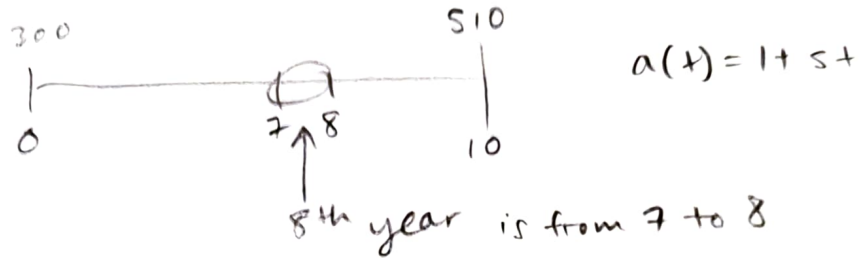


Math 373
Quiz 1
Spring 2024
 January 18, 2024

1. Under a simple interest rate arrangement, an investment grows from 300 to 510 in 10 years. Calculate the effective interest rate in the 8th year.



$$i_8 = i_{[7,8]} = \frac{a(8) - a(7)}{a(7)} = \frac{[1 + (0.07)(8)] - [1 + (0.07)(7)]}{[1 + (0.07)(7)]}$$

$$= \frac{1.56 - 1.49}{1.49} = \frac{0.07}{1.49}$$

① what's s ?

$$300 a(10) = 510$$

$$300 [1 + s(10)] = 510$$

$$s = 0.07$$

$$= \boxed{0.0469799}$$

or

$$i_8 = i_{[7,8]} = \frac{s}{1 + s(n-1)}$$

$$= \frac{0.07}{1 + (0.07)(7)}$$

$$= \frac{0.07}{1.49}$$

2. Raphael invests X in an account that earns 5% compound interest.

Donatello invests 488.67 in an account that earns simple interest of s .

They earn the same effective interest rate in the 6th year and their account balance is the same at the end of the 10th year.

Solve for X .

$$i_6 = i_{[5,6]}$$

Raphael



Donatello



① Find s .

$$0.05 = \frac{a(6) - a(5)}{a(5)} = \frac{s}{1 + 5s}$$

$$0.05 + 0.25s = s$$

$$0.75s = 0.05$$

$$\therefore s = 0.0666\bar{6}$$

$$\textcircled{2} \quad 488.67 a(10) = 488.67 [1 + (10)(0.06\bar{6})] = 814.45$$

$$\textcircled{3} \quad X (1.05)^{10} = 814.45$$

$$X = 500.0016493$$

$$\boxed{500}$$

3. Splinter invests 200 in a fund that has an accumulation function of $a(t) = \alpha + \beta t + \omega t^3$. At the end of one year, Splinter has 215. At the end of three years, Splinter has 250.

Determine the amount that Splinter has at the end of five years.

$$\alpha = 1$$

$$a(0) = 1$$

$$a(0) = \alpha + 0 + 0 = 1 = \alpha$$

$$(1) \quad 200 a(1) = 215$$

$$(2) \quad 200 a(3) = 250$$

$$200[1 + \beta + \omega] = 215$$

$$200[1 + 3\beta + 27\omega] = 250$$

$$\beta + \omega = 0.075$$

$$\beta = 0.075 - \omega$$

substitute

$$1 + 3[0.075 - \omega] + 27\omega = 1.25$$

$$24\omega = 0.025$$

$$\omega = 0.001041667$$

$$\beta = 0.075 - 0.001041667 = 0.07395833$$

$$(3) \quad 200 a(5) = 200 \left[1 + 5 \overbrace{(0.07395833)}^{1.5} + (5)^3 (0.001041667) \right]$$

$$= \boxed{300}$$