MATH 373
Quiz 4
Spring 2024
March 7, 2024

1. You borrow 200,000 to be repaid with level monthly payments of 2,000. You have the choice of paying the loan off with a final balloon payment, B, or a final drop payment, D. The interest rate on your loan is an annual effective rate of 5%.

   need $\frac{i^{(12)}}{12}$, given i

   Calculate $B-D$.

200,000

$\left(1+\frac{i^{(12)}}{12}\right)^{12} = 1.05$

$\frac{i^{(12)}}{12} = 0.004074124$

Find n

$200{,}000 = 2000\, a_{\overline{n}|\frac{i^{(12)}}{12}}$ & solve for n

or on calculator

PV = 200,000
PMT = -2000
I/Y = 0.4074123 78 (enter as a %)
FV = 0
CPT N = 128.695578

∴ B ⓐ 128
D ⓐ 129

Find OLB 128

2ND AMORT P1 = 1 P2 = 128 BAL = 1386.37

∴ B = 2,000 + 1386.37 = 3386.37

D = 1386.37 (1.004074124) = 1392.017

B - D = **1994.35**

or $B = (200{,}000)(1.004074)^{128} - 2000\, \ddot{s}_{\overline{127}|0.004074124}$

$D = (200{,}000)(1.004074)^{129} - 2000\, \ddot{s}_{\overline{128}|0.004074124}$

* should have clarified (b) increases arithmetically, but gave credit if worked correctly for interpreting as geometric

2. You have the option to purchase one of the following:

   a. A perpetuity where the first payment occurs at the beginning of the first year in the amount of 100. Each quarter thereafter, the payment increases by 1%.

   b. A annuity with 20 annual payments. The first payment occurs at the end of the first year in the amount of 500. Each year thereafter, the payment increases by a constant, Q.

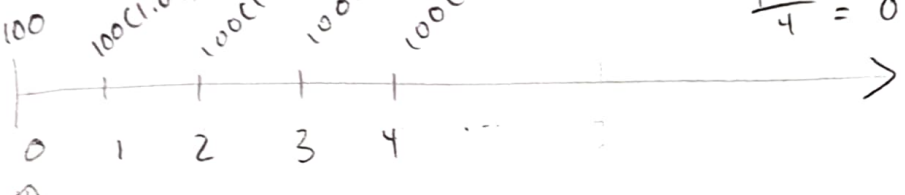
The present value of these options are equivalent today using an annual effective interest rate of 7%.

Calculate Q.

$\left(1+\frac{i^{(4)}}{4}\right)^4 = 1.07$

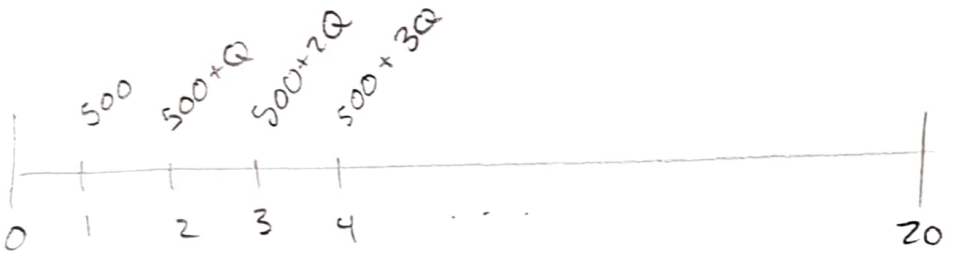
$\frac{i^{(4)}}{4} = 0.0170585$

a)



$PV = \frac{100-0}{1-1.01v} = \frac{100}{1-\left(\frac{1.01}{1.0170585}\right)} = 14,408.938$

b)



$PV = 500\,a_{\overline{20}|0.07} + \frac{Q}{0.07}\left(a_{\overline{20}|0.07} - 20\left(\frac{1}{1.07}\right)^{20}\right)$

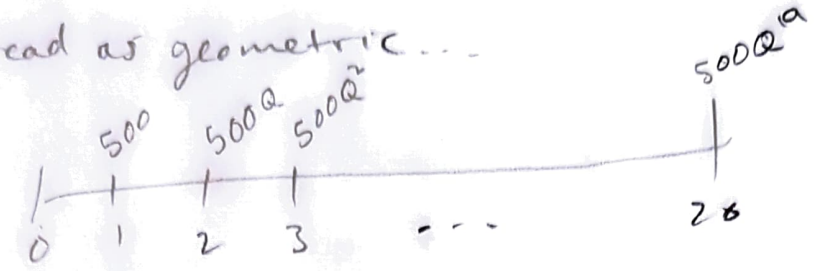
$a_{\overline{20}|0.07} = 10.594014$

$14,408.938 = 5297.007 + 77.50905985\,Q$

$77.50905985\,Q = 9111.9314$

$Q = \boxed{117.5596}$

if read as geometric...



ratio = QV

$$14408.938 = \frac{500V - 500Q^{20}V^{21}}{1 - QV}$$

$$14408.938\left(1 - \frac{Q}{1.07}\right) = \frac{500}{1.07} - 500\frac{Q^{20}}{(1.07)^{21}}$$

... how do you solve?