PVP \implies 500,000 \bar{A}_{90:\overline{20}|}^1 - 0$$

$$500,000 \bar{A}_{90:\overline{10}|}^1 = (500,000) \left(\frac{i}{\delta} \right) (A_{90} - {}_{10}E_{90} \cdot A_{100})$$

$$= (500,000) \left(\frac{0.05}{\ln(1.05)} \right) (0.75317 - (0.9168)(0.87068)) = 345,021$$

A whole life insurance policy is issued to (70) and pays a death benefit of 78,000 at the end of the year of death. The policy has level annual premiums for as long as the insured is alive.

You are given:

- i. Mortality follows the Standard Ultimate Life Table
 - ii. $i = 0.05$
 - iii. The policy pays commissions of 50% for the first year and 5% thereafter.
 - iv. The per policy expenses is 200.
 - v. The maintenance expense for the policy is 40 at the beginning of every year including the first year.
- a. (4 points) Calculate the net premium policy value at the end of 10 years.

Solution:

$$P = \frac{(78,000)A_{70}}{\ddot{a}_{70}} = \frac{(78,000)(0.42818)}{12.0083} = 2781.25$$

$${}_{10}V^n = (78,000)A_{80} - 2781.25\ddot{a}_{80}$$

$$= (78,000)(0.59293) - 2781.25(8.5484) = 22,473.33$$

Richard buys a whole life policy when he is (70). The policy pays a death benefit of 200,000 at the end of the year of death. The premiums are paid annually as long as Richard is alive.

You are given that mortality follows the Standard Ultimate Mortality Table with interest at 5%.

- a. (2 points) The net premium is 7100 to the nearest 100. Calculate the net premium to the nearest 1.

$$P_{70} = \frac{200,000 A_{70}}{\ddot{a}_{70}} = \frac{200,000(0.42818)}{12.0083} = 7,131.4$$

- b. (3 points) Calculate the net premium policy value at the end of the 10th year.

$$_{10}V^n = 200,000 A_{80} - P \ddot{a}_{80} = 200,000(0.59293) - 7131.4(8.5484) = 57,623.94$$

or

$$_{10}V^n = 200,000 \left(1 - \frac{\ddot{a}_{80}}{\ddot{a}_{70}} \right) = 200,000 \left(1 - \frac{8.5484}{12.0083} \right) = 57,625.14$$

(6 points) A whole life policy is issued to (70) with a death benefit of 25,000 paid at the end of the year of death. Premiums are determined by the equivalence principle and are paid annually for the life of the policy.

The policy value basis is the Standard Ultimate Life Table with interest at 5%.

- a. (3 points) Calculate the net premium for this policy.

Solution:

$$PVP = PVB \implies P\ddot{a}_{70} = 25,000A_{70} \implies P(12.0083) = 25,000(0.42818)$$

$$P = \frac{25,000(0.42818)}{12.0083} = 891.425$$

- b. (3 points) Calculate the net premium policy value at the end of the 10th year.

Solution:

$${}_{10}V^n = PVFB - PVFP$$

$$= 25,000A_{80} - 891.425\ddot{a}_{80} = 25,000(0.59293) - 891.425(8.5484) = 7202.99$$

(4 points) A 20-year term insurance policy issued to (55) pays a death benefit of 500,000 at the end of the year of death. You are given the following information about the policy value basis:

- i. The annual net premium is 8500.
- ii. Mortality follows the Standard Ultimate Life Table
- iii. $i = 0.07$

The net premium policy value at the end of the 9th year is 36,500.

Calculate the net premium policy value at the end of the 10th year.

Solution:

$$({}_9V^n + P)(1+i) = (S_{10})q_{x+9} + {}_{10}V^n \cdot p_{x+9}$$

$$(36,500 + 8500)(1.07) = (500,000)(0.005288) + {}_{10}V^n(1 - 0.005288)$$

$${}_{10}V^n = \frac{(36,500 + 8500)(1.07) - (500,000)(0.005288)}{1 - 0.005288} = 45,77.91$$