Chapter 7 – Past Test and Quiz Problems

Conley Life Insurance Company sells a whole life policy to Andrew who is (60). The policy pays a death benefit of 100,000 at the end of the year of death. The premiums for the policy are paid annually.

You are given that:

i. Mortality follows the Standard Ultimate Life Table.

ii. $i = 0.05$

iii. Commissions of 40% of premium in year 1 and 8% of premium thereafter.

iv. Issue expenses of 400 per policy at time 0.

v. Maintenance expenses of 30 at the beginning of each year including year 1.

vi. Expense of paying a death claim is 300 and will be incurred at the end of the year of death.

a. (3 points) Calculate the net premium reserve at the end of the 10th year.

Solution:

$$P = \frac{100,000 A_{60}}{\ddot{a}_{60}} = \frac{(100,000)(0.29028)}{14.9041} = 1947.6520$$

$${}_{10}V^n = 100,000 A_{70} - 1947.6520 \ddot{a}_{70}$$

$$= (100,000)(0.42818) - (1947.6520)(12.0083) = 19,430.01$$

Or

$${}_{10}V^n = 100,000\left(1 - \frac{\ddot{a}_{70}}{\ddot{a}_{60}}\right) = (100,000)\left(1 - \frac{12.0083}{14.9041}\right) = 19,429.55$$

b. (5 points) Calculate the gross premium using the equivalence principle.

Solution:

$$PVP = PVB + PVE$$

$$P\ddot{a}_{60} = 100,000A_{60} + 0.32P + 0.08P\ddot{a}_{60} + 400 + 30\ddot{a}_{60} + 300A_{60}$$

$$P = \frac{100,300A_{60} + 400 + 30\ddot{a}_{60}}{0.92\ddot{a}_{60} - 0.32} = \frac{(100,300)(0.29028) + 400 + (30)(14.9041)}{(0.92)(14.9041) - 0.32} = 2,237.36$$

c. (5 points) Calculate the gross premium reserve at the end of the 10th year.

Solution:

$${}_{10}V^g = PVFB + PVFE - PVFP$$

$$= 100,000A_{70} + (0.08)(2237.36)\ddot{a}_{70} + 30\ddot{a}_{70} + 300A_{70} - 2237.36\ddot{a}_{70}$$

$$= (100,300)(0.42818) - [(0.92)(2237.36) - 30](12.0083) = 18,589.16$$

d. (2 points) Calculate the expense reserve at the end of the 10th year.

Solution:

$${}_{10}V^g = {}_{10}V^n + {}_{10}V^e$$

$${}_{10}V^e = 18,589.16 - 19430.01 = -840.85$$

During the 10th year, actual experience was as follows:

i. Mortality was 90% the Standard Ultimate Life Table.

ii. $i = 0.055$

iii. Commissions 10% of premium.

iv. Maintenance expenses of 42 per policy at the beginning of the year.

v. Expense of paying a death claim was 250.

In calculating profit by source, the company allocates profits first to expenses, then to interest, and finally to mortality.

e. (8 points) Calculate the total profit in the 10th year.

Solution:

$$\text{Profit} = ({}_9V^g + P_9(1-e_9) - X_9^{BOY})(1+i) - (S_{10} + E_{10})q_{x+9} - {}_{10}V^g(1-q_{x+9})$$

$${}_9V^g = 100,000A_{69} + (0.08)(2237.36)\ddot{a}_{69} + 30\ddot{a}_{69} + 300A_{69} - 2237.36\ddot{a}_{69}$$

$$= (100,300)(0.41285) - [(0.92)(2237.36) - 30](12.3302) = 16,398.63$$

$$(16,398.63 + (2237.36)(1-0.1) - 42)(1.055) - (100,000 + 250)(0.9)(0.009294)$$
$$- (18,589.16)(1-(0.9)(0.009294))$$

$$= 108.40$$

f. (8 points) Calculate the profit from interest in the 10th year.

Solution:

$$\mathrm{G}^{Expenses} = (16,398.63 + (2237.36)(1-0.1) - 42)(1.05) - (100,000 + 250)(0.009294) \\ - (18,589.16)(1-(0.009294))$$

$$= -59.35$$

$$\mathrm{G}^{Exp\&Int} = (16,398.63 + (2237.36)(1-0.1) - 42)(1.055) - (100,000 + 250)(0.009294) \\ - (18,589.16)(1-(0.009294))$$

$$= 32.50$$

$$\mathrm{G}^{Interest} = \mathrm{G}^{Exp\&Int} - \mathrm{G}^{Expenses} = 32.50 - (-59.35) = 91.36$$

Danielle buys a 21 year term insurance policy with a death benefit of 250,000 paid at the end of the year of death. Danielle is 44 years old. The premiums for this policy are paid annually.

You are given that:

i. Mortality follows the Standard Ultimate Life Table.

ii. $i = 0.05$

a. (3 Points) Calculate the first year premium under Full Preliminary Term reserves.

Solution:

$$P_1^{FPT} = 250{,}000 v q_{44} = (250{,}000)(1.05)^{-1}(0.00071) = 169.05$$

b. (5 points) Calculate the premium in year 2 and later under Full Preliminary Term reserves.

Solution:

$$P_{x+1}^{FPT} = \frac{(250{,}000)A^{1}_{45:\overline{20|}}}{\ddot{a}_{45:\overline{20|}}} = \frac{(250{,}000)(A_{45:\overline{20|}} - {}_{20}E_{45})}{\ddot{a}_{45:\overline{20|}}} = \frac{(250{,}000)(0.38385 - 0.35994)}{12.9391} = 461.97$$

c. (5 points) Calculate the Full Preliminary Term reserve at the end of the 11th year.

Solution:

$${}_{11}V = 250{,}000 A^{1}_{55:\overline{10|}} - 461.97\ddot{a}_{55:\overline{10|}}$$

$$= (250{,}000)(0.61813 - 0.59342) - (461.97)(8.0192) = 2472.87$$

A 20 year endowment insurance to (50) pays a death benefit of 100,000 at the end of the year of death.

You are given:

a. Mortality follows the Standard Ultimate Life Table.

b. $i = 5\%$

c. Net premiums are determined using the equivalence principle.

Determine the net premium reserve at the end of ten years.

Solution:

$$PVP = PVB$$

$$P\ddot{a}_{50:\overline{20|}} = 100,000A_{50:\overline{20|}}$$

$$P(12.8424) = (100,000)(0.38844)$$

$$P = \frac{38,844.00}{12,8424} = 3024.57$$

$${}_{10}V^{n} = PVFB - PVFP = (100,000)A_{60:\overline{10|}} - 3024.57\ddot{a}_{60:\overline{10|}}$$

$$= (100,000)(0.62116) - (3024.57)(7.9555) = 38,054.03$$

A whole life insurance policy on (70) pays a death benefit of 150,000 at the end of the year of death.

You are given:

a. Mortality follows the Standard Ultimate Life Table.

b. $i = 4\%$

c. ${}_{15}V^n = 70,000.00$

d. ${}_{16}V^n = 74,918.42$

Determine the net premium for this policy.

Solution:

$$({}_{15}V^n + P)(1+i) = (S_{16})q_{x+15} + {}_{16}V^n(1-q_{x+15})$$

$$(70,000 + P)(1.04) = (150,000)(0.057665) + (74,918.42)(1-0.057665)$$

$$P = \frac{(150,000)(0.057665) + (74,918.42)(1-0.057665)}{1.04} - 70,000 = 6200$$

A whole life insurance policy on (80) pays a death benefit of 150,000 at the end of the year of death.

You are given:

a. Mortality follows the Standard Ultimate Life Table.

b. $i = 4\%$

c. ${}_{15}V^{g} = 90,000$

d. The gross premium is 13,000.00.

e. Commissions are 100% in the first year and 8% thereafter

f. Issue Expenses are 1000 at the beginning of the first year.

g. Maintenance expenses are 35 per policy at the beginning of every year including the first year.

Determine ${}_{16}V^{g}$.

Solution:

$$({}_{t}V + P_{t} - e_{t} - X_{t}^{BOY})(1+i) = (S_{t+1} + E_{t+1})(q_{+t}) + {}_{t+1}V(1-q_{+t})$$

$$(90,000 + 13,000 - (0.08)(13,000) - 35)(1.04) = (150,000 + 0)(0.173599) + {}_{16}V(1-0.173599)$$

$${}_{16}V = \frac{(90,000 + 13,000 - (0.08)(13,000) - 35)(1.04) - (150,000 + 0)(0.173599)}{(1-0.173599)}$$

$$= 96,759.50$$

A whole life insurance policy to (60) pays a death benefit of 100,000 at the end of the year of death. The gross annual premium is 2400 payable for the life of the insured. It was not calculated using the equivalence principle.

You are given the following reserve basis:

i. Mortality follows the Standard Ultimate Life Table

ii. $i = 0.05$

iii. Expenses as follows:

1. Issue Expense at time 0 of 925.
2. Maintenance expense of 37 at the start of every year including the first year.
3. Termination expense of 1500 paid at the end of the year of death.
4. Commissions of 40% in the first year and 6.5% thereafter

a. (9 points) Calculate the gross premium reserve at time 10.

Solution:

$${}_{10}V = PVFB + PVFE - PVFP$$

$$= 100,000A_{70} + 37\ddot{a}_{70} + 1500A_{70} + 0.065(2400)\ddot{a}_{70} - (2400)\ddot{a}_{70}$$

$$= (101,500)(0.42818) - \left[(0.935)(2400) - 37\right](12.0083) = 16,957.95$$

b. (6 points) Let the first year Full Preliminary Term premium be P_1^{FPT} and the premiums for years after the first be P_{x+1}^{FPT} . Calculate $P_{x+1}^{FPT} - P_1^{FPT}$.

Solution:

$$P_1^{FPT} = Svq_{60} = (100,000)(1.05)^{-1}(0.003398) = 323.62$$

$$P_{x+1}^{FPT} = \frac{100,000A_{61}}{\ddot{a}_{61}} = \frac{(100,000)(0.30243)}{14.6491} = 2064.50$$

$$P_{x+1}^{FPT} - P_1^{FPT} = 2064.50 - 323.62 = 1740.88$$

c. (6 points) Calculate the Full Preliminary Term reserve at time 10.

Solution:

$${}_{10}V^{FPT} = PVFB - PVFP = 100,000A_{70} - 2064.50\ddot{a}_{70}$$

$$= (100,000)(0.42818) - (2064.50)(12.0083) = 18,026.86$$

A 20 year term insurance policy issued to (55) pays a death benefit of 500,000 at the end of the year of death. You are given the following information about the reserve basis:

i. The annual net premium is 8500.
ii. Mortality follows the Standard Ultimate Life Table
iii. $i = 0.07$
iv. Expenses as follows:
    1. Issue expenses of 1000 at time 0
    2. Maintenance expense of 21 at the beginning of every year including the first year.
    3. Commissions of 38% of premiums in the first year and 4% thereafter.
    4. A termination expense of 152 paid at the end of the year of death.

The net premium reserve at the end of the 9th year is 36,500.

The expense reserve at the end of the 9th year is – 2600. Note that this is a negative reserve.

The expense reserve at the end of the 10 year is – 2423.24. Note that this is a negative reserve.

a. (8 points) Calculate the net premium reserve at the end of the 10th year.

Solution:

$$({}_9V^n + P)(1+i) = (S_{10})q_{x+9} + {}_{10}V^n \cdot p_{x+9}$$

$$(36,500 + 8500)(1.07) = (500,000)(0.005288) + {}_{10}V^n(1 - 0.005288)$$

$${}_{10}V^n = \frac{(36,500 + 8500)(1.07) - (500,000)(0.005288)}{1 - 0.005288} = 45,77.91$$

b. (10 points) Calculate the gross premium.

Solution:

$${}_{10}V^{e} = {}_{10}V^{g} - {}_{10}V^{n} ==> -2423.24 = {}_{10}V^{g} - 45,747.91 ==> {}_{10}V^{g} = 43,324.67$$

$${}_{9}V^{e} = {}_{9}V^{g} - {}_{9}V^{n} ==> -2600.00 = {}_{9}V^{g} - 36,500 ==> {}_{9}V^{g} = 33,900$$

$$({}_{9}V^{g} + P - e_{9} - X_{9}^{BOY})(1+i) = (S_{10} + E_{10})q_{x+9} + {}_{10}V^{g}(1 - q_{x+9})$$

$$33,900 + 0.96P - 21)(1.07) = (500,000 + 152)(0.005288) + (43,324.67)(1 - 0.005288)$$

$$P = \frac{\left[(500,000 + 152)(0.005288) + (43,324.67)(1 - 0.005288)\right](1.07)^{-1} - 33,879}{0.96} = 9238.55$$

Gage who is (70) buys a 21 year term insurance. The death benefit is 400,000 and is paid at the end of the year of death. Annual premiums are paid for 21 years.

The reserves for this policy are determined using the Full Preliminary Term Method. The reserve basis is mortality that follows the Standard Ultimate Life Table and $i = 0.05$.

a. (1 point) Calculate the net premium for the first year of this policy.

Solution:

$$P_1^{FPT} = Svq_x = (400,000)(1.05)^{-1}(0.010413) = 3966.86$$

b. (3 points) Calculate the net premium for years after the first for this policy.

Solution:

$$P_{x+1}^{FPT} = \frac{400,000 A^1_{71:\overline{20|}}}{\ddot{a}_{71:\overline{20|}}} = \frac{400,000(A_{71:\overline{20|}} - {}_{20}E_{71})}{\ddot{a}_{71:\overline{20|}}} = \frac{400,000(0.48039 - 0.15730)}{10.9118} = 11,843.69$$

c. (6 points) Calculate the Full Preliminary Term reserve at the end of 11 years.

Solution:

$${}_{11}V = PVFB - PVFP = (400,000)A^1_{81:\overline{10|}} - 11,843.69\ddot{a}_{81:\overline{10|}}$$

$$= (400,000)(0.68325 - 0.31556) - (11,843.69)(6.6517) = 68,295.31$$

A whole life insurance policy to (70) pays a death benefit of 105,000 at the end of the year of death. The policy has a gross annual premium of 4470.76 determined using the equivalence principle.

You are given the following reserve basis:

i. Mortality follows the Standard Ultimate Life Table
ii. $i = 0.05$
iii. Expenses as follows:
   1. Issue Expense at time 0 of 925.
   2. Maintenance expense of 37 at the start of every year including the first year.
   3. Termination expense of 1500 paid at the end of the year of death.
   4. Commissions of 50% in the first year and 10% thereafter

Actual experience during the 12th year is:

i. Mortality is 105% the Standard Ultimate Life Table.
ii. $i = 0.055$
iii. Expenses as follows:
   1. Maintenance expense of 45 at the start of the year.
   2. Termination expense of 1200 paid at the end of the year of death.
   3. Commissions of 9%.

The reserve at the end of the 11th year is 32,283.50.

a. (12 points) Calculate the total profit in the 12th year.

   Solution:

$$ {}_{12}V = \frac{({}_{11}V + P - e - X)(1+i) - (S+E)q_{81}}{1-q_{81}} $$

$$ = \frac{(32,283.50 + 4470.76(0.9) - 37) - (105,000 + 1500)(0.036607)}{1-0.036607} = 35,484.01 $$

$$ \text{Profit} = ({}_{11}V + P - e - X)(1+i) - (S+E)q_{81} - {}_{12}V(1-q_{81}) $$

$$ = (32,283.50 + 4470.76(0.91) - 45)(1.055) - (105,000 + 1200)(1.05)(0.036607) $$
$$ - (35,484.01)[1 - (1.05)(0.036607)] = 101.63 $$

b. (12 points) The company allocates profits to interest, expenses and mortality in that order. Calculate the profit allocated to expenses.

Solution:

Gain Interest = $({}_{11}V + P - e - X)(1+i) - (S+E)q_{81} - {}_{12}V(1-q_{81})$

$$= (32,283.50 + 4470.76(0.90) - 37)(1.055) - (105,000 + 1500)(0.036607) - (35,484.01)[1-(0.036607)] = 181.35$$

Gain Interest and Expense = $({}_{11}V + P - e - X)(1+i) - (S+E)q_{81} - {}_{12}V(1-q_{81})$

$$= (32,283.50 + 4470.76(0.91) - 45)(1.055) - (105,000 + 1200)(0.036607) - (35,484.01)[1-(0.036607)] = 231.06$$

Gain from Expense = Gain from Interest and Expense - Gain from Interest

$= 231.06 - 181.35 = 49.71$

Kayla who is (65) buys a whole life policy. The death benefit is 200,000 paid at the end of the year of death. Annual premiums are payable for the life of the policy.

The gross premium reserve at the end of the 10th year is 44,699.20. The gross premium reserve at the end of the 11th year is 49,973.11.

Reserves are based on the following assumptions:

i. Mortality follows the Standard Ultimate Life Table.
ii. $i = 0.05$
iii. Expenses:
    1. Commissions of 50% of premiums year 1 and 7% year 2+
    2. Issue Expense of 400 per policy at the start of year 1 only
    3. Maintenance expense of 52 per policy at the start of every year including the first year.

a. (2 points) Calculate the net premium.

Solution:

$$P^n = \frac{(200,000)A_{65}}{\ddot{a}_{65}} = \frac{(200,000)(0.35477)}{13.5498} = 5236.53$$

b. (2 points) Calculate the net premium reserve at the end of 10 years.

Solution:

$${}_{10}V = PVFB - PVFP = 200,000A_{75} - 5236.53\ddot{a}_{75}$$

$$= (200,000)(0.50868) - (5236.53)(10.3178) = 47,706.48$$

or

$${}_{10}V = (200,000)\left(1 - \frac{\ddot{a}_{75}}{\ddot{a}_{65}}\right) = (200,000)\left(1 - \frac{10.3178}{13.5498}\right) = 47,705.50$$

c. (6 points) Calculate the gross premium. The gross premium is not determined using the equivalence principle.

Solution:

$$({}_{10}V^g + P^g - 0.07P^g - 52)(1.05) = (200,000)q_{75} + {}_{11}V^g(1 - q_{75})$$

$$(44,699.20 + (0.93)P^g - 52)(1.05) = (200,000)(0.018433) + (49,973.11)(1 - 0.018433)$$

$$P^g = \frac{[(200,000)(0.018433) + (49,973.11)(1 - 0.018433)](1.05)^{-1} - 44,699.20 + 52}{0.93} = 6000.00$$

Ranya who is (21) purchases a whole life insurance policy with a death benefit of 100,000 payable at the end of the year of death. The policy has annual premiums. **The gross premium for this policy is 360.**

You are given:

i. Mortality follows that Standard Ultimate Life Table.

ii. $i = 0.05$

iii. Deaths are uniformly distributed between integral ages.

a. (3 points) The net premium is 260 to the nearest 10. Calculate the net premium to the nearest 0.01.

Solution:

$$PVP = PVB$$

$$P^n \ddot{a}_{21} == 100,000 A_{21} ==> P^n = \frac{(100,000)(0.051441)}{19.9197} = 258.24$$

b. (4 points) Calculated the net premium reserve at the end of 20 years.

Solution:

$$_{20}V^n = PVFB - PVFP^n = 100,000 A_{41} - 258.24 \ddot{a}_{41}$$

$$= (100,000)(0.12665) - 258.24(18.3403) = 7928.80$$

Or

$$_{20}V^n = (100,000)\left(1 - \frac{\ddot{a}_{41}}{\ddot{a}_{21}}\right) = (100,000)\left(1 - \frac{18.3403}{19.9197}\right) = 7928.83$$

Question Continued . . .

This policy has the following expenses:

i. First year expense of 400 per policy plus 53% of premium

ii. Expense of 50 per policy plus 5% of premium in years 2+

iii. Claim expense of 500 incurred at the end of the year of death

Per policy expenses are incurred at the beginning of the policy year.

c. (4 points) The gross premium reserve at the end of 20 years is 7370 to the nearest 10. Calculate the reserve to the nearest 0.1. Remember that the gross premium is 360.

Solution:

$$ {}_{20}V^g = PVFB + PVFE - PVFP $$

$$ = 100,000A_{41} + 50\ddot{a}_{41} + 0.05P\ddot{a}_{41} + 500A_{41} - P\ddot{a}_{41} $$

$$ = (100,500)(0.12665) - [(0.95)(360) - 50](18.3403) = 7372.96 $$

d. (4 points) Use the recursive formula to find the gross premium reserve at the end of 21 years.

Solution:

$$ {}_{21}V^g = \frac{({}_{20}V^g + 0.95P - 50)(1.05) - (100,500)q_{41}}{1 - q_{41}} $$

$$ = \frac{(7372.96 + (0.95)(360) - 50)(1.05) - (100,500)(0.000565)}{1 - 0.000565} = 7995.94 $$

e. (4 points) Calculate the gross premium reserve at time 20.7 years.

Solution:

$$ {}_{20.7}V = (0.3)(7372.96 + 0.95P - 50) + (0.7)(7995.94) = 7896.65 $$

Question Continued . . .

f. (2 points) Calculate the expense reserve at the end of 20 years.

Solution:

$$ {}_{20}V^{e} = {}_{20}V^{g} - {}_{20}V^{n} = 7372.96 - 7928.80 = -555.76 $$

g. (3 points) Explain why the expense reserve is negative.

Solution:

Expenses are front loaded. That means that the expenses in the first year are higher than the expenses in the second year and later. However, the premium for expenses is level. Initially the present value of the expense premiums is equal to the present value of the expenses. However after the first year, the present value of future expense premiums is greater than the present value of future expenses so the reserve is negative. This occurs because the expenses in the first year are greater than the premium for expenses.

Question Continued . . .

During the 21st year of Ranya's policy, the insurance company has actual experience as follows:

i. Mortality is 110% of the Standard Ultimate Life Table

ii. $i = 0.06$

iii. Expenses are 40 per policy, 6% of premium, and 700 per claim paid.

h. (6 points) Determine the total profit or loss on this policy during the 21st year. Be sure to state if it is a profit or loss.

Solution:

$$\begin{aligned}Total\ \ Gain &= (7372.96+360-0.06(360)-40)(1.06)\\ &\quad -(100,700)(1.1)(0.000565)-(7995.95)[1-(1.1)(0.000565)]=78.08\end{aligned}$$

Profit

The company wants to allocate the gain or loss to the source. The company allocates gains and losses in the following order – First to interest, then to expenses, and finally to mortality.

i. (6 points) Determine the gain or loss from expenses. Be sure to state if it is a gain or loss.

Solution:

$$\begin{aligned}Gain\ Int &= (7372.96+360-0.05(360)-50)(1.06)\\ &\quad -(100,500)(0.000565)-(7995.95)[1-(0.000565)]=76.65\end{aligned}$$

$$\begin{aligned}Gain\ Int\,\&\,Expense &= (7372.96+360-0.06(360)-40)(1.06)\\ &\quad -(100,700)(0.000565)-(7995.95)[1-(0.000565)]=83.32\end{aligned}$$

$$Gain\ Int\,\&\,Expense = 83.32-76.65=6.67 \text{ of Gain}$$

Question Continued . . .

The insurance company decides to hold modified net premium reserves by holding Full Preliminary Term (FPT) reserves. Under FPT reserves, the difference between the FPT premium in years 2 and later and the FPT premium in the first year is called the expense allowance.

j. (4 points) Calculate the expense allowance $(P_{x+1}^{FPT} - {}_1P^{FPT})$ for Ranya's policy.

Solution:

$${}_1P^{FPT} = S \cdot v \cdot q = (100,000)(1.05)^{-1}(0.000253) = 24.10$$

$$P_{x+1}^{FPT} = \frac{100,000A_{22}}{\ddot{a}_{22}} = \frac{(100,000)(0.05378)}{19.8707} = 270.65$$

$$P_{x+1}^{FPT} - {}_1P^{FPT} = 270.65 - 24.10 = 246.55$$

k. (4 points) Calculate the FPT reserve at the end of 20 years on Ranya's policy.

Solution:

$${}_{20}V^{FPT} = 100,000A_{41} - P_{x+1}^{FPT}\ddot{a}_{41} = (100,000)(0.12665) - (270.65)(18.3403) = 7701.20$$

or

$${}_{20}V^{FPT} = 100,000\left(1 - \frac{\ddot{a}_{41}}{\ddot{a}_{21}}\right) = (100,000)\left(1 - \frac{18.3403}{19.8707}\right) = 7701.79$$

l. (3 points) List two reasons that FPT reserves are preferable to net premium reserves or gross premium reserves.

Solution:

FPT reserves a better reflection of economic reserves because some allowance is permitted for expenses.

The FPT reserve maintains the simplicity of the net premium calculations compared to the complexity of the gross premium reserve calculations.

Jake who is (70) purchases a 30 year term insurance policy with a death benefit of 500,000 paid at the moment of death. For this policy, premiums are paid quarterly for 20 years.

You are given:

i. Mortality follows that Standard Ultimate Life Table.

ii. $i = 0.05$

iii. Deaths are uniformly distributed between integral ages.

a. (6 points) The quarterly net premium is 4920 to the nearest 10. Calculate the quarterly net premium to the nearest 0.01 .

Solution:

$$PVP = PVB ==> 4P\ddot{a}^{(4)}_{70:\overline{20|}} = 500,000\bar{A}^{1}_{70:\overline{30|}}$$

$$\ddot{a}^{(4)}_{70:\overline{20|}} = \ddot{a}^{(4)}_{70} - {}_{20}E_{70} \cdot \ddot{a}^{(4)}_{90} = [\alpha(4) \cdot \ddot{a}_{70} - \beta(4)] - {}_{20}E_{70}[\alpha(4) \cdot \ddot{a}_{90} - \beta(4)]$$

$$= [(1.00019)(12.0083) - 0.38272] - (0.38272)[(1.00019)(5.1835) - 0.38272]$$

$$= 10.79653$$

$$\bar{A}^{1}_{70:\overline{30|}} = \left(\frac{i}{\delta}\right)\left(A_{70} - {}_{30}E_{70} \cdot A_{100}\right) = \left(\frac{0.05}{\ln(1.05)}\right)\left(0.42818 - (0.17313)(0.09168)(0.87068)\right)$$

$$= 0.42463$$

$$P = \frac{500,000(0.42463)}{4(10.79653)} = 4916.33$$

b. (5 points) Calculate the net premium reserve at the end of 10 years.

Solution:

$${}_{10}V = PVB - PVP ==> 500,000\bar{A}^{1}_{80:\overline{20}|} - 4P\ddot{a}^{(4)}_{80:\overline{10}|}$$

$$\ddot{a}^{(4)}_{80:\overline{10}|} = \ddot{a}^{(4)}_{80} - {}_{10}E_{80} \cdot \ddot{a}^{(4)}_{90} = [\alpha(4) \cdot \ddot{a}_{80} - \beta(4)] - {}_{10}E_{80}[\alpha(4) \cdot \ddot{a}_{90} - \beta(4)]$$

$$= [(1.00019)(8.5484) - 0.38272] - (0.33952)[(1.00019)(5.1838) - 0.38272]$$

$$= 6.53701$$

$$\bar{A}^{1}_{80:\overline{20}|} = \left(\frac{i}{\delta}\right)\left(A_{80} - {}_{20}E_{80} \cdot A_{100}\right) = \left(\frac{0.05}{\ln(1.05)}\right)\left(0.59293 - (0.03113)(0.87068)\right)$$

$$= 0.57986$$

$${}_{10}V = (500,000)(0.57986) - (4)(4916.33)(6.53701) = 161,377.61$$

c. (2 points) Calculate the net premium reserve at the end of 20 years.

Solution:

$${}_{20}V = PVB - PVP ==> 500,000\bar{A}^{1}_{90:\overline{20}|} - 0$$

$$500,000\bar{A}^{1}_{90:\overline{10}|} = (500,000)\left(\frac{i}{\delta}\right)\left(A_{90} - {}_{10}E_{80} \cdot A_{100}\right)$$

$$= (500,000)\left(\frac{0.05}{\ln(1.05)}\right)\left(0.75317 - (0.9168)(0.87068)\right) = 345,021$$

A fully discrete 20 year term insurance policy to (70) has a death benefit of 50,000. The net premium is calculated using the equivalence principle.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

Calculate the ${}_{10.3}V^n$.

Solution:

$$P = \frac{(50,000)A^{\;1}_{70:\overline{20}|}}{\ddot{a}_{70:\overline{20}|}} = \frac{(50,000)(0.47091 - 0.17313)}{11.1109} = 1340.03$$

$${}_{10}V = PVFB - PVFP = (50,000)A^{\;1}_{80:\overline{10}|} - 1340.03\ddot{a}_{80:\overline{10}|}$$

$$= (50,000)(0.67674 - 0.33952) - 1340.03(6.7885) = 7764.21$$

$${}_{11}V = \frac{(7764.21 + 1340.03)(1.05) - (50,000)(0.032658)}{1 - 0.032658} = 8194.12$$

$${}_{10.3}V = ({}_{10}V + P)(1 - 0.3) + ({}_{11}V)(0.3) = (7764.17 + 1340.03)(0.7) + (8194.12)(0.7) = 8831.18$$

You are given:

a. $1000A_{50} = 200$
b. $1000A_{51} = 210$
c. $v = 0.92$

Let $1000 \cdot {}_1P^{FPT}$ be the first year net premium using the Full Preliminary Term reserve method for a fully discrete whole life policy on (50) with a death 1000. Also let $1000P_{x+1}^{FPT}$ be the net premium in years two and later using the Full Preliminary Term reserve method for a fully discrete whole life policy on (50) with a death 1000.

Calculate $1000P_{x+1}^{FPT} - 1000 \cdot {}_1P^{FPT}$.

Solution:

$${}_1P^{FPT} = Svq_x$$

$$A_{50} = vq_{50} + vp_{50}A_{51} \Longrightarrow 0.2 = (0.92)q_{50} + (0.92)(1-q_{50})(0.21)$$

$$q_{50} = \frac{0.2-(0.92)(0.21)}{(0.92)(1-0.21)} = 0.009356$$

$${}_1P^{FPT} = Svq_x = (1000)(0.92)(0.009356) = 8.61$$

$$1000P_{x+1}^{FPT} = \frac{1000A_{51}}{\ddot{a}_{51}} = \frac{1000(0.21)}{\dfrac{1-0.21}{1-0.92}} = 21.27$$

$$1000P_{x+1}^{FPT} - 1000 \cdot {}_1P^{FPT} = 21.27 - 8.61 = 12.66$$

A whole life insurance policy is issued to (70) and pays a death benefit of 78,000 at the end of the year of death. The policy has level annual premiums for as long as the insured is alive.

You are given:

i. Mortality follows the Standard Ultimate Life Table
ii. $i = 0.05$
iii. The policy pays commissions of 50% for the first year and 5% thereafter.
iv. The per policy expenses is 200.
v. The maintenance expense for the policy is 40 at the beginning of every year including the first year.

a. (5 points) Calculate the net benefit reserve at the end of 10 years.

Solution

$$P = \frac{(78,000)A_{70}}{\ddot{a}_{70}} = \frac{(78,000)(0.42818)}{12.0083} = 2781.25$$

$${}_{10}V^{n} = (78,000)A_{80} - 2781.25\ddot{a}_{80}$$

$$= (78,000)(0.59293) - 2781.25(8.5484) = 22,473.33$$

b. (9 points) The gross premium for this policy is 3200. Calculate the gross premium reserve at the end of 10 years.

Solution:

$${}_{10}V^{g} = PVFB + PVFE - PVFP = (78,000)A_{80} + (0.05)(3200)\ddot{a}_{80} + 40\ddot{a}_{80} - 3200\ddot{a}_{80}$$

$$= (78,000)(0.59293) - (3200 - 160 - 40)(8.5484) = 20,603.34$$

c. (2 points) Calculate the expense premium and the expense reserve at the end of 10 years.

Solution

$$P^e = P^g - P^n = 3200 - 2781.25 = 418.75$$

$${}_{10}V^e = {}_{10}V^g - {}_{10}V^n = 20,603.34 - 22,473.33 = -1869.99$$

d. (4 points) Explain why the expense reserve is negative.

Solution:

Most of the expenses are in the first year. On the other hand, the expense premium is level. At time zero, the present value of future expenses are equal to the present value of future expense premiums. However, after the first year, the present value of future expense premiums are greater the present value of future expenses so the reserve is negative.

Richard buys a whole life policy when he is (70). The policy pays a death benefit of 200,000 at the end of the year of death. The premiums are paid annually as long as Richard is alive.

You are given that mortality follows the Standard Ultimate Mortality Table with interest at 5%.

a. (2 points) The net premium is 7100 to the nearest 100. Calculate the net premium to the nearest 1.

$$P_{70} = \frac{200,000A_{70}}{\ddot{a}_{70}} = \frac{200,000(0.42818)}{12.0083} = 7,131.4$$

b. (3 points) Calculate the net premium reserve at the end of the 10th year.

$${}_{10}V^n = 200,000A_{80} - P\ddot{a}_{80} = 200,000(0.59293) - 7131.4(8.5484) = 57,623.94$$

or

$${}_{10}V^n = 200,000\left(1 - \frac{\ddot{a}_{80}}{\ddot{a}_{70}}\right) = 200,000\left(1 - \frac{8.5484}{12.0083}\right) = 57,625.14$$

The gross premium for this policy is 8308.22. The premium was determined using the equivalence principal.

The reserve basis for the gross premium reserves is:

- Mortality follows the Standard Ultimate Mortality Table
- $i = 0.05$
- Commissions are 65% of premiums in the first year and 8% thereafter
- Issue Expenses are 400 per policy and 1.00 per thousand
- Maintenance expenses are 50 at the beginning of each year including the first year.
- Termination expense of 500 paid at the end of the year of death

c. (4 points) The gross premium reserve at the end of the first year is 284.81. Determine the gross premium reserve after 9 months.

$$_{0.75}V^g = (1-0.75)(_0V^g + P^g - Expenses) + 0.75(_1V^g)$$

$$_0V^g = 0 \text{ because of the equivalence principle}$$

$$_{0.75}V^g = 0.25\left(0 + 8308.22 - 0.65(8308.22) - 400 - 50 - \frac{200{,}000}{1{,}000}\right) + 0.75(284.81) = 778.08$$

d. (4 points) Determine the gross premium reserve at the end of the 10th year.

$$_{10}V^g = PVFB + PVFE - PVFP =$$

$$(200{,}000 + 500)A_{80} - (8308.22 - 0.08(8308.22) - 50)\ddot{a}_{80}$$

$$= 200{,}500(0.59293) - 7593.5624(8.5484) = 53{,}969.66$$

e. (2 points) Determine the expense reserve at the end of the 10th year.

$$_{10}V^e = {}_{10}V^g - {}_{10}V^n = 53{,}969.66 - 57{,}625.14 = -3655.48$$

Jaden who is (25) buys a whole life policy with a death benefit of 75,000 paid at the end of the year of death. The policy has annual gross premiums.

The reserve basis for the gross premium reserves is:

- Mortality follows the Standard Ultimate Mortality Table
- $i = 0.06$ ← Note that this is not $i = 0.05$.
- Expenses are 10% of premium and 40 per policy at the start of each year.
- Termination expense of 250 paid at the end of the year of death

The gross premium reserves for the 9th, 10th, and 11th year are given in the following table:

Time	Gross Premium Reserve
9	1000.00
10	1271.51
11	1558.02

a. (5 points) The gross premium is 300 to the nearest 25. Determine the gross premium to the nearest 1

$$_{10}V = \frac{({}_9V + P - .1P - 40)(1.06) - (75000 + 250)q_{34}}{p_{34}}$$

$$1271.51 = \frac{(960 + 0.9P)(1.06) - (75,250)(0.000372)}{1 - 0.000372}$$

$$.9P = 265.5 ==> P = 295$$

The actual experience during the 10th year of this policy is:

- Mortality is 50% of the Standard Ultimate Mortality Table
- $i = 0.07$
- Expenses are 8% of premium and 50 per policy at the start of the year.
- Termination expense of 275 paid at the end of the year of death

b. (4 points) Determine the total gain for the 10th year of this policy.

$$G = ({}_9V + P - 0.08P - 50)(1.07) = (75,000 + 275)(0.5)q_{34} + {}_{10}V[1-(0.5)q_{34}]$$

$$G = (1000 + 295(.92) - 50)(1.07) - 75,275(0.5)(0.000372) - 1271.51[1-(0.5)(0.000391)]$$

$$G = 21.62$$

Gains by source are determined first for expenses, then for interest, and finally for mortality.

c. (4 points) Determine the gain from expenses.

$$G^e = (1000 + 295(0.92) - 50)(1.06) - 75,275(0.000372) - 1271.51[1-(0.000391)]$$

$$= -4.36$$

d. (4 points) Determine the gain from interest.

$$G^{i\&e} = (1000 + 295(.92) - 50)(1.06) - 75,275(0.000372) - 1271.51[1-(0.000391)]$$

$$G^{i\&e} = 7.86$$

$$G^{i\&e} = G^e + G^i \implies 7.86 = -4.36 + G^i \implies G^i = 12.22$$

e. (4 points) Determine the gain from mortality.

$$G^m = G^{Total} - G^i - G^e$$
$$= 21.62 - 12.22 - (-4.36)$$
$$= 13.76$$

Jacqueline is (44) and purchases an Endowment at Age 65. In other words, this is an endowment that ends in 21 years. The death benefit for this policy is 50,000.

Reserves are calculated using the Full Preliminary Term method with mortality based on the Standard Ultimate Life Table and $i = 0.05$.

Let ${}_tV^{FPT}$ be the Full Preliminary Term reserve at time t on this policy.

a. (3 points) Calculate the net premium for the first year of this policy.

$$ {}_1P^{FPT} = S \cdot v \cdot q_x = 50{,}000(1.05)^{-1}(0.000710) = 33.81 $$

b. (3 points) Calculate ${}_{0.8}V^{FPT}$.

$$ \begin{aligned} {}_{0.8}V &= ({}_0V + {}_1P^{FPT})(1-0.8) + ({}_1V)(0.8) \\ &= (0+33.81)(0.2) + (0)(0.8) \\ &= 6.76 \end{aligned} $$

c. (4 points) Calculate the net premium for year 2 and later for this policy.

$$ P^{FPT}_{45:\overline{20}|} = \frac{50{,}000A_{45:\overline{20}|}}{\ddot{a}_{45:\overline{20}|}} = \frac{50{,}000(A_{45} - {}_{20}E_{45}A_{65} + {}_{20}E_{45})}{a_{45} - {}_{20}E_{45}a_{65}} $$

$$ = \frac{50{,}000(0.15161 - 0.35994 \cdot 0.35477 + 0.35994)}{17.8162 - 0.35994 \cdot 13.5498} = 1{,}483.31 $$

d. (5 points) Calculate ${}_{11}V^{FPT}$.

$$ {}_{11}V^{FPT} = 50{,}000A_{55:\overline{10}|} - 1{,}483.31\ddot{a}_{55:\overline{10}|} $$

$$ = 50{,}000(0.61813) - 1{,}483.31(8.0192) = 19{,}011.54 $$

A life insurance company sells a whole life policy to (80). The death benefit is 100,000 payable at the end of the year of death. Annual premiums are payable for the life of the policy.

You are given the following reserve basis for full preliminary term reserves:

i. $i = 0.05$

ii. Mortality follows the Standard Ultimate Life Table.

a. (3 points) Calculate the first year premium under full preliminary term reserves.

Solution:

$$_{1}P^{FPT} = S \cdot v \cdot q_x = \frac{(100,000)(0.032658)}{1.05} = 3110.29$$

b. (5 points) Calculate the premium for all years after the first under full preliminary term reserves.

Solution:

$$P_{x+1}^{FPT} = \frac{100,000A_{x+1}}{\ddot{a}_{x+1}} = \frac{100,000A_{81}}{\ddot{a}_{81}} = \frac{(100,000)(0.60984)}{8.1934} = 7443.06$$

c. (10 points) Calculate the full preliminary term reserves in the table below:

t	${}_tV^{FPT}$
0	Zero by Definition
0.7	${}_{0.7}V = ({}_0V + {}_1P^{FPT})(1-0.7) + ({}_1V)(0.7) = (0+3110.29)(0.3) + (0)(0.7) = 933.09$
1	Zero by Definition
2	${}_2V = 100,000\left(1-\frac{\ddot{a}_{82}}{\ddot{a}_{81}}\right) = 100,000\left(1-\frac{7.8401}{8.1934}\right) = 4312.01$
2.2	${}_{2.2}V = ({}_2V + P_{x+1}^{FPT})(1-0.2) + ({}_3V)(0.2)$ $= (4312.01+7443.06)(0.8) + (8593.50)(0.2) = 11{,}122.76$
3	${}_3V = 100,000\left(1-\frac{\ddot{a}_{83}}{\ddot{a}_{81}}\right) = 100,000\left(1-\frac{7.4893}{8.1934}\right) = 8593.50$

d. (2 points) Explain why a company would use full preliminary term reserves instead of net premium reserves.

Full Preliminary Term reserves are intended to approximate gross premium reserves. Gross premium reserves are more reflective of the true economic situation that net premium reserves but are more difficult to calculate. FPT reserves maintain the simplicity of the net premium reserves while generating reserves that are closer to economic reality than those of net premium reserves.

A whole life policy is issued to (70) with a death benefit of 25,000 paid at the end of the year of death. Premiums are determined by the equivalence principle and are paid annually for the life of the policy.

The reserve basis is the Standard Ultimate Life Table with interest at 5%.

a. (4 points) Calculate the net premium for this policy.

Solution:

$$PVP = PVB \Longrightarrow P\ddot{a}_{70} = 25,000A_{70} \Longrightarrow P(12.0083) = 25,000(0.42818)$$

$$P = \frac{25,000(0.42818)}{12.0083} = 891.425$$

b. (6 points) Calculate the net premium reserve at the end of the 10th year.

Solution:

$${}_{10}V^{n} = PVFB - PVFP$$

$$= 25,000A_{80} - 891.425\ddot{a}_{80} = 25,000(0.59293) - 891.425(8.5484) = 7202.99$$

A whole life policy is issued to (70) with a death benefit of 25,000 paid at the end of the year of death. The gross premium paid annually for the life of the policy is P. This premium was NOT determined by the equivalence principle.

The reserve basis is the Standard Ultimate Life Table with interest at 6%. (Note that the interest rate is NOT 5%.)

The expenses for this policy are:

- Commissions of 50% of premiums in the first year and 9% thereafter.
- Per policy expenses of 300 in the first year and 50 each year thereafter.
- Claim expense of 350 paid at the end of the year of death.

The gross premium reserve at the end of the 10th year is 6000.00. The gross premium reserve at the end of the 11th year is 6894.60.

Determine P .

Solution:

$$_{11}V^g = \frac{({}_{10}V^g + P_{10} - e_{10} - X_{10}^{BOY})(1+i) - (S_{11} + E_{11})q_{80}}{p_{80}}$$

$$6894.60 = \frac{(6000 + 0.91P - 50)(1.06) - (25,000 + 350)(0.032658)}{(1 - 0.032658)}$$

$$P = \frac{\left[\dfrac{6894.60(1-0.032658) + (25,350)(0.032658)}{1.06} - 5950\right]}{0.91} = 1234$$