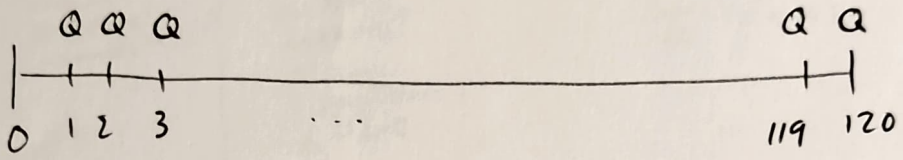
MATH 373
Quiz 3
Spring 2024
February 22, 2024

1. **Poppy borrows 100,000 to be repaid with level monthly payments for the next 10 years. The interest rate on her loan is an annual effective rate of 8%.**

   Calculate Poppy's monthly payment.

$i = 0.08$



100,000

$$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = 1.08$$

$$\frac{i^{(12)}}{12} = \sqrt[12]{1.08} - 1 = 0.00643403$$

$$100{,}000 = Q\, a_{\overline{120}|}$$

$$100{,}000 = Q\left(\frac{1 - \left(\frac{1}{1.00643403}\right)^{120}}{0.00643403}\right) \quad \therefore Q = \boxed{1198.5753}$$

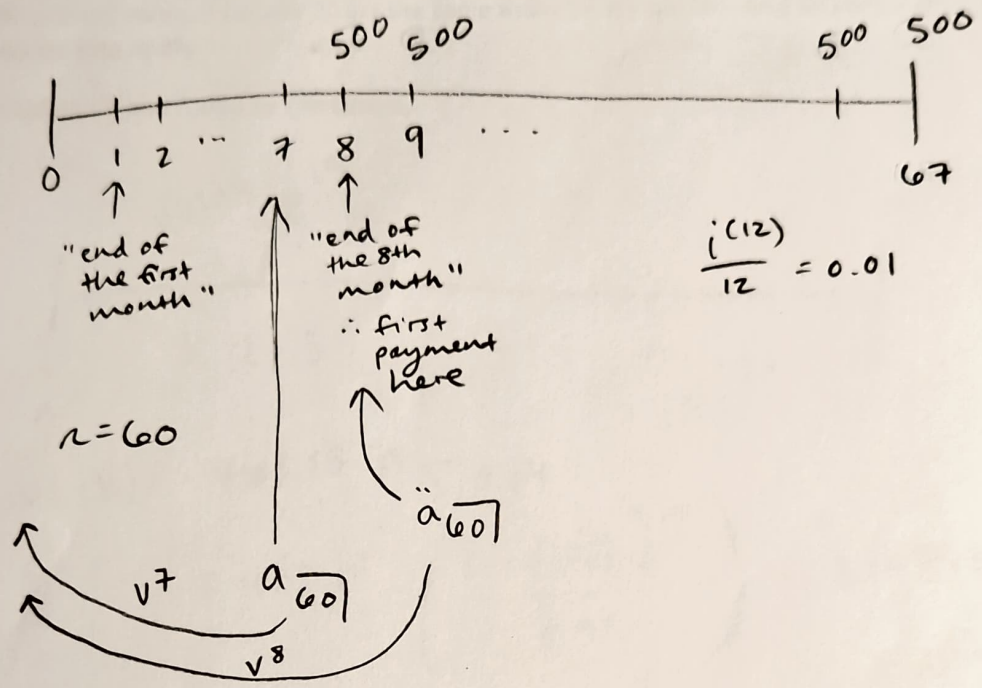
On Calculator

PV = 100,000
I/Y = 0.643403011
FV = 0
N = 120
CPT PMT = 1198.5753

2. Cara is the beneficiary of a deferred annuity. The deferred annuity will pay her 60 monthly payments of 500 with the first payment at the end of 8 months.

Calculate the present value of Cara's annuity at a monthly effective interest rate of 1%.

$\frac{i^{(12)}}{12}$



$\frac{i^{(12)}}{12} = 0.01$

$$PV = \left[500\, a_{\overline{60}|}\right] v^7 = \left[500\, \ddot{a}_{\overline{60}|}\right] v^8$$

$$= \left(\frac{1}{1.01}\right)^7 (500)\left(\frac{1-\left(\frac{1}{1.01}\right)^{60}}{0.01}\right) = \left(\frac{1}{1.01}\right)^{\cancel{8}\,7} (500)\left(\frac{1-\left(\frac{1}{1.01}\right)^{60}}{\frac{0.01}{1.01}}\right)$$

$$= 500\left(\frac{1}{1.01}\right)^7 \left(\frac{1-\left(\frac{1}{1.01}\right)^{60}}{0.01}\right)$$

$$= 500\,(0.932718)(44.955038) = \boxed{20,965.188}$$

3. Leon received a birthday gift from his grandparents today, at his age 20. However, instead of giving it to him as a lump sum, they would like to give him 2 options in which to receive it.

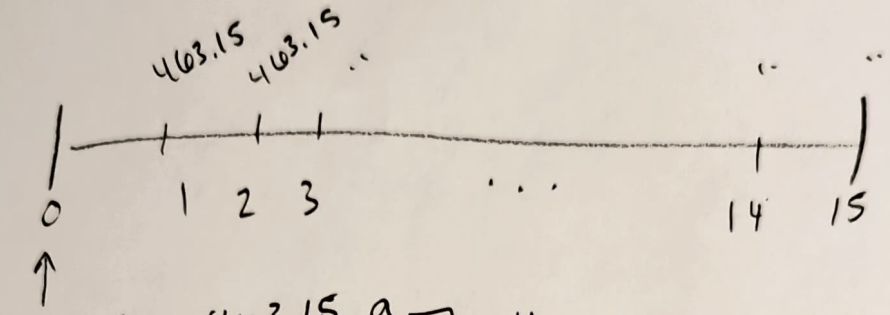
   a. A perpetuity due with semiannual payments of P.

   b. An annuity immediate that pays 463.15 each year for 15 years.

The present value of (a) and (b) are the same today (at his age 20) using an annual effective interest rate of 4%.

Calculate P and round to 2 decimals.

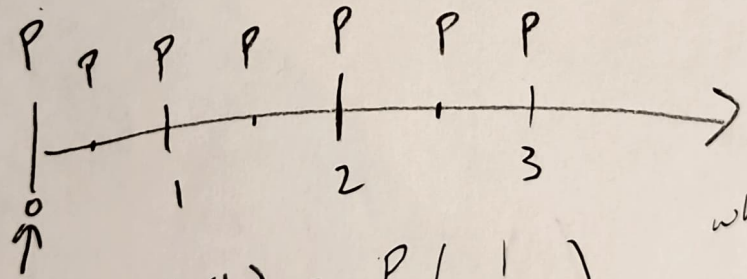
(b)



$$PV(b) = 463.15\, a_{\overline{15}|0.04}$$

$$= 463.15 \left(\frac{1 - \left(\frac{1}{1.04}\right)^{15}}{0.04} \right) = 5149.481139$$

(a)



where $\frac{d^{(2)}}{2}$ = semiannual effective discount rate

$$PV(a) = PV(b) = P\left(\frac{1}{\left[\frac{d^{(2)}}{2}\right]}\right)$$

recall $\left(1.04\right) = \left(1 - \frac{d^{(2)}}{2}\right)^{-2}$

$$\frac{d^{(2)}}{2} = 0.019419324$$

$$5149.481139 = P\left(\frac{1}{0.019419324}\right)$$

$$P = 99.99944 \Longrightarrow \boxed{100}$$