Chapter 5 – Past Test and Quiz Problems – Special Annuities, Annuities with Non-Level Payments

(6 points) Molly is (x) and buys a special 3-year term life annuity due with annual payments. The payment at the beginning of the first year is 50,000. The payment at the beginning of the second year is 30,000. The payment at the beginning of the third year is 10,000. Let $Y_{Special}$ be the present value random variable for the annuity.

You are given that:

- $v = 0.94$
- $q_{x+t} = 0.08 + 0.04t \qquad \text{for } t = 0,1,2,3,4,5$

Calculate the $Var[Y_{Special}]$.

Solution:

Since the payments are not level, we must use first principles.

Cases	Present Value	Probability
Die Year 1	$50,000$	0.08
Die Year 2	$50,000 + 30,000(0.94) = 78,200$	(0.92)(0.12)=0.1104
Live 2 Years	50,000+30,000(0.94)+10,000(0.94)^2 $= 87,036$	$(0.92)(0.88) = 0.8096$

$E[Y_{Special}] = (50,000)(0.08) + (78,200)(0.1104) + (87,036)(0.8096) = 83,097.63$

$E[(Y_{Special})^2] = (50,000)^2(0.08) + (78,200)^2(0.1104) + (87,036)^2(0.8096) = 7,008,057,280$

$Var[(Y_{Special})^2] = 7,008,057,280 - (83,097.63)^{22} = 102,841,849$

(5 points) Jeff is (72). He has 800,000 and wants to buy an annuity to fund his retirement.

Assume that mortality follows the Standard Ultimate Life Table with interest equal to 5%. Further, assume that deaths are uniformly distributed between integral ages.

He is considering a certain and life annuity due with annual payments. The payments for first 15 years are guaranteed to be made. Payments after 15 years will continue if Jeff is alive. Determine the annual payment under this annuity.

Solution:

$$800,000 = P\ddot{a}_{\overline{\overline{72:\overline{15}|}}}$$

$$P = \frac{800,000}{\ddot{a}_{\overline{15}|} + {}_{15}E_{72}\ddot{a}_{87}}$$

$$P = \frac{800,000}{\dfrac{1-(1.05)^{-15}}{\left(\dfrac{0.05}{1.05}\right)} + (1.05)^{-15}\left(\dfrac{53,934.7}{89,082.1}\right)(6.1308)}$$

$$P = 63,070.97$$

(6 points) Bailey is (92) and buys a whole life annuity with non-level annual payments. A payment of 1000 is made at age 92 if Bailey is alive. A payment of 2000 is made at age 93 if Bailey is alive. A payment of 3000 is made at age 94 if Bailey is alive. The payments continue to increase in the same pattern for the rest of Bailey's life.

You are given:

a. $v = 0.9$

b. $q_{92+t} = \dfrac{(t+1)}{3}$ for $t = 0,1,2$ and $q_{92+t} = 1$ for $t > 2$

If Y is the present value random variable for Bailey's annuity, determine the $Var[Y]$.

Solution:

Case	Y	Probability
Die Year 1	1000	1/3=3/9
Die Year 2	1000+2000v=2800	(2/3)(2/3)=4/9
Die Year 3	1000+2000v+3000v^2=5230	(2/3)(1/3)(1)=2/9

$$Var[Y] = E[Y^2] - E[Y]^2$$

$$E[Y] = 1000\left(\frac{3}{9}\right) + 2800\left(\frac{4}{9}\right) + 5230\left(\frac{2}{9}\right) = 2740$$

$$E[Y^2] = 1000^2\left(\frac{3}{9}\right) + 2800^2\left(\frac{4}{9}\right) + 5230^2\left(\frac{2}{9}\right) = 9,896,200$$

$$Var[Y] = 9,896,200 - 2740^2 = 2,388,600$$

(4 points) Brett is (35) and is receiving a whole life annuity due with annual payments. The payments at the beginning of the first 10 years are 10,000. The payments during the second 10 years are 20,000. After 20 years, the payments are 5000 for as long as Brett is alive.

You are given that mortality follows the Standard Ultimate Life Table with interest equal to 5%.

Determine the actuarial present value of Brett's annuity.

Solution:

$$APV = 10,000\ddot{a}_{35} + 10,000\,{}_{10}E_{35}\ddot{a}_{45} - 15,000\,{}_{20}E_{35}\ddot{a}_{55}$$

$$= (10,000)(18.9728) + (10,000)(0.61069)(17.8162) - (15,000)(0.37041)(16.0599)$$

$$= 209,298.54$$

(5 points) Jeff is (68). He has 1,200,000 and wants to purchase an annuity.

Assume that mortality follows the Standard Ultimate Life Table with interest equal to 5%. Use the two term Woolhouse formula to calculate the monthly annuities.

He decides he wants to purchase a deferred annuity due. The deferred annuity will beginning making monthly payments at age 80. Determine the amount of the monthly payment.

Solution:

$$1,200,000 = 12P \cdot {}_{12|}\ddot{a}_{68}^{(12)} = 12P\left({}_{12}E_{68}\ddot{a}_{80}^{(12)}\right)$$

$$P = \frac{1,200,000}{12\left(\dfrac{l_{80}}{l_{68}}v^{12}\right)\left(a_{80} - \dfrac{11}{24}\right)} = \frac{1,200,000}{44.11677} = 27,200.54$$

(3 points) Jeff (65) has the option to receive a 20-year deferred whole life annuity due. This annuity would begin making level annual payments to Jeff at age 85. If Jeff died prior to age 85, no benefits would be paid. The actuarial present value of this benefit is 1,381,408.10.

Calculate the annual payment.

Solution:

$$1{,}381{,}408.10 = P \cdot {}_{20|}\ddot{a}_{65} = P \cdot {}_{20}E_{65} \cdot \ddot{a}_{85}$$

$$P = \frac{1{,}381{,}408.10}{(0.24381)(6.7993)} = 833{,}309.40$$