Chapter 2

1. You are given:

$$F_0(t) = 1 - \left(1 - \frac{t}{125}\right)^{\frac{1}{5}}, 0 \le t \le 125$$

   Calculate:

   a. $S_0(t)$
   b. $\Pr[T_0 \le t]$
   c. $\Pr[T_0 > t]$
   d. $S_x(t)$
   e. Probability that a newborn will live to age 25.
   f. Probability that a person age 25 will live to age 75.
   g. Probability that a person age 25 will die between age 50 and age 75.
   h. ω
   i. μ_x
   j. μ_{25}
   k. μ_{100}
   l. ${}_t p_x$
   m. ${}_{10} p_{50}$
   n. ${}_t q_x$
   o. ${}_{10} q_{50}$
   p. ${}_{10} p_{50} + {}_{10} q_{50}$
   q. p_{50}
   r. ${}_{u|t} q_x$
   s. $f_x(t)$
   t. $E[T_x]$
   u. $\mathring{e}_x$
   v. $Var[T_x]$
   w. Standard Deviation of T_{50}
   x. $E[K_{120}]$
   y. $Var[K_{120}]$

2. You are given that mortality follows Gompertz Law with B = 0.00027 and c = 1.1. Calculate:

   a. μ_x
   b. μ_{25}
   c. μ_{100}
   d. $S_0(t)$
   e. $\Pr[T_0 \le t]$
   f. $\Pr[T_0 > t]$
   g. $S_x(t)$
   h. Probability that a newborn will live to age 25.
   i. Probability that a person age 25 will live to age 75.
   j. Probability that a person age 25 will die between age 50 and age 75.
   k. ω
   l. ${}_t p_x$
   m. ${}_{10} p_{50}$
   n. ${}_t q_x$
   o. ${}_{10} q_{50}$
   p. ${}_{10} p_{50} + {}_{10} q_{50}$
   q. p_{50}
   r. ${}_{u|t} q_x$
   s. $f_x(t)$

3. You are given that that $\mu_x = c$ for all $x \geq 0$ where c is a constant. This mortality law is known as a constant force of mortality.

    a. ${}_t p_x$
    b. ${}_t q_x$
    c. ω
    d. $\overset{\circ}{e}_x$
    e. $Var[T_x]$
    f. e_x
    g. ${}_{10} p_{10}$
    h. ${}_{10} p_{100}$
    i. ${}_{10} p_{500}$
    j. Would this be a reasonable model for human mortality? Why or why not?

4. You are given ${}_t q_0 = \dfrac{t^2}{10,000}$ for 0 < t < 100. Calculate:

    a. $F_0(x)$
    b. $S_0(x)$
    c. $S_x(t)$
    d. $f_0(x)$
    e. $E[T_0]$
    f. $Var[T_0]$
    g. ${}_{40} q_0$
    h. ${}_{40} p_0$
    i. $\Pr(40 < T_0 < 60)$
    j. μ_x
    k. μ_{75}
    l. ${}_t p_x$
    m. ${}_t q_x$
    n. ${}_t q_{75}$
    o. ${}_t p_{75}$
    p. $E[T_x]$
    q. $E[T_{75}]$
    r. $\overset{\circ}{e}_x$
    s. $\overset{\circ}{e}_0$
    t. $\overset{\circ}{e}_{75}$
    u. $\overset{\circ}{e}_{75:\overline{10}|}$
    v. ${}_{u|t} q_x$
    w. ${}_{10|5} q_{75}$

5. You are given that $\mu_x = \dfrac{2}{100 - x}$ for $0 \leq x < 100$. Calculate $F_0(x)$ and ${}_{10} p_{50}$.

6. Book Exercise 2.6

7. You need to calculate the following values using the mortality basis described below.

- $\mathring{e}_{50}$ using the trapezium rule with a span of 0.1 ($h = 0.1$)
- $Var(T_{50})$ using the trapezium rule with a span of 0.1 ($h = 0.1$)

Assume that mortality follows Gompertz Law with the following parameters:

B	**c**
0.00032	1.080

Email your spreadsheet to sallyray@purdue.edu in support of your work.

8. You are given the following mortality table:

x	q_x for males	q_x for females
90	0.20	0.10
91	0.25	0.15
92	0.30	0.20
93	0.40	0.25
94	0.50	0.30
95	0.60	0.40
96	1.00	1.00

a. Calculate the probability that a male exact age 91 will die at age 93 or 94.

b. Calculate the amount that the curtate life expectancy for a female age 90 exceeds the curtate life expectancy for a male age 90.

c. For females, calculate $e_{91:\overline{3}|}$.

9. You are given the following:

a. $e_{40:\overline{20}|} = 18$

b. $e_{60} = 25$

c. ${}_{20}q_{40} = 0.2$

d. $q_{40} = 0.003$

Calculate e_{41}

10. (SWAQ) You are given that ${}_t p_x = 1 - \frac{t^3}{125}$.

Your boss calculates the complete expectation of life for (x) using the relationship that the complete expectation of life is approximately equal to the curtate expectation of life plus one half of a year.

(2 points) Calculate the complete expectation of life for (x) as computed by your boss.

(2 points) Calculate the actual complete expectation of life.

(1 point) Write a short paragraph explaining to your boss why his calculation gets a different answer. Identify the error in his approximation.

Answers

1.

a. $\left(1-\frac{t}{125}\right)^{\frac{1}{5}}$

b. $1-\left(1-\frac{t}{125}\right)^{\frac{1}{5}}$

c. $\left(1-\frac{t}{125}\right)^{\frac{1}{5}}$

d. $\left(\frac{125-x-t}{125-x}\right)^{\frac{1}{5}}$

e. 0.95635

f. 0.87055

g. 0.07354

h. 125

i. $\frac{1}{625-5x}$

j. 0.002

k. 0.008

l. $\left(\frac{125-x-t}{125-x}\right)^{\frac{1}{5}}$

m. 0.97179

n. $1-\left(\frac{125-x-t}{125-x}\right)^{\frac{1}{5}}$

o. 0.02821

p. 1

q. 0.99732

r. $\frac{(125-x-u)^{\frac{1}{5}}-(125-x-u-t)^{\frac{1}{5}}}{(125-x)^{\frac{1}{5}}}$

s. $\frac{1}{625-5x}\left(\frac{125-x-t}{125-x}\right)^{\frac{-4}{5}}$

t. $\frac{5(125-x)}{6}$

u. $\frac{5(125-x)}{6}$

v. $\frac{25(125-x)^2}{396}$

w. 18.84446

x. 3.41656

y. 1.22830

2.

a. $(0.00027)(1.1)^x$
b. 0.0029254
c. 3.72077
d. $\exp\left(\frac{-0.00027}{\ln(1.1)}(1.1^t - 1)\right)$
e. $1 - \exp\left(\frac{-0.00027}{\ln(1.1)}(1.1^t - 1)\right)$
f. $\exp\left(\frac{-0.00027}{\ln(1.1)}(1.1^t - 1)\right)$
g. $\exp\left(\frac{-0.00027}{\ln(1.1)}(1.1)^x(1.1^t - 1)\right)$
h. 0.97252
i. 0.028088
j. 0.71135
k. ∞
l. $\exp\left(\frac{-0.00027}{\ln(1.1)}(1.1)^x(1.1^t - 1)\right)$
m. 0.58860
n. $1 - \exp\left(\frac{-0.00027}{\ln(1.1)}(1.1)^x(1.1^t - 1)\right)$
o. 0.41140
p. 1
q. 0.96729
r. $\exp\left(\frac{-0.00027}{\ln(1.1)}(1.1)^x(1.1^u - 1)\right) - \exp\left(\frac{-0.00027}{\ln(1.1)}(1.1)^x(1.1^{u+t} - 1)\right)$
s. $\exp\left(\frac{-0.00027}{\ln(1.1)}(1.1)^x(1.1^t - 1)\right) \bullet (0.00027)(1.1)^{x+t}$

3.

a. $\exp(-tc)$
b. $1 - \exp(-tc)$
c. ∞
d. $\frac{1}{c}$
e. $\frac{1}{c^2}$
f. $\frac{1}{e^c - 1}$
g. e^{-10c}
h. e^{-10c}
i. e^{-10c}

4.

a. $\frac{x^2}{10,000}$

b. $1-\frac{x^2}{10,000}$

c. $\frac{10,000-(x+t)^2}{10,000-x^2}$

d. $\frac{x}{5000}$

e. 66.66667

f. 555.56

g. 0.16

h. 0.84

i. 0.20

j. $\frac{2x}{10,000-x^2}$

k. $\frac{6}{175}$

l. $\frac{10,000-(x+t)^2}{10,000-x^2}$

m. $\frac{2xt+t^2}{10,000-x^2}$

n. $\frac{150t+t^2}{4375}$

o. $\frac{4375-150t-t^2}{4375}$

p. $\frac{(100-x)(200+x)}{300+3x}$

q. 13.09524

r. $\frac{(100-x)(200+x)}{300+3x}$

s. 66.66667

t. 13.09524

u. 8.20952

v. $\frac{2(x+u)t+t^2}{10,000-x^2}$

w. 0.2

5. $F_0(x)=1-\left(\frac{100-x}{100}\right)^2$ *and* ${}_{10}p_{50}=0.64$

6.

a. 0.98

b. 0.97515

c. 0.96939

d. 0.95969

e. 0.03031

7.

a. 19..6347

b. 103.3139

8.

a. 0.3675

b. 1.00168

c. 2.04000

9. 37.11434

10. Answer Not Given