1. Complete the following table:

x	q_x	l_x	d_x
50		20,000	800
51			
52		18,000	
53	0.100		
54	0.125	14,985	

$$q_{50} = \frac{d_{50}}{l_{50}} = \frac{800}{20000} = 0.04$$

$$l_{51} = l_{50} - d_{50} = 20000 - 800 = 19200$$

$$d_{51} = l_{51} - l_{52} = 19200 - 18000 = 1200$$

$$q_{51} = \frac{d_{51}}{l_{51}} = \frac{1200}{19200} = 0.0625$$

$$l_{54} = l_{53} \cdot (1 - q_{53})$$

$$\Rightarrow l_{53} = \frac{l_{54}}{1 - q_{53}} = \frac{14985}{1 - 0.1} = 16650$$

$$d_{53} = l_{53} - l_{54} = 16650 - 14985 = 1665$$

$$d_{52} = l_{52} - l_{53} = 18000 - 16650 = 1350$$

$$q_{52} = \frac{d_{52}}{l_{52}} = \frac{1350}{18000} = 0.075$$

$$d_{54} = p_{54} \cdot l_{54} = 0.125 \times 14985 = 1873.125$$

$\therefore$ The table becomes

x	q_x	l_x	d_x
50	0.040	20,000	800
51	0.0625	19,200	1,200
52	0.075	18,000	1,350
53	0.100	16,650	1,665
54	0.125	14,985	1873.125

2.

3. L_t is distributed as a binomial with parameters n and ${}_t p_x$

a. $E[L_t] = np = 1000({}_{22} p_{58}) = 1000\left(\dfrac{75,657.2}{97,195.6}\right) = 778.40$

b. $\text{Var}[L_t] = npq = 1000({}_{22} p_{58})({}_{22} q_{58}) =$

$$1000\left(\frac{75,657.2}{97,195.6}\right)\left(1 - \frac{75,657.2}{97,195.6}\right) = 172.49$$

4. Assume that mortality follows the Standard Ultimate Life Table for integral ages. Assume that deaths are uniformly distributed (UDD) between integral ages. Calculate:

   a. ${}_{0.5}q_{80}$

   $$ {}_{0.5}q_{80} = \frac{l_{80} - l_{80.5}}{l_{80}} = 1 - \frac{0.5l_{80} - +0.5l_{81}}{l_{80}} = 1 - \frac{(0.5)(75,657.2 + 73,186.3)}{75,657.2} = 0.01633 $$

   b. ${}_{0.5}p_{80}$

   $$ {}_{0.5}p_{80} = \frac{l_{80.5}}{l_{80}} = \frac{0.5l_{80} + 0.5l_{81}}{l_{80}} = \frac{(0.5)(75,657.2 + 73,186.3)}{75,657.2} = 0.98367 $$

   or

   $$ {}_{0.5}p_{80} = 1 - {}_{0.5}q_{80} = 1 - 0.01633 = 0.98367 $$

   c. $\mu_{80.5}$

   $$ \mu_{80.5} = \frac{q_{80}}{1 - 0.5 \cdot q_{80}} = \frac{0.032658}{1 - 0.5 \cdot 0.032658} = 0.03320 $$

   d. ${}_{1.5}p_{80}$

   $$ {}_{1.5}p_{80} = \frac{l_{81.5}}{l_{80}} = \frac{0.5l_{81} + 0.5l_{82}}{l_{80}} = \frac{(0.5)(73,186.3 + 70,507.2)}{75,657.2} = 0.94964 $$

   e. ${}_{1.5}q_{80}$

   $$ {}_{1.5}q_{80} = 1 - {}_{1.5}p_{80} = 1 - 0.94964 = 0.05036 $$

   f. ${}_{0.5}q_{80.5}$

   $$ {}_{0.5}q_{80.5} = \frac{l_{80.5} - l_{81}}{l_{80.5}} = \frac{0.5l_{80} + 0.5l_{81} - l_{81}}{0.5l_{80} + 0.5l_{81}} = \frac{(0.5)(75,657.2 + 73,186.3) - 73,186.3}{(0.5)(75,657.2 + 73,186.3)} = 0.01660 $$

   g. ${}_{0.5}q_{80.25}$

   $$ {}_{0.5}q_{80.25} = \frac{l_{80.25} - l_{80.75}}{l_{80.25}} = 1 - \frac{l_{80.75}}{l_{80.25}} = 1 - \frac{0.25l_{80} + 0.75l_{81}}{0.75l_{80} + 0.25l_{81}} $$

   $$ = 1 - \frac{(0.25)(75,657.2) + (0.75)(73,186.3)}{(0.75)(75,657.2) + (0.25)(73,186.3)} = 0.01646 $$

h.

$_{3.2\,|\,2.4}q_{80.5}$

$$_{3.2|2.4}q_{80.5} = \frac{l_{80.5+3.2} - l_{80.5+3.2+2.4}}{l_{80.5}} = \frac{l_{83.7} - l_{86.1}}{l_{80.5}} = \frac{\left(0.3l_{83} + 0.7l_{84}\right) - \left(0.9l_{86} + 0.1l_{87}\right)}{0.5l_{80} + 0.5l_{81}}$$

$$= \frac{(0.3)(67,614.6) + (0.7)(64,506.5) - (0.9)(57,656.7) - (0.1)(53,934.7)}{(0.5)(75,657.2 + 73,186.3)} = 0.10957$$

5. *For a certain mortality table, you are given:

   i. $\mu_{80.5} = 0.0202$
   ii. $\mu_{81.5} = 0.0408$
   iii. $\mu_{82.5} = 0.0619$
   iv. Deaths are uniformly distributed between integral ages.

   Calculate $_2q_{80.5}$

$$\mu_{x+s} = \frac{q_x}{1 - s \cdot q_x}$$

$$\left(1 - s \cdot q_x\right)\mu_{x+s} = q_x$$

$$\mu_{x+s} - s \cdot q_x \cdot \mu_{x+s} = q_x$$

$$\mu_{x+s} = \left(1 + s \cdot \mu_{x+s}\right) \cdot q_x$$

$$q_x = \frac{\mu_{x+s}}{1 + s \cdot \mu_{x+s}}$$

$$q_{80} = \frac{\mu_{80.5}}{1 + 0.5 \times \mu_{80.5}} = \frac{0.0202}{1 + 0.5 \times 0.0202} = 0.019998$$

$$q_{81} = \frac{\mu_{81.5}}{1 + 0.5 \times \mu_{81.5}} = \frac{0.0408}{1 + 0.5 \times 0.0408} = 0.039984$$

$$q_{82} = \frac{\mu_{82.5}}{1 + 0.5\mu_{82.5}} = \frac{0.0619}{1 + 0.5 \times 0.0619} = 0.060042$$

Suppose $l_{80} = 1,000,000$

Then

$$l_{81} = l_{80} \cdot (1 - q_{80}) = 980,001.98$$

$$l_{82} = l_{81} \cdot (1 - q_{81}) = 940,817.27$$

$$l_{83} = l_{82} \cdot (1 - q_{82}) = 884,328.99$$

$$_{2}q_{80.5} = 1 - \frac{l_{82.5}}{l_{80.5}} = 1 - \frac{0.5l_{82} + 0.5l_{83}}{0.5l_{80} + 0.5l_{81}} = 0.078210$$

6. Assume that mortality follows the Standard Ultimate Life Table for integral ages. Assume that probability of survival is geometrically distributed (Constant Force) between integral ages. Calculate:

   a. $_{0.5}q_{80}$

   $$_{0.5}q_{80} = \frac{l_{80} - l_{80.5}}{l_{80}} = 1 - \frac{l_{80}^{\ 0.5} \times l_{81}^{\ 0.5}}{l_{80}} = 1 - \frac{\sqrt{(75,657.2)(73,186.3)}}{75,657.2} = 0.01647$$

   b. $_{0.5}p_{80}$

   $$_{0.5}p_{80} = \frac{l_{80.5}}{l_{80}} = \frac{l_{80}^{\ 0.5} \times l_{81}^{\ 0.5}}{l_{80}} = \frac{\sqrt{(75,657.2)(73,186.3)}}{75,657.2} = 0.98353$$

   or

   $$_{0.5}p_{80} = p_{80}^{\ 0.5} = \sqrt{1 - 0.032658} = 0.95354 \qquad \text{Difference due to rounding}$$

   or

   $$_{0.5}p_{80} = 1 - {}_{0.5}q_{80} = 1 - 0.01647 = 0.98353$$

   c. $\mu_{80.5}$

   $$\mu_{80.5} = -\ln[p_{80}] = -\ln(1 - 0.032658) = 0.03320$$

   d. $_{1.5}p_{80}$

   $$_{1.5}p_{80} = \frac{l_{81.5}}{l_{80}} = \frac{l_{81}^{\ 0.5} \times l_{82}^{\ 0.5}}{l_{80}} = \frac{\sqrt{(73,186.3)(70,507.2)}}{75,657.2} = 0.94947$$

   e. $_{1.5}q_{80}$

   $$_{1.5}q_{80} = 1 - {}_{1.5}p_{80} = 1 - 0.94947 = 0.05053$$

   f. $_{0.5}q_{80.5}$

   $$_{0.5}q_{80.5} = \frac{l_{80.5} - l_{81}}{l_{80.5}} = 1 - \frac{l_{81}}{l_{80}^{\ 0.5} \times l_{81}^{\ 0.5}} = 1 - \frac{73,186.3}{\sqrt{(75,657.2)(73,186.3)}} = 0.01647$$

g. $\,_{0.5}q_{80.25}$

$$\,_{0.5}q_{80.25} = \frac{l_{80.25} - l_{80.75}}{l_{80.25}} = 1 - \frac{l_{80.75}}{l_{80.25}} = 1 - \frac{l_{80}^{\ 0.25} \times l_{81}^{\ 0.75}}{l_{80}^{\ 0.75} \times l_{81}^{\ 0.25}}$$

$$= 1 - \frac{(75,657.2)^{0.25}(73,186.3)^{0.75}}{(75,657.2)^{0.75}(73,186.3)^{0.25}} = 0.016465$$

h. $\,_{3.2\,|\,2.4}q_{80.5}$

$$\,_{3.2|2.4}q_{80.5} = \frac{l_{80.5+3.2} - l_{80.5+3.2+2.4}}{l_{80.5}} = \frac{l_{83.7} - l_{86.1}}{l_{80.5}} = \frac{l_{83}^{\ 0.3} \cdot l_{84}^{\ 0.7} - l_{86}^{\ 0.9} \cdot l_{87}^{\ 0.1}}{l_{80}^{\ 0.5} \cdot l_{81}^{\ 0.5}}$$

$$= \frac{(67,614.6)^{0.3}(64,506.5)^{0.7} - (57,656.7)^{0.9}(53,934.7)^{0.1}}{(75,657.2)^{0.5}(73,186.3)^{0.5}} = 0.10953$$

7. You are given $q_{80} = 0.06$ and $q_{81} = 0.09$. Calculate:

Suppose $l_{80} = 10000$

Then

$l_{81} = l_{80} \cdot (1 - q_{80}) = 9400$

$l_{82} = l_{81} \cdot (1 - q_{81}) = 8554$

a. ${}_{0.5}q_{80}$ given UDD

$${}_{0.5}q_{80}^{UDD} = 1 - \frac{l_{80.5}^{UDD}}{l_{80}} = 1 - \frac{0.5l_{80} + 0.5l_{81}}{l_{80}} = 1 - \frac{10000 + 9400}{2 \times 10000} = 0.03$$

b. ${}_{0.5}q_{80}$ given CFM

$${}_{0.5}q_{80}^{CFM} = 1 - \frac{l_{80.5}^{CFM}}{l_{80}} = 1 - \frac{l_{80}{}^{0.5} \cdot l_{81}{}^{0.5}}{l_{80}} = 1 - \frac{\sqrt{10000 \times 9400}}{10000} \approx 0.030464$$

c. ${}_{0.5}q_{80.75}$ given UDD

$${}_{0.5}q_{80.75}^{UDD} = 1 - \frac{l_{81.25}^{UDD}}{l_{80.75}^{UDD}} = 1 - \frac{0.75l_{81} + 0.25l_{82}}{0.25l_{80} + 0.75l_{81}} \approx 0.037853$$

d. ${}_{0.5}q_{80.75}$ given CFM

$${}_{0.5}q_{80.75}^{CFM} = 1 - \frac{l_{81.25}^{CFM}}{l_{80.75}^{CFM}} = 1 - \frac{l_{81}{}^{0.75} \cdot l_{82}{}^{0.25}}{l_{80}{}^{0.25} \cdot l_{81}{}^{0.75}} \approx 0.038294$$

8. You are given
$_{t|}q_x = 0.05$ for $t = 0,1,2,\ldots,19.$

Calculate $_4q_{x+8}$.

Suppose $l_x = 100$
Then
$$d_x = q_x \cdot l_x = _{0|}q_x \cdot l_x = 0.05 \times 100 = 5$$
$$d_{x+1} = _{1|}q_x \cdot l_x = 0.05 \times 100 = 5$$

In general,
$$d_{x+t} = _{t|}q_x \cdot l_x = 0.05 \times 100 = 5$$

$$l_{x+8} = l_x - \sum_{t=0}^{t=7} d_{x+t} = 100 - 8 \times 5 = 60$$
$$l_{x+12} = l_x - \sum_{t=0}^{t=11} d_{x+t} = 100 - 12 \times 5 = 40$$
$$_4q_{x+8} = 1 - \frac{l_{x+12}}{l_{x+8}} \approx 0.33333$$

9. (SWAQ) David and Adam both work for Lauren who is the Chief Actuary at Baugh Life Insurance Company. Lauren calls both David and Adam into her office and asks them to each calculate $\mu_{80.3}$ and ${}_{0.3}q_{80.4}$.

   David assumes uniform distribution of deaths between integral ages and calculates $\mu_{80.3} = 0.1277126$ and ${}_{0.3}q_{80.4}$.

   Adam assumes a constant force of mortality between integral ages and calculates $\mu_{80.3}$ and ${}_{0.3}q_{80.4}$.

   i. (2 points) Calculate the value of ${}_{0.3}q_{80.4}$ determined by David.

   $$\mu_{x+s}^{UDD} = \frac{q_x}{1 - s \cdot q_x}$$

   $$\Rightarrow q_x = \frac{\mu_{x+s}^{UDD}}{1 + s \cdot \mu_{x+s}^{UDD}} = 0.123$$

   Suppose $l_{80} = 1000$

   Then $l_{81} = (1 - q_{80}) l_{80} = 877$

   $${}_{0.3}q_{80.4}^{UDD} = 1 - \frac{l_{80.7}^{UDD}}{l_{80.4}^{UDD}} = 1 - \frac{0.3 l_{80} + 0.7 l_{81}}{0.6 l_{80} + 0.4 l_{81}} \approx 0.038809$$

   ii. (2 points) Calculate the value of $\mu_{80.3}$ and ${}_{0.3}q_{80.4}$ determined by Adam.

   $${}_{0.3}q_{80.4}^{CFM} = 1 - \frac{l_{80.7}^{CFM}}{l_{80.4}^{CFM}} = 1 - \frac{l_{80}^{\ 0.3} \cdot l_{81}^{\ 0.7}}{l_{80}^{\ 0.6} \cdot l_{81}^{\ 0.4}} \approx 0.038609$$

   $$\mu_{80.3}^{CFM} = -\ln(p_{80}) = -\ln(1 - q_{80}) \approx 0.13125$$

   iii. (2 points) After David and Adam provide their answers to Lauren, she calls them into her office to explain the difference in the answers to explain which number she should use and why. What should David and Adam tell Lauren?

   The difference in their answers comes from the different assumptions they have made. David assumes UDD: the death is uniformly distributed within integral ages; Adam assumes CFM: the force of mortality is identical within integral ages. Both assumptions are fairly accurate. However, UDD is more realistic as it assumes that μ_{x+s} and ${}_{s}q_x$ increase as s increases which is what happens in real life.

10. You are given the following select and ultimate mortality table of q_x's.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
50	0.020	0.031	0.043	0.056	53
51	0.025	0.037	0.050	0.065	54
52	0.030	0.043	0.057	0.072	55
53	0.035	0.049	0.065	0.091	56
54	0.040	0.055	0.076	0.113	57
55	0.045	0.061	0.090	0.140	58

Calculate:

a. $p_{[54]}$

$p_{[54]} = 1 - q_{[54]} = 1 - 0.040 = 0.960$

b. $p_{[53]+1}$

$p_{[53]+1} = 1 - q_{[53]+1} = 1 - 0.049 = 0.951$

c. $p_{[52]+2}$

$p_{[52]+2} = 1 - q_{[52]+2} = 1 - 0.057 = 0.943$

d. $p_{[51]+3}$

$p_{[51]+3} = 1 - q_{[51]+3} = 1 - 0.065 = 0.935$

e. p_{54}

$p_{54} = 1 - q_{54} = 1 - 0.065 = 0.935$

f. ${}_5p_{[54]}$

$${}_5p_{[54]} = \left(1 - q_{[54]}\right)\left(1 - q_{[54]+1}\right)\left(1 - q_{[54]+2}\right)\left(1 - q_{57}\right)\left(1 - q_{58}\right)$$
$$= (1 - 0.04)\cdot(1 - 0.055)\cdot(1 - 0.076)\cdot(1 - 0.113)\cdot(1 - 0.140)$$
$$\approx 0.63944$$

g. $_{2|2}q_{[52]}$

Suppose $l_{[52]} = 1{,}000{,}000$

then

$$l_{[52]+1} = \left(1 - q_{[52]}\right) l_{[52]} = (1 - 0.03) \times 1{,}000{,}000 = 970{,}000$$

$$l_{[52]+2} = \left(1 - q_{[52]+1}\right) l_{[52]+1} = (1 - 0.043) \times 970{,}000 = 928{,}290$$

$$l_{55} = \left(1 - q_{[52]+2}\right) l_{[52]+2} = (1 - 0.057) \times 928{,}290 = 875{,}377.47$$

$$l_{56} = (1 - q_{55}) l_{55} = (1 - 0.072) \times 875{,}377.47 \approx 812{,}350.29$$

$$_{2|2}q_{[52]} = \frac{l_{[52]+2} - l_{[52]+4}}{l_{[52]}} = \frac{l_{[52]+2} - l_{56}}{l_{[52]}} \approx 0.11594$$

h. A life policy insurance policy was issued two years ago to (52). Calculate the probability that this person will live to age 59.

Continue the assumption made in part g.

$$l_{57} = (1 - q_{56}) l_{56} = (1 - 0.091) \times 812{,}350.29 \approx 738{,}426.42$$

$$l_{58} = (1 - q_{57}) l_{57} = (1 - 0.113) \times 738{,}426.42 \approx 654{,}984.23$$

$$l_{59} = (1 - q_{58}) l_{58} = (1 - 0.140) \times 654{,}984.23 \approx 563{,}286.44$$

$$_5 p_{[52]+2} = \frac{l_{59}}{l_{[52]+2}} = \frac{563{,}286.44}{928{,}290} \approx 0.60680$$

i. Clair is 54 and just purchased a life insurance policy. Raf is 54 and purchased a life insurance policy at age 50. How much larger is the probability that Raf will die during the next 4 years than the probability that Clair will die.

$$\Pr[\text{Clair will die during the next 4 years}]$$

$$= {}_4 q_{[54]} = 1 - {}_4 p_{[54]} = 1 - \left(1 - q_{[54]}\right)\left(1 - q_{[54]+1}\right)\left(1 - q_{[54]+2}\right)(1 - q_{57})$$

$$= 1 - (1 - 0.04) \cdot (1 - 0.055) \cdot (1 - 0.076) \cdot (1 - 0.113)$$

$$\approx 0.25647$$

$$\Pr[\text{Raf will die during the next 4 years}]$$

$$= {}_4 q_{[50]+4} = {}_4 q_{54} = 1 - {}_4 p_{54} = 1 - (1 - q_{54})(1 - q_{55})(1 - q_{56})(1 - q_{57})$$

$$= 1 - (1 - 0.065) \cdot (1 - 0.072) \cdot (1 - 0.091) \cdot (1 - 0.113)$$

$$\approx 0.30040$$

$$\Delta \Pr = 0.30040 - 0.25647 = 0.04393$$

11.

12. You are given the following select mortality table.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
90	0.04	0.10	0.17	0.20	93
91	0.06	0.14	0.18	0.30	94
92	0.08	0.16	0.27	0.40	95
93	0.14	0.24	0.36	0.50	96
94	0.21	0.32	0.45	0.70	97
95	0.28	0.40	0.63	0.90	98
96	0.35	0.56	0.81	1.00	99

Calculate $e_{[94]}$ and e_{94}.

$$e_{[94]} = \sum_{t=1}^{\infty} {}_t p_{[94]} = {}_1 p_{[94]} + {}_2 p_{[94]} + {}_3 p_{[94]} + {}_4 p_{[94]} + {}_5 p_{[94]}$$
$$= p_{[94]}$$
$$+ p_{[94]} \cdot p_{[94]+1}$$
$$+ p_{[94]} \cdot p_{[94]+1} \cdot p_{[94]+2}$$
$$+ p_{[94]} \cdot p_{[94]+1} \cdot p_{[94]+2} \cdot p_{97}$$
$$+ p_{[94]} \cdot p_{[94]+1} \cdot p_{[94]+2} \cdot p_{97} \cdot p_{98}$$
$$= p_{[94]} \left(1 + p_{[94]+1} \left(1 + p_{[94]+2} \left(1 + p_{97} \left(1 + p_{98}\right)\right)\right)\right)$$
$$= 0.79\left(1 + 0.68 \times \left(1 + 0.55 \times \left(1 + 0.3 \times \left(1 + 0.1\right)\right)\right)\right)$$
$$\approx 1.72016$$

Similarly,

$$e_{94} = p_{94} \left(1 + p_{95} \left(1 + p_{96} \left(1 + p_{97} \left(1 + p_{98}\right)\right)\right)\right)$$
$$= 0.7\left(1 + 0.6 \times \left(1 + 0.5 \times \left(1 + 0.3 \times \left(1 + 0.1\right)\right)\right)\right)$$
$$= 1.3993$$

You are given the following select and ultimate mortality table of q_x 's to be used for Numbers 13-15.

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
50	0.020	0.031	0.043	0.056	53
51	0.025	0.037	0.050	0.065	54
52	0.030	0.043	0.057	0.072	55
53	0.035	0.049	0.065	0.091	56
54	0.040	0.055	0.076	0.113	57
55	0.045	0.061	0.090	0.140	58

13. If deaths are uniformly distributed between integral ages.
    Calculate ${}_{1.5}q_{[53]+2}$.

    Suppose $l_{[53]+2} = 1,000,000$

    Then

$$l_{56} = \left(1 - q_{[53]+2}\right) l_{[53]+2} = \left(1 - 0.065\right) \times 1,000,000 = 935,000$$
$$l_{57} = \left(1 - q_{56}\right) l_{56} = \left(1 - 0.091\right) \times 935,000 = 849,915$$
$${}_{1.5}q_{[53]+2} = 1 - \frac{l_{56.5}}{l_{[53]+2}} = 1 - \frac{0.5 l_{56} + 0.5 l_{57}}{l_{[53]+2}} \approx 0.10754$$

14. If $l_{[51]} = 100,000$, calculate $l_{[50]}$.

$$l_{[51]+1} = \left(1 - q_{[51]}\right) l_{[51]} = (1 - 0.025) \times 100,000 = 97,500$$

$$l_{[51]+2} = \left(1 - q_{[51]+1}\right) l_{[51]+1} = (1 - 0.037) \times 97,500 = 93,892.5$$

$$l_{54} = \left(1 - q_{[51]+2}\right) l_{[51]+2} = (1 - 0.050) \times 93,892.5 = 89,197.875$$

$$l_{[50]+3} = l_{53} = \frac{l_{54}}{1 - q_{53}} = \frac{89,197.875}{1 - 0.056} \approx 94,489.274$$

$$l_{[50]+2} = \frac{l_{53}}{1 - q_{[50]+2}} = \frac{94,489.274}{1 - 0.043} \approx 98,734.874$$

$$l_{[50]+1} = \frac{l_{[50]+2}}{1 - q_{[50]+1}} = \frac{98,734.874}{1 - 0.031} \approx 101,893.575$$

$$l_{[50]} = \frac{l_{[50]+1}}{1 - q_{[50]}} = \frac{101,893.575}{1 - 0.02} \approx 103,973.036$$

15. Trout Life Insurance Company has two cohorts of policyholders.

    Cohort A has 1000 insured lives who are all age 53 and were just underwritten today.
    Cohort B has 1000 insured lives who are all age 53 and were underwritten 3 years ago.

    Calculate the total number of insured lives that will still be alive after 2 years.

    Cohort A:

    $l_{[53]} = 1,000$

    $l_{[53]+1} = \left(1 - q_{[53]}\right) l_{[53]} = (1 - 0.035) \times 1000 = 965$

    $l_{[53]+2} = \left(1 - q_{[53]+1}\right) l_{[53]+1} = (1 - 0.049) \times 965 = 917.715$

    Cohort B:

    $l_{[50]+3} = l_{53} = 1,000$

    $l_{54} = (1 - q_{53}) l_{53} = (1 - 0.056) \times 1000 = 944$

    $l_{55} = (1 - q_{54}) l_{54} = (1 - 0.065) \times 944 = 882.64$

    $l_{total} = l_{[53]+2} + l_{55} = 917.715 + 882.64 = 1800.355$

16. For a two year select and ultimate table, you are given:
    i. $q_{[x]} = 0.50 q_x$
    ii. $q_{[x]+1} = 0.75 q_{x+1}$

Complete the following table:

$[x]$	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}
105	1650	1600	1500
106	1473.1458		1200
107			800
108			400
109			100

$$q_{107} = 1 - \frac{l_{108}}{l_{107}} = 1 - \frac{1200}{1500} = 0.2$$

$$q_{[106]+1} = 0.75 q_{107} = 0.15$$

$$l_{[106]+1} = \frac{l_{108}}{1 - q_{[106]+1}} = \frac{1200}{1 - 0.15} = 1411.76471$$

$$q_{108} = 1 - \frac{l_{109}}{l_{108}} = 1 - \frac{800}{1200} = 0.33333$$

$$q_{[107]+1} = 0.75 q_{107} = 0.25$$

$$l_{[107]+1} = \frac{l_{109}}{1 - q_{[107]+1}} = \frac{800}{1 - 0.25} = 1066.66667$$

$$q_{[107]} = 0.5 q_{107} = 0.1$$

$$l_{[107]} = \frac{l_{[107]+1}}{1 - q_{[107]}} = 1185.18519$$

$$q_{109} = 1 - \frac{l_{110}}{l_{109}} = 1 - \frac{400}{800} = 0.5$$

$$q_{[108]+1} = 0.75 q_{109} = 0.375$$

$$l_{[108]+1} = \frac{l_{110}}{1 - q_{[108]+1}} = 640$$

$$q_{[108]} = 0.5q_{108} = 0.16667$$

$$l_{[108]} = \frac{l_{[108]+1}}{1-q_{[108]}} = \frac{640}{1-0.16667} = 768$$

$$q_{110} = 1 - \frac{l_{111}}{l_{110}} = 1 - \frac{100}{400} = 0.75$$

$$q_{[109]+1} = 0.75q_{110} = 0.5625$$

$$l_{[109]+1} = \frac{l_{111}}{1-q_{[109]+1}} = \frac{100}{1-0.5625} \approx 228.57143$$

$$q_{[109]} = 0.5q_{109} = 0.25$$

$$l_{[109]} = \frac{l_{[109]+1}}{1-q_{[109]}} = \frac{228.57143}{1-0.25} \approx 304.76190$$

Therefore, the table becomes

$[x]$	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}
105	1650	1600	1500
106	1518	1411.76	1200
107	1185.19	1066.67	800
108	768	640	400
109	304.76	228.57	100

17. *For a 2-year select and ultimate mortality model, you are given:

iii. $q_{[x]+1} = 0.80q_{x+1}$

iv. $l_{51} = 100,000$

v. $l_{52} = 99,000$

Calculate $l_{[50]+1}$.

$$q_{51} = 1 - \frac{l_{52}}{l_{51}} = 1 - \frac{99,000}{100,000} = 0.01$$

$$q_{[50]+1} = 0.8q_{51} = 0.008$$

$$l_{[50]+1} = \frac{l_{52}}{1 - q_{[50]+1}} \approx 99,798.3871$$

18. *You are given:

    vi. $p_x = 0.95$

    vii. $p_{x+1} = 0.92$

    viii. $e_{x+1.6} = 12$

    ix. Deaths are uniformly distributed between ages x and $x+1$.

    x. The force of mortality is constant between ages $x+1$ and $x+2$.

    Calculate $e_{x+0.6}$.

    Suppose $l_x = 1,000$

    $$l_{x+1} = p_x \cdot l_x = 0.95 \times 1,000 = 950$$

    $$l_{x+2} = p_{x+1} \cdot l_{x+1} = 0.92 \times 950 = 874$$

    $$p_{x+0.6} = \frac{l_{x+1+0.6}}{l_{x+0.6}} = \frac{{l_{x+1}}^{0.4} \cdot {l_{x+2}}^{0.6}}{0.4 l_x + 0.6 l_{x+1}} = \frac{950^{0.4} \cdot 874^{0.6}}{0.4 \times 1,000 + 0.6 \times 950} \approx 0.93159$$

    $$e_{x+0.6} = \sum_{t=1}^{\infty} {}_t p_{x+0.6}$$

    $$= p_{x+0.6} + \sum_{t=2}^{\infty} {}_t p_{x+0.6}$$

    $$= p_{x+0.6} + \sum_{k=1}^{\infty} p_{x+0.6} \cdot {}_k\, p_{x+1.6}$$

    $$= p_{x+0.6} + p_{x+0.6} \cdot e_{x+1.6}$$

    $$= p_{x+0.6}\left(1 + e_{x+1.6}\right)$$

    $$= 0.93159 \times (1+12)$$

    $$= 12.11066$$

19. You are given the following two year select and ultimate mortality table:

x	$q_{[x]}$	$q_{[x-1]+1}$	q_x
70	0.01	0.03	0.06
71	0.02	0.05	0.09
72	0.04	0.07	0.12
73	0.06	0.12	0.15
74	0.10	0.18	0.20
75	0.15	0.22	0.25

Calculate

i. $_3p_{[70]}$

$$_3p_{[70]} = \frac{l_{[70]+3}}{l_{[70]}}$$

$$l_{[70]} = 1000$$

$$l_{[70]+1} = 1000(1-0.01) = 990$$

$$l_{[70]+2} = 990(1-0.05) = 940.5$$

$$l_{[70]+3} = 940.5(1-0.12) = 827.64$$

$$_3p_{[70]} = \frac{827.64}{1000} = 0.82764$$

ii. $_2q_{[71]+1}$

$$_2q_{[71]+1} = \frac{l_{[71]+1} - l_{[71]+3}}{l_{[71]+1}}$$

I will create new l's here that are not related to the l's in Part a.

$$l_{[71]+1} = 1000; l_{[71]+3} = (1000)(1-0.07)(1-0.15) = 790.5$$

$$_2q_{[71]+1} = \frac{l_{[71]+1} - l_{[71]+3}}{l_{[71]+1}} = \frac{1000-790.5}{1000} = 0.2095$$

iii. ${}_{1|2}q_{[70]}$

Using the l's from Part a, ${}_{1|2}q_{[70]} = \dfrac{l_{[70]+1} - l_{[70]+3}}{l_{[70]}} = \dfrac{990 - 827.64}{1000} = 0.16236$