Chapters 5 & 6 – Past Test and Quiz Problems

The Chung Life Insurance Company sells life insurance policies to people who are age 70 only.

The Company uses the Standard Ultimate Life Table and 5% interest to calculate all premiums. They also assume that deaths are uniformly distributed between integral ages.

All premiums are calculated using the Equivalence Principle.

All policies are sold to insureds whose death is independent of the death of any other insured.

a. (3 points) The annual net premium for a whole life policy with a death benefit of 100,000 paid at the end of the year of death is 3600 to the nearest 100. Calculate the net premium to the nearest 1.

Solution:

$$PVP = PVB \implies P\ddot{a}_{70} = 100,000A_{70}$$

$$P = \frac{(100,000)(0.42818)}{12.0083} = 3565.70$$

Let L_0^n be the loss at issue random variable based on the net premium.

b. (8 points) For a single policy, calculate the probability that the policy generates a loss.

Solution:

$$L_0^n = 100,000v^{K_{70}+1} - 3565.70\ddot{a}_{\overline{K_{70}+1|}}$$

$$100,000v^{K_{70}+1} - 3565.70\left(\frac{1-v^{K_{70}+1}}{d}\right) = 0 \implies 1.335471691v^{K_{70}+1} = 1 - v^{K_{70}+1}$$

$$v^{K_{70}+1} = 0.428179 \implies K_{70}+1 = \frac{\ln(0.3428179)}{\ln[(1.05)^{-1}]} = 17.38 \implies 17$$

A loss is generated by an early death so probability of a loss is

$${}_{17}q_{70} = 1 - \frac{l_{87}}{l_{70}} = 1 - \frac{53,934.7}{91,082.4} = 0.40785$$

c. (5 points) Calculate the $\sqrt{Var[L_0^n]}$ for this whole life. It is 31,000 to the nearest 1000. Calculate it to the nearest 1.

Solution:

$$Var[L_0^n] = \left(100,000 + \frac{3565.70}{(0.05/1.05)}\right)\left({}^2A_{70} - \left[A_{70}\right]^2\right)$$

$$= (174,879.70)^2\left(0.21467 - \left[0.42818\right]^2\right)$$

$$\sqrt{Var[L_0^n]} = \sqrt{(174,879.70)^2\left(0.21467 - \left[0.42818\right]^2\right)} = 30,956$$

d. (7 points) Chung Life sells 625 whole life policies. Using the normal distribution, calculate the 90% confidence interval for L_0^n.

Solution:

$E[L_0^n] = 0$ due to the equivalence principle

$$CI = 0 \pm 1.645\sqrt{625}(30,956) ==> (-1,273,024;1,273,024)$$

e. (7 points) Calculate the monthly net premium for a 20 year term insurance policy with a death benefit of 1,000,000 paid at the moment of death.

Solution:

$$PVP = PVB ==> 12P\ddot{a}^{(12)}_{70:\overline{20|}} = 1,000,000\bar{A}^{\,1}_{70:\overline{20|}}$$

$$12P\left(\alpha(12)\ddot{a}_{70} - \beta(12) - {}_{20}E_{70}\left[\alpha(12)\ddot{a}_{90} - \beta(12)\right]\right)$$
$$= 1,000,000\left(\frac{i}{\delta}\right)A^{\,1}_{70:\overline{20|}} = 1,000,000\left(\frac{i}{\delta}\right)(A_{70:\overline{20|}} - {}_{20}E_{70})$$

$$12P\left(\alpha(12)\ddot{a}_{70:\overline{20|}} - \beta(12)(1 - {}_{20}E_{70})\right) = 1,000,000\left(\frac{i}{\delta}\right)(A_{70:\overline{20|}} - {}_{20}E_{70})$$
$$12P\left((1.00020)(11.1109) - (0.46651)(1 - 0.17313)\right) = 1,000,000(1.02480)(0.47091 - 0.17313)$$

$$P = \frac{11,000,000(1.02480)(0.47091 - 0.17313)}{12\left((1.00020)(11.1109) - (0.46651)(1 - 0.17313)\right)} = 2370.60$$

f. (2 points) Without doing any calculations, state whether the annual premium of the policy in Part e would be more or less than 12 times the monthly premium. Explain why.

Solution:

The annual premium would be less than 12 times the monthly premium. This is for two reasons. The primary reason is that if premiums are paid annually, the insurance company can earn more interest during the year since they have the premium for the whole year whereas under monthly premiums, the premiums are spread throughout the year. Secondly, in the year of death, the annual premium is paid for the full year since it is paid at the start of the year of death. However, with monthly premiums, only the premiums prior to death in the year of death are collected.

The Arissa Assurance Company sells annuities to people who are age 61 only.

The Company uses the Standard Ultimate Life Table and 5% interest to calculate all premiums.

Jeff, (61), buys two annuities from Arissa.

The first annuity is a whole life annuity due that pays 1000 at the beginning of every year. Let Y_{WL} be the present value random variable for the annuity.

a. (7 points) Calculate the $Var[Y_{WL}]$.

Solution:

$$Var[Y_{WL}] = (1000)^2 \left[\frac{{}^2A_{61} - (A_{61})^2}{d^2} \right] = 441,000,000[0.11644 - (0.30243)^2]$$

$$= 11,014,458$$

The second annuity is a 10 year certain and life annuity due with monthly payments of 500. The first 10 years of payments are guaranteed.

b. (8 points) Using the two factor Woolhouse formula, the actuarial present value of this annuity is 86,300 to the nearest 100. Calculate the actuarial present value to the nearest 1.

Solution:

$$APV = (500)(12)\ddot{a}^{(12)}_{\overline{61:\overline{10|}}} = (500)(12)(\ddot{a}^{(12)}_{\overline{10|}} + {}_{10}E_{61}\ddot{a}^{(12)}_{71})$$

$$= 6000\left(\frac{1-(1.05)^{-10}}{d^{(12)}} + (0.57457)(11.6803 - 11/24) \right)$$

$$= 6000\left(\frac{1-(1.05)^{-10}}{0.04869} + (0.57457)(11.6803 - 11/24) \right) = 86,264$$

(9 points) Molly is (x) and buys a special three year term life annuity due with annual payments. The payment at the beginning of the first year is 50,000. The payment at the beginning of the second year is 30,000. The payment at the beginning of the third year is 10,000. Let $Y_{Special}$ be the present value random variable for the annuity.

You are given that:

- $v = 0.94$
- $q_{x+t} = 0.08 + 0.04t \qquad \text{for } t = 0,1,2,3,4,5$

Calculate the $Var[Y_{Special}]$.

Solution:

Since the payments are not level, we must use first principles.

Cases	Present Value	Probability
Die Year 1	$50,000$	0.08
Die Year 2	$50,000 + 30,000(0.94) = 78,200$	(0.92)(0.12)=0.1104
Live 2 Years	$50{,}000+30{,}000(0.94)+10{,}000(0.94)^2 = 87,036$	$(0.92)(0.88) = 0.8096$

$$E[Y_{Special}] = (50,000)(0.08) + (78,200)(0.1104) + (87,036)(0.8096) = 83,097.63$$

$$E[(Y_{Special})^2] = (50,000)^2(0.08) + (78,200)^2(0.1104) + (87,036)^2(0.8096) = 7,008,057,280$$

$$Var[(Y_{Special})^2] = 7,008,057,280 - (83,097.63)^{22} = 102,841,849$$

Jeff is (72). He has 800,000 and wants to buy an annuity to fund his retirement.

Assume that mortality follows the Standard Ultimate Life Table with interest equal to 5%. Further, assume that deaths are uniformly distributed between integral ages.

a. (4 points) If Jeff decides to buy a whole life annuity due with annual payments, the annual payment will be 70,500 to the nearest 100. Calculate it to the nearest 1.

Solution:

$$800,000 = P\ddot{a}_{72}$$

$$P = \frac{800,000}{11.3468} = 70,504$$

b. (6 points) If Y is the present value random variable for the annuity in a., determine the $\sqrt{Var[Y]}$.

Solution:

$$\sqrt{Var[Y]} = \sqrt{(70,504)^2 \frac{{}^2A_{72} - A_{72}^2}{d^2}} = \sqrt{(70,504)^2 \frac{0.24324 - 0.45968^2}{\left(\frac{0.05}{1.05}\right)^2}} = 264,582.88$$

c. (8 points) Jeff is also considering a certain and life annuity due with annual payments. The payments for first 15 years are guaranteed to be made. Payments after 15 years will continue if Jeff is alive. Determine the annual payment under this annuity.

Solution:

$$800,000 = P\ddot{a}_{\overline{\overline{72:15}|}}$$

$$P = \frac{800,000}{\ddot{a}_{\overline{15}|} + {}_{15}E_{72}\ddot{a}_{87}}$$

$$P = \frac{800,000}{\dfrac{1-(1.05)^{-15}}{\left(\dfrac{0.05}{1.05}\right)} + (1.05)^{-15}\left(\dfrac{53,934.7}{89,082.1}\right)(6.1308)}$$

$$P = 63,070.97$$

d. (4 points) The annual payment under the annuity in c. is less than the annual payment under the annuity in a. Given that this is that case, why would Jeff consider annuity in c.

Solution:

The annuity payments in C are guaranteed for 15 years, so Jeff would receive the first 15 payments even if he dies; whereas in part A, if he dies within the first 15 years the payments will stop. So, if Jeff thinks he may within 15 years he might consider the annuity in C to ensure he gets at least 15 annuity payments.

e. (8 points) Jeff decides he wants to maximize his income over the next twelve years so he buys a 12 year term life annuity due with monthly payments. Determine the amount of the monthly payment.

Solution:

$$800,000 = 12P\ddot{a}^{(12)}_{72:\overline{12}|} \implies P = \frac{800,000}{12\ddot{a}^{(12)}_{72:\overline{12}|}}$$

$$\ddot{a}^{(12)}_{72:\overline{12}|} = \ddot{a}^{(12)}_{72} - {}_{12}E_{72}\ddot{a}^{(12)}_{84} \implies \ddot{a}^{(12)}_{72:\overline{12}|} = \left[\alpha(12)\ddot{a}_{72} - \beta(12)\right] - {}_{12}E_{72}\left[\alpha(12)\ddot{a}_{84} - \beta(12)\right]$$

$$= [(1.0002)(11.3468) - 0.46651] - (1.05)^{-12}\left(\frac{64,506.5}{89,082.1}\right)[(1.0002)(7.1421) - 0.46651]$$

$$= 10.88256 - 2.6923 = 8.190256$$

$$P = \frac{800,000}{12(8.190256)} = 8,139.75$$

(10 points) Bailey is (92) and buys a whole life annuity with non-level annual payments. A payment of 1000 is made at age 92 if Bailey is alive. A payment of 2000 is made at age 93 if Bailey is alive. A payment of 3000 is made at age 94 if Bailey is alive. The payments continue to increase in the same pattern for the rest of Bailey's life.

You are given:

a. $v = 0.9$

b. $q_{92+t} = \dfrac{(t+1)}{3}$ for $t = 0,1,2$ and $q_{92+t} = 1$ for $t > 2$

If Y is the present value random variable for Bailey's annuity, determine the $Var[Y]$.

Solution:

Case	Y	Probability
Die Year 1	1000	1/3=3/9
Die Year 2	1000+2000v=2800	(2/3)(2/3)=4/9
Die Year 3	1000+2000v+3000v^2=5230	(2/3)(1/3)(1)=2/9

$$Var[Y] = E[Y^2] - E[Y]^2$$

$$E[Y] = 1000\left(\frac{3}{9}\right) + 2800\left(\frac{4}{9}\right) + 5230\left(\frac{2}{9}\right) = 2740$$

$$E[Y^2] = 1000^2\left(\frac{3}{9}\right) + 2800^2\left(\frac{4}{9}\right) + 5230^2\left(\frac{2}{9}\right) = 9{,}896{,}200$$

$$Var[Y] = 9{,}896{,}200 - 2740^2 = 2{,}388{,}600$$

(10 points) Ranya is (60) and purchases a 20 year term insurance. The term insurance has a death benefit of 100,000 paid at the end of the year of death. Premiums for the policy are paid monthly for 5 years.

You are given:

a. Mortality follows the Standard Ultimate Life Table with $i = 5\%$.
b. Monthly annuities are determined using the two term Woolhouse formula.

Calculate the monthly premium for this policy.

Solution:

$$12P\ddot{a}^{(12)}_{60:\overline{5}|} = 100,000A^{\,1}_{60:\overline{12}|} \text{ ==> } P = \frac{100,000A^{\,1}_{60:\overline{12}|}}{12\ddot{a}^{(12)}_{60:\overline{5}|}}$$

$$A^{\,1}_{60:\overline{12}|} = A_{60:\overline{20}|} - {}_{20}E_{60} = 0.41040 - 0.29508 = 0.11532$$

$$\ddot{a}^{(12)}_{60:\overline{5}|} = \ddot{a}^{(12)}_{60} - {}_{5}E_{60}\ddot{a}^{(12)}_{65}$$

$$\ddot{a}^{(12)}_{60} = \ddot{a}_{60} - \frac{12-1}{12(2)} = 14.9041 - \frac{11}{24} = 14.44576667$$

$$\ddot{a}^{(12)}_{65} = \ddot{a}_{65} - \frac{12-1}{12(2)} = 13.5498 - \frac{11}{24} = 13.09146667$$

$$\ddot{a}^{(12)}_{60:\overline{5}|} = (14.44576667) - (0.76687)(13.09146667) = 4.406313625$$

$$P = \frac{100,000(0.11532)}{12(4.406313625)} = 218.10$$

(8 points) Taylen is (70) and buys a whole life insurance policy with a death benefit of 50,000 to be paid at the end of the year of death. She will pay premiums annually for life.

You are given:

a. $v = 0.95$

b. $q_{70} = 0.010$

c. $q_{71} = 0.012$

d. $\ddot{a}_{71} = 11.5$

Calculate the premium for Taylen's policy.

Solution:

$$P\ddot{a}_{70} = 50,000A_{70} ==> P = \frac{50,000A_{70}}{\ddot{a}_{70}}$$

$$\ddot{a}_{70} = 1 + v \cdot p_{70} \cdot \ddot{a}_{71} = 1 + (0.95)(1 - .01)(11.5) = 11.81575$$

$$A_{70} = 1 - d \cdot \ddot{a}_{70} = 1 - (1 - 0.95)(11.81575) = 0.4092125$$

$$P = \frac{50,000(0.4092125)}{11.81575} = 1,731.64$$

Beau is (45) and buys a whole life policy with a death benefit of 70,000 paid at the end of the year of death. Gross premiums are paid annually.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

The expenses for Beau's policy are:

i. Commissions of 60% of gross premium in the first year and 8% of gross premium in all years after the first;

ii. Issue expense of 74.

iii. Maintenance expense of 34 at the beginning of all years including the first year.

iv. A claim expense of 750 paid at the end of the year of death.

a. (8 points) The gross premium based on the equivalence principle is 720 to the nearest 10. Calculate the gross premium based on 1.

Solution:

$$P\ddot{a}_{45} = 70,000A_{45} + 0.52P + 0.08P\ddot{a}_{45} + 74 + 34\ddot{a}_{45} + 750A_{45}$$

$$P = \frac{70,750A_{45} + 74 + 34\ddot{a}_{45}}{0.92a_{45} - 0.52} = \frac{(70,750)(0.15161) + 74 + (34)(17.8162)}{(0.92)(17.8162) - 0.52} = 718.68$$

b. (6 points) If the gross premium was 750, determine the expected gain or loss at issue of the policy. Be sure to state whether it is a gain or loss.

Solution:

$$L_0^g = 70,750A_{45} + 0.52(750) + 0.08(750)\ddot{a}_{45} + 74 + 34\ddot{a}_{45} - 750\ddot{a}_{45}$$

$$= (70,750)(0.15161) + (0.52)(750) + (0.08)(750)(17.8162)$$
$$+ 74 + (34)(17.8162) - (750)(17.8162)$$

$$= -497.02$$

Gain of $497.02

c. (8 points) Let L_0^G be the loss at issue random variable based on a gross premium of 750. It is possible to write an expression for L_0^G as $Av^{K_{45}+1} + B\ddot{a}_{\overline{K_{45}+1|}} + C$. Determine $A,\ B,$ and $C.$

Solution:

$$L_0^G = 70,750v^{k_{45}+1} + (0.52)(750) + (0.08)(750)\ddot{a}_{\overline{k_{45}+1|}} + 74 + 34\ddot{a}_{\overline{k_{45}+1|}} - 750\ddot{a}_{\overline{k_{45}+1|}}$$

$$L_0^G = 70,750v^{k_{45}+1} - 656\ddot{a}_{\overline{k_{45}+1|}} + 464$$

$$A = 70,750$$
$$B = -656$$
$$C = 464$$

d. (8 points) Determine the $\sqrt{Var[L_0^G]}.$

Solution:

$$Var[L_0^G] = Var\left[70,750v^{k_{45}+1} - 656\ddot{a}_{\overline{k_{45}+1|}} + 464\right] = Var\left[70,750v^{k_{45}+1} - 656\left(\frac{1-v^{k_{45}+1}}{d}\right)\right]$$

$$= Var\left[70,750v^{k_{45}+1} + \frac{656}{d}v^{k_{45}+1}\right] = \left(70,750 + \frac{656}{d}\right)^2 Var[v^{k_{45}+1}]$$

$$= \left(70,750 + \frac{656}{d}\right)^2 [{}^2A_{45} - (A_{45})^2] = 831,915,156.62$$

$$\sqrt{Var[L_0^G]} = 9,121.14$$

e. Without doing any additional calculations, state whether the $Var[L_0^G]$ would increase, decrease or remain the same, if each of the following occurred without any other changes to the policy assumptions or premiums?

   i. (2 points) The gross premium was increased.

      Solution:

      Increase

   ii. (2 points) The claims expense was decreased.

      Solution:

      Decrease

   iii. (2 points) The first year commission was increased to 65%

      Solution:

      remain the same

(6 points) Brett is (35) and is receiving a whole life annuity due with annual payments. The payments at the beginning of the first 10 years are 10,000. The payments during the second 10 years are 20,000. After 20 years, the payments are 5000 for as long as Brett is alive.

You are given that mortality follows the Standard Ultimate Life Table with interest equal to 5%.

Determine the actuarial present value of Brett's annuity.

Solution:

$$APV = 10{,}000\ddot{a}_{35} + 10{,}000\,{}_{10}E_{35}\ddot{a}_{45} - 15{,}000\,{}_{20}E_{35}\ddot{a}_{55}$$

$$= (10{,}000)(18.9728) + (10{,}000)(0.61069)(17.8162) - (15{,}000)(0.37041)(16.0599)$$

$$= 209{,}298.54$$

(10 points) Maddie is (50) and purchases a 20 year endowment insurance. The endowment insurance has a death benefit of 50,000 paid at the end of the year of death. Premiums for the policy are paid monthly for 5 years.

You are given:

a. Mortality follows the Standard Ultimate Life Table with $i = 5\%$.

b. Deaths are uniformly distributed between integral ages.

Calculate the monthly premium for this policy.

Solution:

$$50,000A_{50:\overline{20}|} = 12P\ddot{a}^{(12)}_{50:\overline{5}|} ==> P = \frac{50,000A_{50:\overline{20}|}}{12\ddot{a}^{(12)}_{50:\overline{5}|}}$$

$$\ddot{a}^{(12)}_{50:\overline{5}|} = \ddot{a}^{(12)}_{50} - {}_5E_{50}\ddot{a}^{(12)}_{55} = \left[\alpha(12)\ddot{a}_{50} - \beta(12)\right] - {}_5E_{50}\left[\alpha(12)\ddot{a}_{55} - \beta(12)\right]$$

$$[(1.0002)(17.0245) - 0.46651] - 0.77772[(1.0002)(16.0599) - 0.46651] = 4.431605608$$

$$P = \frac{50,000A_{50:\overline{20}|}}{12\ddot{a}^{(12)}_{50:\overline{5}|}} = \frac{(50,000)(0.38844)}{(12)(4.431605608)} = 365.22$$

(8 points) Lorenzo is (71) and buys a whole life insurance policy with a death benefit of 50,000 that will be paid at the end of the year of death. He will pay premiums annually for life.

You are given:

a. $v = 0.94$

b. $q_{70} = 0.010$

c. $q_{71} = 0.012$

d. $\ddot{a}_{70} = 11$

Calculate the premium for Lorenzo's policy.

Solutions:

$$P\ddot{a}_{71} = 50,000A_{71}$$

$$\ddot{a}_{70} = 1 + v \cdot p_{70} \cdot \ddot{a}_{71} \Rightarrow \ddot{a}_{71} = \frac{a_{70} - 1}{v \cdot p_{70}} = 10.74575543$$

$$A_{71} = 1 - d \cdot \ddot{a}_{71} = 1 - (1 - 0.94)(10.74575573) = 0.355254674$$

$$P = \frac{50,000(0.355254674)}{10.74575543} = 1,653$$

Amir is (55) and buys a whole life policy with a death benefit of 100,000 paid at the end of the year of death. Gross premiums are paid annually.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

The expenses for Amir's policy are:

i. Commissions of 50% of gross premium in the first year and 12% of gross premium in all years after the first;

ii. Issue Expense of 78;

iii. Maintenance expense of 48 at the beginning of all years including the first year; and

iv. A claim expense of 570 paid at the end of the year of death.

a. (8 points) The gross premium based on the equivalence principle is 1780 to the nearest 10. Calculate the gross premium to the nearest 1.

Solution:

$$P\ddot{a}_{55} = 100,570A_{55} + 0.38P + 0.12P\ddot{a}_{55} + 78 + 48\ddot{a}_{55}$$

$$P = \frac{100,570A_{55} + 78 + 48\ddot{a}_{55}}{0.88a_{55} - 0.38} = \frac{24506.962}{13.752712} = 1,782$$

b. (6 points) If the gross premium was 1800, determine the expected gain or loss at issue of the policy. State whether it is a gain or a loss.

Solution:

$$L_0^g = 100,570A_{55} + 0.38(1800) + 48\ddot{a}_{55} + 78 + 0.88(1800)\ddot{a}_{55}$$

$$= 25,190.962 - 25,438.8816 = -247.92$$

Gain of 247.92

c. (8 points) Let L_0^G be the loss at issue random variable based on a gross premium of 1800. It is possible to write an expression for L_0^G as $Av^{K_{55}+1} + B\ddot{a}_{\overline{K_{55}+1}|} + C$. Determine $A,\ B,$ and $C.$

Solution:

$$L_0^G = 100,570v^{k_{55}+1} + 0.38(1800) + 0.12(1800)\ddot{a}_{\overline{k_{55}+1}|} + 78 + 48\ddot{a}_{\overline{k_{55}+1}|} - (1800)\ddot{a}_{\overline{k_{55}+1}|}$$

$$L_0^G = 100,570v^{k_{55}+1} - 1532\ddot{a}_{\overline{k_{55}+1}|} + 762$$

$$A = 100,570$$
$$B = -1,536$$
$$C = 762$$

d. (8 points) Determine the $\sqrt{Var[L_0^G]}$.

Solution:

$$Var[L_0^G] = Var\left[100,570v^{k_{55}+1} - 1536\ddot{a}_{\overline{k_{55}+1}|} + 762\right]$$

$$= Var\left[100,570v^{k_{55}+1} - 1536\left(\frac{1-v^{k_{55}+1}}{d}\right)\right]$$

$$= Var\left[100,570v^{k_{55}+1} + \frac{1536}{d}v^{k_{55}+1}\right] = \left(100,570 + \frac{1536}{d}\right)^2 Var\left[v^{k_{55}+1}\right]$$

$$= \left(100,570 + \frac{1536}{d}\right)^2 ({}^2A_{55} - A_{55})^2 = 343,894,922.8$$

$$\sqrt{Var[L_0^G]} = 18,544.4$$

e. Without doing any additional calculations, state whether the $Var[L_0^G]$ would increase, decrease or remain the same, if each of the following occurred without any other changes to the policy assumptions or premiums?

i. (2 points) The gross premium was increased.

Solution:

Increase

ii. (2 points) The claims expense was decreased.

Solution:

Decrease

iii. (2 points) The first year commission was increased to 55%

Solution:

Remain the same

Jeff is (68). He has 1,200,000 and wants to purchase an annuity.

Assume that mortality follows the Standard Ultimate Life Table with interest equal to 5%. Use the two term Woolhouse formula to calculate the monthly annuities.

a. (4 points) If Jeff decides to buy a whole life annuity due with annual payments, the annual payment will be 95,000 to the nearest 1000. Calculate it to the nearest 1.

Solution:

$$1,200,000 = P\ddot{a}_{68}$$

$$P = \frac{1,200,000}{12.6456} = 94,895$$

b. (6 points) If Y is the present value random variable for the annuity in a., determine the $\sqrt{Var[Y]}$.

Solution:

$$\sqrt{Var[Y]} = \sqrt{94,845^2 \frac{{}^2A_{68} - A_{68}^2}{d^2}} = \sqrt{94,845^2 \frac{0.03035129}{\left(\frac{0.05}{1.05}\right)^2}} = 347,177.21$$

c. (8 points) Jeff is also considering a certain and life annuity due with annual payments. The payments for first 15 years are guaranteed to be made. Payments after 15 years will continue if Jeff is alive. Determine the annual payment under this annuity.

Solution:

$$1,200,000 = P\ddot{a}_{\overline{68:\overline{15|}}} \Longrightarrow P = \frac{1,200,000}{\ddot{a}_{\overline{15|}} + {}_{15}E_{68}\ddot{a}_{83}}$$

$$P = \frac{1,200,000}{\dfrac{1-(1.05)^{-15}}{\left(\dfrac{0.05}{1.05}\right)} + (0.52781)(0.66217)(7.4873)} = 88,717.57$$

d. (8 points) Jeff decides he wants to purchase a deferred annuity due. The deferred annuity will beginning making monthly payments at age 80. Determine the amount of the monthly payment.

Solution:

$$1,200,000 = 12P \cdot {}_{12|}\ddot{a}_{68}^{(12)} = 12P\left({}_{12}E_{68}\ddot{a}_{80}^{(12)}\right)$$

$$P = \frac{1,200,000}{12\left(\dfrac{l_{80}}{l_{68}}v^{12}\right)\left(a_{80} - \dfrac{11}{24}\right)} = \frac{1,200,000}{44.11677} = 27,200.54$$

e. (4 points) From Jeff's perspective, state one possible disadvantage of a deferred annuity and one possible advantage of a deferred annuity.

Solution:

A disadvantage is that you might die before you begin to receive the payments

An advantage is that premiums are less expensive

(10 points) Mallory is (92) and buys a whole life annuity with non-level annual payments. A payment of 2000 is made at age 92 if Mallory is alive. A payment of 4000 is made at age 93 if Mallory is alive. A payment of 6000 is made at age 94 if Mallory is alive. The payments continue to increase in the same pattern for the rest of Mallory's life.

You are given:

b. $v = 0.9$

c. $q_{92} = 0.3$
$q_{93} = 0.7$
$q_{94} = 1$

If Y is the present value random variable for Bailey's annuity, determine the $Var[Y]$.

Solution:

Case	Y	Probability
Die Year 1	2000	0.3
Die Year 2	2000+4000v=5600	(1-0.3)(0.7)=0.49
Die Year 3	2000+4000v+6000v^2=10,460	(1-0.3)(1-0.7)(1)=0.21

$$Var[Y] = E[Y^2] - E[Y]^2$$

$$E[Y] = 2000(0.3) + 5600(0.49) + 10460(0.21) = 5540.60$$

$$E[Y^2] = 2000^2(0.3) + 5600^2(0.49) + 10460^2(0.21) = 39,542,836$$

$$Var[Y] = 39,542,836 - (5540.60)^2 = 8,844,587.64$$

(6 points) Michael (45) buys a whole life annuity due with annual payments. The payments for the first 10 years are 20,000. The payments for the next 10 years are 10,000. The payments after 20 years are 15,000 for as long as Michael is alive.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

Calculate the actuarial present value of the annuity.

Solution:

$$APV = 20,000\ddot{a}_{45} - 10,000\,{}_{10}E_{45}\ddot{a}_{55} + 5,000\,{}_{20}E_{45}\ddot{a}_{65}$$

$$APV = 20,000(17.8162) - 10,000(0.60655)(16.0599) + 5,000(0.35994)(13.5478)$$

$$APV = 283,298.2516$$

The Lewis Life Insurance Company sells a whole life insurance policy to (75). The policy has a death benefit of 20,000 paid at the end of the year of death. The premiums are paid annually for the life of the policy.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

The policy has the following expenses. All expenses occur at the beginning of the policy year.

- Commissions of 50% of premium the first year and 8% of premiums in year 2 and later.
- Issue expenses of 500 per policy in the first year only.
- Maintenance expenses of 40 per policy in all years.
- Termination expense of 1000 per policy paid at the end of the year of death.

a. (6 points) Calculate the annual gross premium using the equivalence principle.

Solution:

$$PVP = PVB + PVE$$

$$P\ddot{a}_{75} = 21,000A_{75} + 500 + 40\ddot{a}_{75} + 0.42P + 0.08P\ddot{a}_{75}$$

$$P = \frac{(21,000)(0.50868) + 500 + 40(10.3177)}{0.92(10.3178) - 0.42} = 1278.05$$

Lewis decides to set the annual gross premium equal to 1325. (Note this is not the gross premium determined part a.) The loss at issue random variable L_0^g is calculated using the gross premium of 1325. L_0^g can be written in the form of $Av^{K_{75}+1} + B + C\ddot{a}_{\overline{K_{75}+1}|}$.

b. (8 points) Determine $A, B,$ and C .

Solution:

$$L_0^g = 21{,}000v^{K_{75}+1} + 500 + (0.42)(1325) + (40 - (0.92)(1325))\ddot{a}_{\overline{K_{75}+1}|}$$

$$= 21{,}000v^{K_{75}+1} + 1056.50 - (1179)\ddot{a}_{\overline{K_{75}+1}|}$$

$$==> A = 21{,}000; B = 1056.50; C = -1179$$

c. (5 points) Calculate $E[L_0^g]$.

Solution:

$$E[L_0^g] = E[21{,}000v^{K_{75}+1} + 1056.501 - (1179)\ddot{a}_{\overline{K_{75}+1}|}]$$

$$= 21{,}000A_{75} + 1056.50 - 1179\ddot{a}_{75}$$

$$= 21{,}000(0.50868) + 1056.50 - 1179(10.3178) = -425.91$$

d. (8 points) Calculate $\sqrt{Var[L_0^g]}$

Solution:

$$Var[L_0^g] = Var[21,000v^{K_{75}+1} + 1056.501 - (1179)\ddot{a}_{\overline{K_{75}+1|}}]$$

$$= Var\left[21,000v^{K_{75}+1} - (1179)\left(\frac{1-v^{K_{75}+1}}{d}\right)\right]$$

$$= Var\left[\left(21,000 + \frac{1179}{0.05/1.05}\right)v^{K_{75}+1}\right] = \left(21,000 + \frac{1179}{0.05/1.05}\right)^2 Var\left[v^{K_{75}+1}\right]$$

$$= \left(21,000 + \frac{1179}{0.05/1.05}\right)^2 \left({}^2A_{75} - (A_{75})^2\right)$$

$$\sqrt{Var[L_0^g]} = \left(21,000 + \frac{1179}{0.05/1.05}\right)\sqrt{0.29079 - (0.50868)^2} = 8190.05$$

(8 points) A special whole life insurance policy to (65) pays the death benefit at the moment of death. The death benefit is 100,000 for the first 10 years of the policy. The death benefit is 250,000 for death during the second 10 years of the policy. The death benefit is 50,000 for death after 20 years.

The annual net premium for this policy is paid for 10 years and determined using the Equivalence Principle.

You are given that mortality follows the Standard Ultimate Life Table with $i = 5\%$. Deaths are assumed to be uniformly distributed between integral ages.

Determine the annual net premium.

Solution:

$$PVP = PVB$$

$$P\ddot{a}_{65:\overline{10}|} = 100,000\bar{A}_{65} + 150,000\,{}_{10}E_{65}\bar{A}_{75} - 200,000\,{}_{20}E_{65}\bar{A}_{85}$$

$$P = \left(\frac{100,000(0.35477) + 150,000(0.55305)(0.50868) - 200,000(0.24381)(0.67622)}{7.8435}\right)(1.02480)$$

$$= 5840.58$$

(9 points) You are given:

a. $\ddot{a}_{50} = 11$

b. $i = 0.05$

c. $q_{50} = 0.01$

d. Deaths are uniformly distributed between integral ages.

Let P be the net annual premium for a whole life policy on **(51)** with a death benefit of 30,000 paid at the moment of death.

Determine P.

Solution:

$$PVP = PVB$$

$$P\ddot{a}_{51} = 30,000\bar{A}_{51}$$

$$\ddot{a}_{50} = 1 + vp_{50}\ddot{a}_{51} \Longrightarrow \ddot{a}_{51} = \frac{\ddot{a}_{50} - 1}{vp_{50}} = \frac{11-1}{(1.05)^{-1}(1-0.01)} = 10.6060606$$

$$A_{51} = 1 - d\ddot{a}_{51} = 1 - (0.05/1.05)(10.6060606) = 0.4949495$$

$$P = \frac{(30,000)(1.02480)(0.4949495)}{10.6060606} = 1434.72$$

Megan is (45). She purchases a whole life policy with death benefit of 250,000 payable at the moment of death.

You are given that mortality follows the Standard Ultimate Life Table and $i = 0.05$. You are also given that deaths are uniformly distributed between integral ages.

a. (4 points) The actuarial present value of Megan's death benefit is 38,800 to the nearest 100. Calculate it to the nearest 1.

Solution:

$$APV = 250,000\bar{A}_{45} = (250,000)\left(\frac{i}{\delta}\right)A_{45} = (250,000)\left(\frac{0.05}{\ln(1.05)}\right)(0.15161) = 38,842$$

Megan wants to explore different premium payment patterns. All premiums are calculated using the equivalence principle.

b. (4 points) Calculate the net annual premium if premiums are paid annually during her lifetime.

Solution:

$$PVP = PVB ==> P\ddot{a}_{45} = 38,842$$

$$P = \frac{38,842}{17.8162} = 2180.15$$

c. (7 points) Calculate the net annual premium if premiums are only payable for 15 years.

Solution:

$$PVP = PVB ==> P\ddot{a}_{45:\overline{15|}} = 38,842$$

$$P = \frac{38,842}{\ddot{a}_{45} - {}_{15}E_{45} \cdot \ddot{a}_{60}} = \frac{38,842}{17.8162 - (1.05)^{-15}\left(\frac{96,634.1}{99,033.9}\right)(14.9041)} = 3589.57$$

d. (7 points) Calculate the net quarterly premium if premiums are payable for life.

Solution:

$$PVP = PVB ==> 4P\ddot{a}_{45}^{(4)} = 38,842$$

$$P = \frac{38,842}{4P\ddot{a}_{45}^{(4)}} = \frac{38,842}{4\{\alpha(4)\ddot{a}_{45} - \beta(4)\}} = \frac{38,842}{4\{(1.00019)(17.8162) - 0.38272\}} = 556.89$$

e. (4 points) List two reasons that the annual premium in part b is less than four times the quarterly premium in part d.

Solution:

One reason is the time value of money. The annual premium is paid at the beginning of the year and therefore can earn interest during the entire year. The quarter premiums are paid at the start of each quarter. The premium paid at the beginning of the first quarter earns interest for the whole year. However the premium paid at the start of the second quarter only earns ¾ of a year of interest. The premium paid at the start of the third quarter only earns interest for ½ year. The last quarterly premium only earns interest for ¼ year. Therefore, the quarterly premiums earn less interest so they must be larger.

In the year of death, the entire annual premium will be collected but 4 quarterly premiums may not be collected as premiums are not collected after the death of the insured. For example, if the insured dies during the first quarter the entire annual premium would be collected but if the premiums were quarterly, only one quarterly premium would be collected.

f. (7 points) Megan decides that she wants to pay annual premiums for 30 years. The premiums will not be level. The premiums for the first 10 years will be P . The premiums during the second 10 years will be $2P$. The premiums during the final 10 years will be $3P$.

Determine P .

Solution:

$$PVP = PVB$$

$$P(\ddot{a}_{45} + {}_{10}E_{45} \cdot \ddot{a}_{55} + {}_{20}E_{45} \cdot \ddot{a}_{65} - 3{}_{30}E_{45} \cdot \ddot{a}_{75}) = 38,842$$

$$P = \frac{38,842}{17.8162 + (0.60655)(16.0599) + (0.35994)(13.5498) - (3)(0.35994)(0.55305)(10.3178)}$$

$$= 1478.42$$

Kaitlyn who is (70) buys a whole life insurance policy with a death benefit of 500,000 which is payable at the end of the year of death.

You are given that mortality follows the Standard Ultimate Life Table and $i = 0.05$.

The expenses associated with Kaitlyn's policy are:

i. Commissions of 55% of premium in year 1 and 8% of premium thereafter;

ii. Issue Expense of 300 per policy in the first year only;

iii. Maintenance expenses of 40 per policy in all years including the first;

iv. Termination expense of 1000 paid at the end of the year of death.

a. (6 points) The gross premium based on the equivalence principle is 20,350 to the nearest 10. Calculate it to the nearest 0.01.

Solution:

$$PVP = PVB + PVE$$

$$P\ddot{a}_{70} = 500,000A_{70} + 0.47P + 0.08P\ddot{a}_{70} + 300 + 40\ddot{a}_{70} + 1000A_{70}$$

$$P = \frac{501,000A_{70} + 300 + 40\ddot{a}_{70}}{0.92\ddot{a}_{70} - 0.47} = \frac{(501,000)(0.42818) + 300 + 40(12.0083)}{0.92(12.0083) - 0.47} = 20,354.12$$

b. (7 points) The loss at issue random variable is L_0^g which can be expressed as $Av^{K_{70}+1}+B+C\ddot{a}_{\overline{K_{70}+1}|}$. Determine $A,\ B,$ and C.

Solution:

$$L_0^g = 500,000v^{K_{70}+1} + (0.47)(20,354.12) + 300 + 1000v^{K_{70}+1} + [40 - 0.92(20,354.12)]a_{\overline{K_{70}+1}|}$$

$$= 501,000v^{K_{70}+1} + 9866.4364 - 18,685.7904a_{\overline{K_{70}+1}|}$$

$$A = 501,000$$
$$B = 9866.4364$$
$$C = -18,685.7904$$

c. (7 points) Calculate the $\sqrt{Var[L_0^g]}$.

Solution:

$$Var[L_0^g] = Var[501,000v^{K_{70}+1} + 9866.4364 - 18,685.7904a_{\overline{K_{70}+1}|}]$$

$$= Var\left[501,000v^{K_{70}+1} + 9866.4364 - 18,685.7904\left(\frac{1-v^{K_{70}+1}}{d}\right)\right]$$

$$= Var\left[\left(501,000 + \frac{18,685.7904}{d}\right)v^{K_{70}+1} + 9866.4364 - \frac{18,685.7904}{d}\right]$$

$$Var\left[\left(501,000 + \frac{18,685.7904}{d}\right)v^{K_{70}+1}\right] = \left(501,000 + \frac{18,685.7904}{d}\right)^2 Var\left[v^{K_{70}+1}\right]$$

$$\left(501,000 + \frac{18,685.7904}{d}\right)^2 \left({}^2A_{70} - (A_{70})^2\right)$$

$$\sqrt{Var[L_0^g]} = \left(501,000 + \frac{18,685.7904}{(0.05/1.05)}\right)\sqrt{0.21467 - (0.42818)^2} = 158,139.37$$

Ian who is (90) buys a special 3 year life annuity due with non-level payments. The payment in the first year is 50,000. The payment in the second year is 40,000. The payment in the third year is 20,000.

You are given that $v = 0.94$. You are given that $q_{89+t} = 0.1t \quad \text{for } t = 1,\ 2,\ 3,\ 4,\ and\ 5$.

Let Y be the present value random variable for this annuity.

a. (5 points) The $E[Y] = 97,000$ to the nearest 1000. Calculate the $E[Y]$ to the nearest 1.

Solution:

Case	Y	**Probability**
Die Year 1	50,000	0.10
Die Year 2	50,000+40,000v = 87,600	(0.9)(0.2) = 0.18
Live 2 Years	50,000+40,000v+20,000v^2 = 105,272	(0.9)(0.8) = 0.72

$$E[Y] = (50,000)(0.1) + (87,600)(0.18) + (105,272)(0.72) = 96,563.84$$

b. (7 points) Calculate the $Var[Y]$.

Solution:

$$Var[Y] = E[Y^2] - (E[Y])^2$$

$$E[Y] = 96,563.84$$

$$E[Y] = (50,000)^2(0.1) + (87,600)^2(0.18) + (105,272)^2(0.72) = 9,610,456,468$$

$$Var[Y] = 9,610,456,468 - (96,563.84)^2 = 285,850,372$$

c. (4 points) If the payments were reversed (year 1 of 20,000, year 2 of 40,000 and year 3 of 50,000), explain why the $E[Y]$ is lower and the $Var[Y]$ is higher.

Solution:

The payment of 50,000 is now only made for those who are alive at the end of 2 years so it gets discounted for both two years of interest and for the probability that Ian is alive at that time. The second payment has the exact same present value. The payment of 20,000 now being paid at time 0 has a higher present value but since it is a smaller amount (substantially less than 50,000), it will not increase in value as much the 50,000 payment will decrease in value.

The variance increases because the largest payment is no longer certain and this adds variability to the present value whereas before the largest payment was certain to be made so it added no variance.

(7 points) Let Y be the present value random variable for a whole life annuity due with annual payments of 100 issued to (84).

You are given:

i. $\ddot{a}_{85} = 8$

ii. ${}^2A_{84} = 0.4$

iii. $i = 0.05$

iv. $q_{84} = 0.040$

v. $q_{85} = 0.045$

Calculate the $Var[Y]$.

Solution:

$$Var[Y] = (100)^2 \left(\frac{{}^2A_{84} - \{A_{84}\}^2}{d^2} \right)$$

$$\ddot{a}_{84} = 1 + vp_{84}\ddot{a}_{85} = 1 + (1.05)^{-1}(1-0.04)(8) = 8.314285714$$

$$A_{84} = 1 - d\ddot{a}_{84} = 1 - \left(\frac{0.05}{1.05} \right)(8.314285714) = 0.604081633$$

$$Var[Y] = (100)^2 \left(\frac{0.4 - \{0.60401633\}^2}{(0.05/1.05)^2} \right) = 154,727$$

Jeff retires from Purdue at age 65. He is entitled to a pension payout of 100,000 annually at the beginning of the each year. The standard payout is based on a 10 year certain and life thereafter payout.

You are given that mortality follows the Standard Ultimate Life Table and $i = 0.05$.

a. (7 points) Calculate the Actuarial Present Value of Jeff's pension payments.

Solution:

$$APV = 100,000\ddot{a}_{\overline{65:\overline{10|}}} = (100,000)(\ddot{a}_{\overline{10|}} + {}_{10}E_{65} \cdot \ddot{a}_{75})$$

$$= (100,000)\left(\frac{1-(1.05)^{-10}}{(0.05/1.05)} + (0.55305)(10.3178)\right) = 1,381,408.10$$

Jeff has the option to receive a 20 year deferred whole life annuity due with no certain period instead of the 10 year certain and life payout in part a. This annuity would begin making level payments to Jeff at age 85. If Jeff died prior to age 85, no benefits would be paid. The payment under the deferred annuity option will be actuarially equivalent (which means it will have the same actuarial present value).

b. (7 points) Calculate the payment if Jeff chooses this option.

Solution:

$$1,381,408.10 = P \cdot {}_{20|}\ddot{a}_{65} = P \cdot {}_{20}E_{65} \cdot \ddot{a}_{85}$$

$$P = \frac{1,381,408.10}{(0.24381)(6.7993)} = 833,309.40$$

c. (4 points) List two advantages and one disadvantage to the option in part b.

Solution

Advantages

- Jeff receives a much larger payment if he survives 20 years.
- A deferred annuity such as this provides longevity insurance as it assures Jeff that he will not outlive his savings

Disadvantage

- If Jeff dies in the next 20 years, he will not receive any payments.

(10 points) Andrew is (45) and purchases a whole life policy with a death benefit of 200,000 paid at the moment of death. Premiums are paid annually during Andrew's lifetime on the policy.

You are given:

i. Mortality follows the Standard Ultimate Life Table.

ii. Deaths are uniformly distributed between integral ages.

iii. $i = 0.05$

Calculate the premium for Andrew's policy.

Solution:

$$PVP = PVB \implies P\ddot{a}_{45} = 200,000\bar{A}_{45} = (200,000)(i/\delta)A_{45}$$

$$\implies P(17.8162) = 200,000(1.02480)(0.15161)$$

$$P = 1744.14$$

(10 points) Trevor is (60) and is receiving a life annuity with payments at the beginning of each year. The payments are not level and are as follows:

i. Payments of 10,000 for the first 10 payments;

ii. Payments of 25,000 for the next 10 payments; and

iii. Payments of 5000 for any payments after the first 20.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

Calculate the Actuarial Present Value of this annuity.

Solution:

$$APV = 10,000\ddot{a}_{60} + 15,000 \cdot {}_{10}E_{60} \cdot \ddot{a}_{70} - 20,000 \cdot {}_{20}E_{60} \cdot \ddot{a}_{80}$$

$$= (10,000)(14.9041) + (15,000)(0.57864)(12.0083) - (20,000)(0.29508)(8.5484)$$

$$= 202,819.00$$

(10 points) A 3 year temporary life annuity due to (x) makes payment a payment of 300 at the beginning of year one, 400 at the beginning of year 2, and 200 at the beginning of year 3.

Let Y be the present value for this annuity.

You are given:

a. $q_x = 0.2$

b. $q_{x+1} = 0.3$

c. $q_{x+2} = 0.4$

d. $v = 0.92$

Calculate the $Var(Y)$.

Solution:

Case	**Present Value**	**Probability**
Die Year 1	300	0.2
Die Year 2	300 + 400v = 668	(0.8)(0.3) = 0.24
Live 2 Years	300 + 400v +200v^2 = 837.28	(0.8)(0.7) = 0.56

$$E[Y] = 300(0.2) + 668(0.24) + 837.28(0.56) = 689.1968$$

$$E[Y^2] = (300)^2(0.2) + (668)^2(0.24) + (837.28)^2(0.56) = 517,674.9271$$

$$Var[Y] = 517,674.9271 - (689.1968)^2 = 42,682.70$$

Diego is (65) and is the recipient of an annuity that pays 5000 at the beginning of each year for the rest of his life.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

Let Y be the present value random variable for Diego's annuity.

a. (1 point) Calculate the $E[Y]$.

   Solution:

   $$E[Y] = 5000\ddot{a}_{65} = (5000)(13.5498) = 67,749$$

b. (7 points) Calculate the $Var[Y]$.

   Solution:

   $$Var[Y] = (5000)^2 \left[\frac{{}^2A_{65} - (A_{65})^2}{d^2} \right] = (5000)^2 \left[\frac{0.15420 - (0.35477)^2}{(0.05/1.05)^2} \right] = 312,429,174$$

Lin Life Insurance Company has sold 625 annuities identical to Diego's annuity to 625 independent lives.

c. (7 points) Calculate the 90% confidence interval for the present value of this annuity portfolio using the normal distribution.

   Solution:

$$E[Port] = 625(67,749) = 42,343,125$$

$$\sqrt{Var[Port]} = \sqrt{(625)(312,429,174)} = 441,891.65$$

$$CI = 42,343,125 \pm (1.645)(441,891.65)$$

$$(41,616,213\ ;\ 43,070,037)$$

(10 points) Sam who is (35) buys a 15 year temporary life annuity that pays 2000 at the beginning of each a year that she is alive. The payments stop at the end of 15 years even if Sam is still alive.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

Calculate the actuarial present value of Sam's annuity.

Solution:

$$APV = 2000\ddot{a}_{35:\overline{15}|} = (2000)(\ddot{a}_{35} - {}_{15}E_{35} \cdot \ddot{a}_{50})$$

$$= (2000)\left[18.9728 - (1.05)^{-15}\left(\frac{98,576.4}{99,556.7}\right)(17.0245)\right] = 21,728.72$$

(10 points) Dylan who is (68) receives an annuity as his pension benefit. The annuity is a 10 year certain and life annuity which pays 50,000 at the beginning of each year. The first 10 payments are guaranteed to be made even if Dylan dies prior to the payment. After 10 payments, payments are only made if Dylan is alive.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

Calculate the Actuarial Present Value of Dylan's pension benefit.

Solution:

$$APV = 50,000\ddot{a}_{\overline{\overline{68:\overline{10}|}}} = (50,000)(\ddot{a}_{\overline{10}|} + {}_{10}E_{68} \cdot \ddot{a}_{78})$$

$$= (50,000)\left(\frac{1-(1.05)^{-10}}{0.5/1.05} + (0.52981)(9.2598)\right) = 650,687.82$$

Jackson is (50) and has 1,000,000 that he wants to invest in an annuity that makes quarterly payments of X at the beginning quarter for the rest of his life.

Jackson approaches Gigli Life Insurance Company. Gigli calculates the amount of the benefit using the two term Woolhouse formula with mortality equal to the Standard Ultimate Life Table and interest equal to 5%.

a. (6 points) Calculate X from Gigli.

Solution:

$$1,000,000 = 4X\ddot{a}_{50}^{(4)} = 4X\left(\ddot{a}_{50} - 3/8\right) = 4X\left(17.0245 - 3/8\right)$$

$$X = 15,015.47$$

Jackson also approaches Balson Life Insurance Company. Balson calculates the amount of the benefit assuming uniform distribution of deaths between integral ages with mortality equal to the Standard Ultimate Life Table and interest equal to 5%.

b. (6 points) Calculate X from Balson.

Solution:

$$1,000,000 = 4X\ddot{a}_{50}^{(4)} = 4X\left(\alpha(4)\ddot{a}_{50} - \beta(4)\right) = 4X\left((1.00019)(17.0245) - 0.38272\right)$$

$$X = 15,019.51$$

c. (3 points) In real life, life insurance companies use the 2 term Woolhouse formula. List three reasons that Woolhouse is used.

Solution:

Valid Reasons would include:

- Simplicity of Calculation
- Produces a smaller payment that 3 term woolhouse or UDD
- Mortality does not actually follow UDD
- Woolhouse tends to provide a more accurate estimate than UDD

(10 points) You are given:

a. $A_{70} = 0.400$

b. $i = 0.06$

c. $q_{70} = 0.02$ and $q_{71} = 0.025$

Calculate $\ddot{a}_{71}$.

Solution:

$$\ddot{a}_{71} = \frac{1 - A_{71}}{d} = \frac{1 - 0.412244898}{0.06/1.06} = 10.3837$$

$$\boxed{\begin{aligned} &A_{70} = vq_{70} + v(1 - q_{70})A_{71} \\ &0.400 = (1.06)^{-1}(0.02) + (1.06)^{-1}(1 - 0.02)A_{71} \\ &A_{71} = \frac{0.400 - (1.06)^{-1}(0.02)}{(1.06)^{-1}(1 - 0.02)} = 0.412244898 \end{aligned}}$$

The Pierce Life Insurance Company sells a 20 year term policy with a death benefit of 80,000 to (63). The death benefit is payable at the end of the year of death. The premiums for the policy are paid annually for 10 years.

You are given:

i. Mortality follows the Standard Ultimate Life Table

ii. $i = 0.05$

iii. The expenses for the policy:

1. 30% of premium the first year and 5% of premium thereafter. This expense stops when premiums stop.

2. 100 per policy in the first year and 30 per policy in years 2 and later.

a. (6 points) Calculate the gross premium for this policy if the gross premium is determined using the equivalence principle.

Solutions:

$$PVP = PVB + PVE$$

$$P\ddot{a}_{63:\overline{10}|} = 80,000A^{\;1}_{63:\overline{20}|} + 0.25P + 0.05P\ddot{a}_{63:\overline{10}|} + 70 + 30\ddot{a}_{63:\overline{20}|}$$

$$P = \frac{80,000A^{\;1}_{63:\overline{20}|} + 70 + 30\ddot{a}_{63:\overline{20}|}}{0.95\ddot{a}_{63:\overline{10}|} - 0.25}$$

$$= \frac{80,000(0.42298 - 0.26674) + 70 + 30(12.1174)}{0.95(7.8960) - 0.25} = 1783.53$$

Pierce decides to charge a premium of 1800. Let L_0^g be the loss at issue random variable based on the gross premium for this policy.

b. (6 points) Calculate the $E(L_0^g)$.

Solutions:

$$E(L_0^g) = PVB + PVE - PVP$$

$$= 80,000A^{\,1}_{63:\overline{20}|} + 0.25(1800) + 0.05(1800)\ddot{a}_{63:\overline{10}|} + 70 + 30\ddot{a}_{63:\overline{20}|} - (1800)\ddot{a}_{63:\overline{10}|}$$

$$80,000(0.42298 - 0.26674) + 0.25(1800) + 0.05(1800)(7.8960) + 70 + 30(12.1174) - (1800)(7.8960)$$

$$= -119.44$$

c. (4 points) Explain why the expected value is negative.

Solution:

The premium has increased and is greater than the premium under the equivalence principle where the expected value is equal to zero. Therefore, we expect a gain which is a negative loss.

d. (4 points) If Pierce wanted an expected profit of 500 on the policy, determine the gross premium that should be charged. Note that a profit of 500 is equivalent to $E(L_0^g) = -500$.

Solutions:

$$E(L_0^g) = PVB + PVE - PVP$$

$$-500 = 80,000A^{\,1}_{63:\overline{20}|} + 0.25(P) + 0.05(P)\ddot{a}_{63:\overline{10}|} + 70 + 30\ddot{a}_{63:\overline{20}|} - (P)\ddot{a}_{63:\overline{10}|}$$

$$-500 = 80,000(0.42298 - 0.26674) + 0.25(P) + 0.05(P)(7.8960) + 70 + 30(12.1174) - (P)(7.8960)$$

$$P = \frac{500 + 80,000(0.42298 - 0.26674) + 70 + 30(12.1174)}{0.95(7.8960) - 0.25} = 1852.48$$