Math 373
Quiz 1
Spring 2024
January 18, 2024

1. Under a simple interest rate arrangement, an investment grows from 300 to 510 in 10 years. Calculate the effective interest rate in the 8th year.

$a(t) = 1 + st$

8th year is from 7 to 8

$$i_8 = i_{[7,8]} = \frac{a(8) - a(7)}{a(7)} = \frac{[1 + (0.07)(8)] - [1 + (0.07)(7)]}{[1 + (0.07)(7)]}$$

$$= \frac{1.56 - 1.49}{1.49} = \frac{0.07}{1.49}$$

$$= 0.04697987$$

① what's s?

$300\, a(10) = 510$

$300\,[1 + s(10)] = 510$

$s = 0.07$

or

$$i_8 = i_{[7,8]} = \frac{s}{1 + s(n-1)}$$

$$= \frac{0.07}{1 + (0.07)(7)}$$

$$= \frac{0.07}{1.49}$$

2. Raphael invests X in an account that earns 5% compound interest.

Donatello invests 488.67 in an account that earns simple interest of s.

They earn the same effective interest rate in the 6th year and their account balance is the same at the end of the 10th year.

Solve for X.

$i_6 = i_{[5,6]}$

Raphael

Donatello

(1) find s.

$$0.05 = \frac{a(6)-a(5)}{a(5)} = \frac{s}{1+5s}$$

$$0.05 + 0.25s = s$$

$$0.75s = 0.05$$

$$\therefore s = 0.0666\overline{6}$$

(2) $488.67\, a(10) = 488.67\left[1 + (10)(0.0\overline{6})\right] = 814.45$

(3) $X(1.05)^{10} = 814.45$

$$X = 500.0016493$$

$$\boxed{500}$$

$= 1 + \beta t + \omega t^3$

3. Splinter invests 200 in a fund that has an accumulation function of $a(t) = \alpha + \beta t + \omega t^3$. At the end of one year, Splinter has 215. At the end of three years, Splinter has 250.

Determine the amount that Splinter has at the end of five years.

$\alpha = 1$ $\quad a(0) = 1$

$a(0) = \alpha + 0 + 0 = 1 = \alpha$

① $200\,a(1) = 215$

$200\left[1 + \beta + \omega\right] = 215$

$\beta + \omega = 0.075$

$\beta = 0.075 - \omega$

② $200\,a(3) = 250$

$200\left[1 + 3\beta + 27\omega\right] = 250$

substitute

$1 + 3\left[0.075 - \omega\right] + 27\omega = 1.25$

$24\omega = 0.025$

$\omega = 0.00104166\overline{7}$

$\beta = 0.075 - 0.00104166\overline{7} = 0.0739583\overline{3}$

1.5

③ $200\,a(5) = 200\left[1 + 5\left(0.0739583\overline{3}\right) + (5)^3\left(0.00104166\overline{7}\right)\right]$

$= \boxed{300}$