Chapters 5 – Past Test and Quiz Problems

Whole Life and Temporary Life Annuity Due (Expected Value, Variance, and Clyde)

1. (5 points) Diego is (65) and is the recipient of an annuity that pays 5000 at the beginning of each year for the rest of his life.

   You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

   Let Y be the present value random variable for Diego's annuity.

   a. (1 point) Calculate the $E[Y]$.

      Solution:

      $$E[Y] = 5000\ddot{a}_{65} = (5000)(13.5498) = 67,749$$

   b. (4 points) Calculate the $Var[Y]$.

      Solution:

      $$Var[Y] = (5000)^2 \left[\frac{{}^2A_{65} - (A_{65})^2}{d^2} \right] = (5000)^2 \left[\frac{0.15420 - (0.35477)^2}{(0.05/1.05)^2} \right] = 312,429,174$$

2. (12 points) Sam who is (35) buys a 15-year temporary life annuity that pays 2000 at the beginning of each year that she is alive. The payments stop at the end of 15 years even if Sam is still alive.

You are given that mortality follows the Standard Ultimate Life Table with interest at 5%.

a. (2 points) Write an expression for Y.

$$Y = 2000\ddot{a}_{\overline{\min(15, K_{35}+1)|}}$$

b. (5 points) Calculate the actuarial present value of Sam's annuity.

$$APV = 2000\ddot{a}_{35:\overline{15|}} = (2000)\left(\frac{1 - A_{35:\overline{15|}}}{d}\right)$$

$$= (2000)\left(\frac{1 - \left(A_{35} - \left(_{15}E_{35}\right)A_{50} + {}_{15}E_{35}\right)}{d}\right) = (2000)\left(\frac{1 - \left(A_{35} - \left(_{10}E_{35}\right)\left(_{5}E_{45}\right)A_{50} + \left(_{10}E_{35}\right)\left(_{5}E_{45}\right)\right)}{d}\right)$$

$$= (2000)\left(\frac{1 - \left(0.09653 - (0.61069)(0.77991)(0.18931) + (0.61069)(0.77991)\right)}{\dfrac{0.05}{1.05}}\right) = 21,728.78$$

$$= (2000)\left(\frac{1 - (0.482648095)}{\dfrac{0.05}{1.05}}\right) = 21,728.78$$

c. (5 points) Calculate the variance of Sam's annuity.

$$Var(Y) = (2000)^2\frac{{}^2A_{35:\overline{15|}} - \left(A_{35:\overline{15|}}\right)^2}{d^2} = (2000)^2\frac{\left({}^2A_{35} - v^{15}\left(_{15}E_{35}\right){}^2A_{50} + v^{15}\left(_{15}E_{35}\right)\right) - \left(A_{35:\overline{15|}}\right)^2}{\left(\dfrac{0.05}{1.05}\right)^2}$$

$$= (2000)^2\frac{\left[0.01601 - \left(\dfrac{1}{1.05}\right)^{15}(0.476283)(0.05108) + \left(\dfrac{1}{1.05}\right)^{15}(0.476283)\right] - (0.482648)^2}{\left(\dfrac{0.05}{1.05}\right)^2}$$

$= 809,043$ *Differences due to rounding likely for this variance calculation

3. (6 points) You are given:

     a. $A_{70} = 0.400$

     b. $i = 0.06$

     c. $q_{70} = 0.02$ and $q_{71} = 0.025$

Calculate $\ddot{a}_{71}$.

Solution:

$$\ddot{a}_{71} = \frac{1 - A_{71}}{d} = \frac{1 - 0.412244898}{0.06/1.06} = 10.3837$$

$$A_{70} = vq_{70} + v(1 - q_{70})A_{71}$$

$$0.400 = (1.06)^{-1}(0.02) + (1.06)^{-1}(1 - 0.02)A_{71}$$

$$A_{71} = \frac{0.400 - (1.06)^{-1}(0.02)}{(1.06)^{-1}(1 - 0.02)} = 0.412244898$$