MATH 373

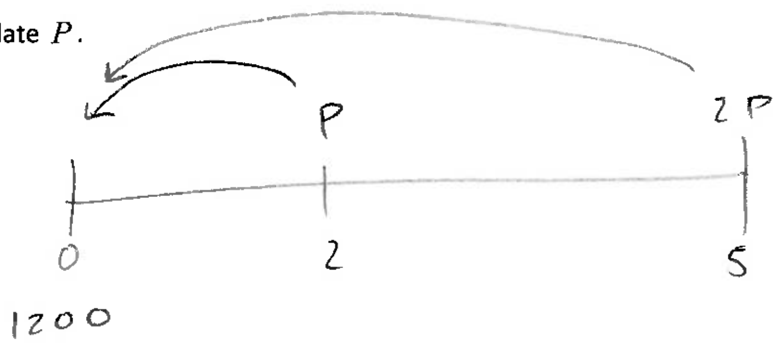
Test 1

Spring 2024

February 8, 2024

1. Pete borrows 1200 to build a motorized couch. He will repay the loan with payments of P at time 2 and $2P$ at time 5. The loan has an annual effective interest rate of 6.0%.

Calculate P.



$i = 0.06$

$$1200 = \frac{P}{(1.06)^2} + \frac{2P}{(1.06)^5}$$

$$1200 = 2.3845128P$$

$$P = \boxed{503.25}$$

money out

2. Painter Automotive is building a new factory. Painter will invest 100 million at time 0 to build the factory. Additionally, Painter expects to receive the following profits from the factory over the next four years:

Time	Amount
1	20 million
2	X million
3	40 million
4	36 million

After 4 years, the factory will be obsolete and will no longer be used.

→ by definition, where NPV=0

This factory will result in an Internal Rate of Return of 10% based on the above cash flows.

Calculate the Net Present Value at an interest rate of 12%.

$$NPV = -100 + \frac{20}{1.10} + \frac{x}{(1.1)^2} + \frac{40}{(1.1)^3} + \frac{36}{(1.1)^4} = 0$$

on calculator

CF0 = -100
C01 = 20
F01 = 1
C02 = 0
F02 = 1
C03 = 40
F03 = 1
C04 = 36
F04 = 1

I = 10
↓ CPT NPV = -27.1771

$$NPV = -27.1771 + \frac{x}{(1.1)^2} = 0$$

$$x = 27.1771(1.1)^2 = 32.88429752$$

→ add in C02 = 32.88429752
I = 12
↓ CPT NPV = $\boxed{-4.57783576}$ million

3. You are given that:

a. $v(t) = \dfrac{1}{\alpha + \beta t}$

b. $\delta_{10} = \dfrac{2}{45}$

Calculate the effective interest rate in the 10th year.

$a(t) = \alpha + \beta t$

recall $a(0) = 1$

$\therefore \; a(0) = \alpha + \beta(0) = 1$

$\alpha = 1$

$a(t) = 1 + \beta t \qquad a'(t) = \beta$

$$\delta_{10} = \frac{a'(10)}{a(10)} = \frac{\beta}{1+\beta(10)} = \frac{2}{45}$$

$45\beta = 2 + 20\beta$

$25\beta = 2$

$\beta = \dfrac{2}{25} = 0.08$

need i_{10}

$$i_{10} = \frac{a(10) - a(9)}{a(9)} = \frac{(\cancel{1} + (0.08)(10)) - (\cancel{1} + (0.08)(9))}{(1 + (0.08)(9))}$$

or recognize $a(t)$ is acc. function for simple interest and jump straight to here

$$= \frac{0.08}{1 + (0.08)(9)} = \frac{0.08}{1.72} = \boxed{0.046511628}$$

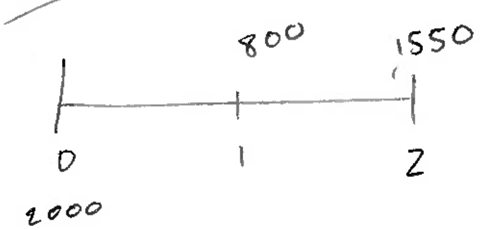
4. Project A requires an investment of 2000 today. The investment pays 800 one year from today and 1550 two years from today.

Project B requires an investment of 1000 immediately and X two years from today. The investment pays 400 at the end of one year and 1400 at the end of three years.

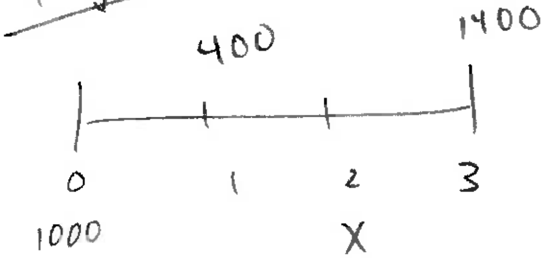
The two projects have equal net present values when calculated using 6% annual effective interest rate.

Find the absolute value of the difference in the net present values of the two projects if they are calculated using an 8% annual effective interest rate.

Project A



Project B



$$NPV(A) = NPV(B) \quad @ \quad i = 0.06$$

$$-2000 + \frac{800}{1.06} + \frac{1550}{(1.06)^2} = \boxed{-1000 + \frac{400}{1.06}} - \frac{X}{(1.06)^2} + \boxed{\frac{1400}{(1.06)^3}}$$

CF0 = -2000
C01 = 800
F01 = 1
C02 = 1550
F02 = 1
I = 6, NPV = 134.21146

CF0 = -1000
C01 = 400
F01 = 1
C02 = 0
F02 = 1
C03 = 1400
F03 = 1
I = 6, NPV = 552.825

recalc @ 8%

NPV = 69.6159

$$134.21146 = 552.825 - \frac{X}{(1.06)^2}$$

$$X = 470.354717$$

recalc @ 8% after add C02 = -470.354717

NPV = 78.48215

$$|69.6159 - 78.48215| = \boxed{8.8662374}$$

5. Using the Rule of 72 and a compound interest rate of 6%, it can be estimated that 10 will grow to 20 in n years.

Under simple interest, assuming a simple interest rate equal to 6%, 10 will grow to 20 in t years.

Calculate $t - n$.

Rule of 72

$$n = \frac{72}{6} = 12$$

Simple Interest

$$a(t) = 1 + 0.06t$$

$$10\,a(t) = 20$$

$$10(1 + 0.06t) = 20$$

$$1 + 0.06t = 2$$

$$0.06t = 1$$

$$t = 16.\overline{6}$$

$$t - n = 16.\overline{6} - 12 = \boxed{4.\overline{6}}$$

6. You are given the following:

   i. $a(t) = 1 + 0.02t^2$

   ii. $\delta_5 = \delta_t$ where t is an integer greater than 5. (Hint: this expression means that there is some other t value at which the force of interest for the t[th] year matches the force of interest in the fifth year).

Determine t.

$$\delta_t = \frac{a'(t)}{a(t)} = \frac{0.02(2)t}{1+0.02t^2} = \frac{0.04t}{1+0.02t^2}$$

$$\delta_5 = \frac{0.04(5)}{1+0.02(5)^5} = \frac{0.2}{1.5} = 0.1\overline{3}$$

$$\frac{0.2}{1.5} = \frac{0.04t}{1+0.02t^2}$$

$$0.2 + 0.004t^2 = 0.06t$$

$$t^2 - 15t + 50 = 0$$

quadratic formula

$$t = \frac{15 \pm \sqrt{(-15)^2 - 4(50)}}{2}$$

$$= \frac{15 \pm 5}{2} = 10, 5$$

$$\boxed{10}$$

7. Ben has the option to invest in either Account A or Account B:

   a. Account A pays a rate equivalent to $d^{(12)}$ for ten years. $i^{(4)} = 0.06$

   b. Account B pays nominal annual interest rate of 6% compounded quarterly for the first six years and then earns a force of interest of $\delta_t = 0.004t$ for the next four years.

If Ben invests 600 in either account, he will have the same amount at the end of 10 years.

Determine $d^{(12)}$.

$e^{0.128}$

Account A — Account B

$$\cancel{600}\left[\left(1-\frac{d^{(12)}}{12}\right)^{-12}\right]^{10} = \cancel{600}\left[\left(1+\frac{0.06}{4}\right)^{4}\right]^{6} e^{\int_6^{10} 0.004t\,dt}$$

$$= (1.4295028)\, e^{\left.\frac{0.004t^2}{2}\right|_6^{10}}$$

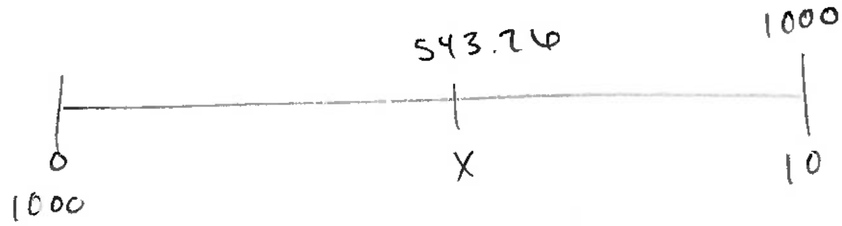
$$\left(1-\frac{d^{(12)}}{12}\right)^{-120} = (1.4295028)(1.136553)$$

$$\left(1-\frac{d^{(12)}}{12}\right) = \sqrt[120]{\frac{1}{(1.4295028)(1.136553)}}$$

$$d^{(12)} = \boxed{0.0484346}$$

8. Harvey borrows 1000 to buy a new fridge. He will repay the loan with payments of 543.26 at time X and 1000 at time 10. The loan has an annual effective interest rate of 5.0%.

Calculate X.



$i = 0.05$

$$1000 = \frac{543.26}{(1.05)^x} + \frac{1000}{(1.05)^{10}}$$

$$1000 - 613.91325 = \frac{543.26}{(1.05)^x}$$

$$(1.05)^x (386.0867) = 543.26$$

$$(1.05)^x = 1.407093$$

$$x \ln(1.05) = \ln(1.407093)$$

$$x = 6.99989$$

$$\approx \boxed{7}$$

9. You are given that $i = 6\%$.

Let X be the nominal annual interest rate compounded every other year. $X = i^{(1/2)}$

Let Y be the effective monthly discount rate. $Y = \frac{d^{(12)}}{12}$

Calculate $1000(X - Y)$.

$$(1+i) = \left(1 + \frac{i^{(1/2)}}{1/2}\right)^{1/2} = \left(1 - \frac{d^{(12)}}{12}\right)^{-12}$$

Find X

$$1.06 = \left(1 + \frac{X}{1/2}\right)^{1/2}$$

$$(1.06)^2 = 1 + 2X$$

$$X = 0.0618$$

Find Y

$$1.06 = (1 - Y)^{-12}$$

$$\sqrt[12]{\frac{1}{1.06}} = (1-Y)$$

$$Y = 0.004843972$$

$$\Rightarrow \quad 1000(X-Y) = \boxed{56.95603}$$

10. You invest 1000 in Bank ABC. Bank ABC pays a simple interest rate of s. In the 10th year, you earn an annual effective interest rate of 5%. $i_{10} = 0.05$

Calculate the amount of money you will have at the end of the 10th year.

s

$$i_{10} = \frac{a(10) - a(9)}{a(9)} = \frac{s}{1 + 9s} = 0.05$$

$$0.05(1 + 9s) = s$$

$$0.05 = 0.55s$$

$$s = 0.\overline{09}$$

Calculate X

$$a(t) = 1 + (0.\overline{09})t$$

→ linear function for simple interest

$$X = 1000\left(1 + (0.\overline{09})(10)\right)$$

$$= \boxed{1909.09091}$$

11. Phineas invests in private equity. You are given:

a. The account earns simple interest at an annual rate of 10%. $s = 0.10$

b. $i_n = 4.0\%$

Ferb needs the same accumulated value at time n as Phineas, but he wants to invest in an account that earns compound interest at an annual effective interest rate of $X\%$.

Both invest the same amount at time 0.

Calculate X.

$$i_n = 0.04 = \frac{a(n)-a(n-1)}{a(n-1)} = \frac{0.10}{1+0.10(n-1)}$$

$$0.04(1+0.10n-0.10) = 0.10$$

$$0.004n = 0.064$$

$$n = \underline{16}$$

$i = X\%$

$$\cancel{K}\left(1+\frac{X}{100}\right)^{16} = \cancel{K}\left(1+16(0.10)\right)$$

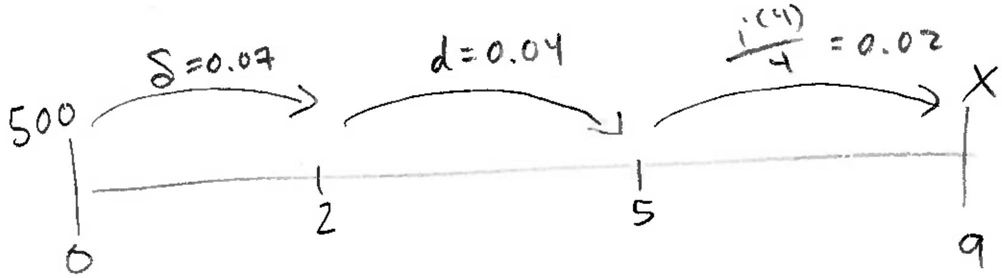
$$\sqrt[16]{\left(1+\frac{X}{100}\right)^{16}} = \sqrt[16]{2.6}$$

$$\frac{X}{100} = 0.0615387$$

$$X = \boxed{6.1538706}\ \%$$

12. Specter invests 500 in a fund for nine years. During the first two years, Specter earns a force of interest of 7%. For the next three years, Specter earns an annual effective discount rate of 4%. For the last four years, Specter earns a quarterly effective interest rate of 2%. $\frac{i^{(4)}}{4}$

Determine the annual effective interest rate that Specter earned over the nine year period



$$X = 500\, e^{0.07(2)} (1-0.04)^{-3} \left[(1+0.02)^4\right]^4$$

$$= 500\,(1.1502738)(1.13028067)(1.3727857)$$

$$= 892.401478$$

$$892.401478 = 500(1+i)^9$$

$$i = \boxed{0.0664843}$$