**Chapter 6 – Past Test and Quiz Problems – Net Premium**

The Chung Life Insurance Company sells life insurance policies to people who are age 70 only.

The Company uses the Standard Ultimate Life Table and 5% interest to calculate all net premiums. They also assume that deaths are uniformly distributed between integral ages.

All premiums are calculated using the Equivalence Principle.

All policies are sold to insureds whose death is independent of the death of any other insured.

a. (2 points) The annual net premium for a whole life policy with a death benefit of 100,000 paid at the end of the year of death is 3600 to the nearest 100. Calculate the net premium to the nearest 1.

   **Solution:**

   $$PVP = PVB ==> P\ddot{a}_{70} = 100,000 A_{70}$$

   $$P = \frac{(100,000)(0.42818)}{12.0083} = 3565.70$$

b. (7 points) Calculate the monthly net premium for a 20 year term insurance policy with a death benefit of 1,000,000 paid at the moment of death.

   **Solution:**

   $$PVP = PVB ==> 12P\ddot{a}^{(12)}_{70:\overline{20|}} = 1,000,000\bar{A}^{\;1}_{70:\overline{20|}}$$

   $$12P\left(\alpha(12)\ddot{a}_{70} - \beta(12) - {}_{20}E_{70}\left[\alpha(12)\ddot{a}_{90} - \beta(12)\right]\right)$$
   $$= 1,000,000\left(\frac{i}{\delta}\right)A^{\;1}_{70:\overline{20|}} = 1,000,000\left(\frac{i}{\delta}\right)(A_{70:\overline{20|}} - {}_{20}E_{70})$$

   $$12P\left(\alpha(12)\ddot{a}_{70:\overline{20|}} - \beta(12)(1 - {}_{20}E_{70})\right) = 1,000,000\left(\frac{i}{\delta}\right)(A_{70:\overline{20|}} - {}_{20}E_{70})$$
   $$12P\left((1.00020)(11.1109) - (0.46651)(1 - 0.17313)\right) = 1,000,000(1.02480)(0.47091 - 0.17313)$$

   $$P = \frac{11,000,000(1.02480)(0.47091 - 0.17313)}{12\left((1.00020)(11.1109) - (0.46651)(1 - 0.17313)\right)} = 2370.60$$

(7 points) Ranya is (60) and purchases a 20 year term insurance. The term insurance has a death benefit of 100,000 paid at the end of the year of death. Premiums for the policy are paid monthly for 5 years.

You are given:

a. Mortality follows the Standard Ultimate Life Table with $i = 5\%$.

b. Monthly annuities are determined using the two term Woolhouse formula.

Calculate the monthly net premium for this policy.

**Solution:**

$$12P\ddot{a}^{(12)}_{60:\overline{5}|} = 100,000A^{\;1}_{60:\overline{20}|} \implies P = \frac{100,000A^{\;1}_{60:\overline{20}|}}{12\ddot{a}^{(12)}_{60:\overline{5}|}}$$

$$A^{\;1}_{60:\overline{20}|} = A_{60:\overline{20}|} - {}_{20}E_{60} = 0.41040 - 0.29508 = 0.11532$$

$$\ddot{a}^{(12)}_{60:\overline{5}|} = \ddot{a}^{(12)}_{60} - {}_5E_{60}\ddot{a}^{(12)}_{65}$$

$$\ddot{a}^{(12)}_{60} = \ddot{a}_{60} - \frac{12-1}{12(2)} = 14.9041 - \frac{11}{24} = 14.44576667$$

$$\ddot{a}^{(12)}_{65} = \ddot{a}_{65} - \frac{12-1}{12(2)} = 13.5498 - \frac{11}{24} = 13.09146667$$

$$\ddot{a}^{(12)}_{60:\overline{5}|} = (14.44576667) - (0.76687)(13.09146667) = 4.406313625$$

$$P = \frac{100,000(0.11532)}{12(4.406313625)} = 218.10$$

(5 points) Taylen is (70) and buys a whole life insurance policy with a death benefit of 50,000 to be paid at the end of the year of death. She will pay premiums annually for life.

You are given:

a. $v = 0.95$

b. $q_{70} = 0.010$

c. $q_{71} = 0.012$

d. $\ddot{a}_{71} = 11.5$

Calculate the net premium for Taylen's policy.

**Solution:**

$$P\ddot{a}_{70} = 50,000A_{70} \implies P = \frac{50,000A_{70}}{\ddot{a}_{70}}$$

$$\ddot{a}_{70} = 1 + v \cdot p_{70} \cdot \ddot{a}_{71} = 1 + (0.95)(1 - .01)(11.5) = 11.81575$$

$$A_{70} = 1 - d \cdot \ddot{a}_{70} = 1 - (1 - 0.95)(11.81575) = 0.4092125$$

$$P = \frac{50,000(0.4092125)}{11.81575} = 1,731.64$$

(7 points) Maddie is (50) and purchases a 20-year endowment insurance. The endowment insurance has a death benefit of 50,000 paid at the end of the year of death. Premiums for the policy are paid monthly for 5 years.

You are given:

a. Mortality follows the Standard Ultimate Life Table with $i = 5\%$.

b. Deaths are uniformly distributed between integral ages.

Calculate the monthly net premium for this policy.

**Solution:**

$$50,000A_{50:\overline{20}|} = 12P\ddot{a}^{(12)}_{50:\overline{5}|} \text{ ==> } P = \frac{50,000A_{50:\overline{20}|}}{12\ddot{a}^{(12)}_{50:\overline{5}|}}$$

$$\ddot{a}^{(12)}_{50:\overline{5}|} = \ddot{a}^{(12)}_{50} - {}_5E_{50}\ddot{a}^{(12)}_{55} = \left[\alpha(12)\ddot{a}_{50} - \beta(12)\right] - {}_5E_{50}\left[\alpha(12)\ddot{a}_{55} - \beta(12)\right]$$

$$[(1.0002)(17.0245) - 0.46651] - 0.77772[(1.0002)(16.0599) - 0.46651] = 4.431605608$$

$$P = \frac{50,000A_{50:\overline{20}|}}{12\ddot{a}^{(12)}_{50:\overline{5}|}} = \frac{(50,000)(0.38844)}{(12)(4.431605608)} = 365.22$$

(5 points) You are given:

a. $\ddot{a}_{50} = 11$

b. $i = 0.05$

c. $q_{50} = 0.01$

d. Deaths are uniformly distributed between integral ages.

Let $P$ be the net annual premium for a whole life policy on **(51)** with a death benefit of 30,000 paid at the moment of death.

Determine $P$.

**Solution:**

$$PVP = PVB$$

$$P\ddot{a}_{51} = 30,000\bar{A}_{51}$$

$$\ddot{a}_{50} = 1 + vp_{50}\ddot{a}_{51} \Longrightarrow \ddot{a}_{51} = \frac{\ddot{a}_{50} - 1}{vp_{50}} = \frac{11-1}{(1.05)^{-1}(1-0.01)} = 10.6060606$$

$$A_{51} = 1 - d\ddot{a}_{51} = 1 - (0.05/1.05)(10.6060606) = 0.4949495$$

$$P = \frac{(30,000)(1.02480)(0.4949495)}{10.6060606} = 1434.72$$

Megan is (45). She purchases a whole life policy with death benefit of 250,000 payable at the moment of death.

You are given that mortality follows the Standard Ultimate Life Table and $i = 0.05$ . You are also given that deaths are uniformly distributed between integral ages.

a. (2 points) The actuarial present value of Megan's death benefit is 38,800 to the nearest 100. Calculate it to the nearest 1.

**Solution:**

$$APV = 250,000\bar{A}_{45} = (250,000)\left(\frac{i}{\delta}\right)A_{45} = (250,000)\left(\frac{0.05}{\ln(1.05)}\right)(0.15161) = 38,842$$

Megan wants to explore different premium payment patterns. All premiums are calculated using the equivalence principle.

b. (2 points) Calculate the net annual premium if premiums are paid annually during her lifetime.

**Solution:**

$$PVP = PVB ==> P\ddot{a}_{45} = 38,842$$

$$P = \frac{38,842}{17.8162} = 2180.15$$

c. (4 points) Calculate the net annual premium if premiums are only payable for 15 years.

**Solution:**

$$PVP = PVB ==> P\ddot{a}_{45:\overline{15|}} = 38,842$$

$$P = \frac{38,842}{\ddot{a}_{45} - {}_{15}E_{45} \cdot \ddot{a}_{60}} = \frac{38,842}{17.8162 - (1.05)^{-15}\left(\frac{96,634.1}{99,033.9}\right)(14.9041)} = 3589.57$$

d. (7 points) Calculate the net quarterly premium if premiums are payable for life.

**Solution:**

$$PVP = PVB \Longrightarrow 4P\ddot{a}_{45}^{(4)} = 38,842$$

$$P = \frac{38,842}{4P\ddot{a}_{45}^{(4)}} = \frac{38,842}{4\{\alpha(4)\ddot{a}_{45} - \beta(4)\}} = \frac{38,842}{4\{(1.00019)(17.8162) - 0.38272\}} = 556.89$$

e. (4 points) Megan decides that she wants to pay annual premiums for 30 years. The premiums will not be level. The premiums for the first 10 years will be $P$ . The premiums during the second 10 years will be $2P$ . The premiums during the final 10 years will be $3P$.

Determine $P$ .

**Solution:**

$$PVP = PVB$$

$$P(\ddot{a}_{45} + {}_{10}E_{45} \cdot \ddot{a}_{55} + {}_{20}E_{45} \cdot \ddot{a}_{65} - 3\,{}_{30}E_{45} \cdot \ddot{a}_{75}) = 38,842$$

$$P = \frac{38,842}{17.8162 + (0.60655)(16.0599) + (0.35994)(13.5498) - (3)(0.35994)(0.55305)(10.3178)}$$

$$= 1478.42$$

(3 points) Andrew is (45) and purchases a whole life policy with a death benefit of 200,000 paid at the moment of death. Premiums are paid annually during Andrew's lifetime on the policy.

You are given:

i. Mortality follows the Standard Ultimate Life Table.

ii. Deaths are uniformly distributed between integral ages.

iii. $i = 0.05$

Calculate the premium for Andrew's policy.

**Solution:**

$$PVP = PVB \implies P\ddot{a}_{45} = 200,000\bar{A}_{45} = (200,000)(i/\delta)A_{45}$$

$$\implies P(17.8162) = 200,000(1.02480)(0.15161)$$

$$P = 1744.14$$