non-MC

3.75

imm

a)

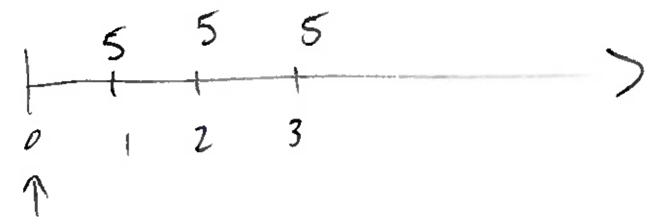
1 ~~6~~. At an annual effective interest rate of i, the present value of a perpetuity paying 5 at the end of each year, with the first payment at the end of year 1, is 40.

b)

due At the same annual effective rate of i, the present value of a perpetuity paying 1 at the beginning of the first quarter (and each subsequent payment being 1% higher than the preceding), with the first payment at time 0, is X. } re-word

Calculate X.

a)



PV=40

$$40 = 5\,a_{\overline{\infty}|i}$$

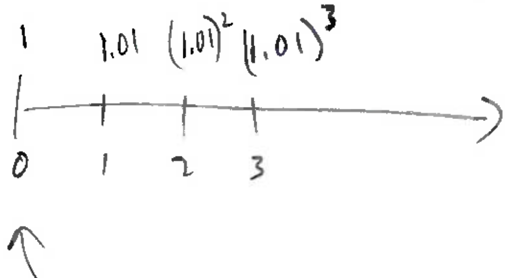
$$40 = 5\left(\frac{1-0}{i}\right)$$

$$\therefore\ i = 0.125$$

b) need $\frac{i^{(4)}}{4}$

$$1.125 = \left(1+\frac{i^{(4)}}{4}\right)^4$$

$$\frac{i^{(4)}}{4} = 0.0298835$$



$$X = PV = \frac{1-0}{1-1.01v} = \frac{1}{1-\frac{1.01}{1.0298835}} = \boxed{51.7957}$$

non-MC

2 ~~12~~. You are receiving a continuous annuity with payments of $10t$ at time t for 18 years. It is also given that $a(t)=\dfrac{1}{(1-0.005t^2)}$.

2

$f(t)=10t$

$v(t)=1-0.005t^2$

Calculate the present value of your annuity.

$$PV=\int_0^n f(t)\,v(t)\,dt$$

$$=\int_0^{18} 10t(1-0.005t^2)\,dt$$

$$=\int_0^{18}\left(10t-0.05t^3\right)dt$$

$$=\left[\frac{10t^2}{2}-\frac{0.05t^4}{4}\right]\Bigg|_0^{18}$$

$$=\frac{10(18)^2}{2}-\frac{0.05(18)^4}{4}$$

$$=1620-1312.20=\boxed{307.80}$$

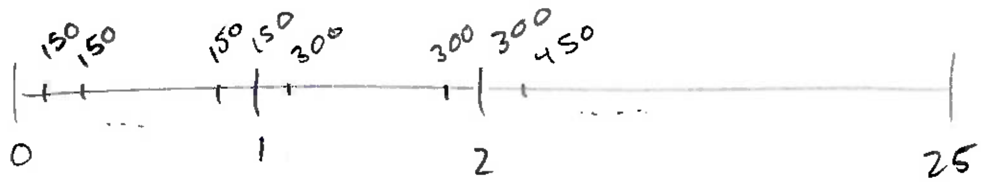
non-MC

5

3 You purchase a 25-year annuity that pays at the end of each month. The first year the monthly payments are each 150. In subsequent years, the payment increases by 150 over what it was in the previous year. The first year, each monthly payment is 150. The second year, each monthly payment is 300. The third year, each monthly payment is 450. Each year's monthly payments continue to increase in the same pattern with each year of monthly payments being 150 greater than the prior year's monthly payments.

immediate

Calculate the purchase price of your annuity (the present value) at an interest rate of 6% compounded monthly.



$i^{(12)} = 0.06$

$\frac{i^{(12)}}{12} = 0.005$

ENTDNFTR

$$PV = 150 \left(\frac{\ddot{a}_{\overline{25}|i} - 25v^{25}}{\frac{i^{(12)}}{12}} \right)$$

corr. to freq. of incr.

corr. to freq. of pmt

need i for numerator

$$1+i = \left(1+\frac{0.06}{12}\right)^{12}$$

$$i = 0.0616778$$

$$PV = 150 \left(\frac{\dfrac{1-\left(\frac{1}{1.0616778}\right)^{25}}{\frac{0.0616778}{1.0616778}} - 25\left(\frac{1}{1.0616778}\right)^{25}}{0.005} \right)$$

$$= 150 \left(\frac{13.3581 - 5.59914}{0.005} \right) = \boxed{232,768.76}$$

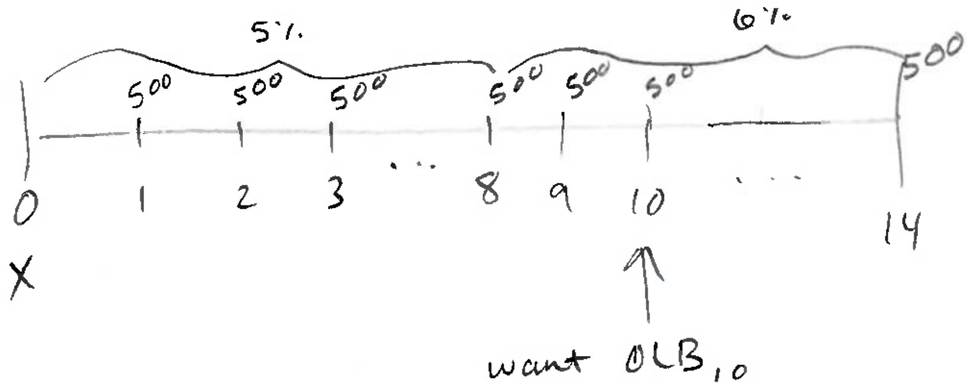
non-MC

2.75

ann. imm.

4 ~~10~~. You have taken out a loan of X to purchase a car. You will repay the loan with 14 level annual payments of 500. The annual effective interest rate on the loan is 5% for the first 8 years and 6% for the final 6.

Find the outstanding loan balance immediately after the 10th payment.



Prospective Approach

$$OLB_{10} = 500\, a_{\overline{4}|0.06}$$

$$= 500 \underbrace{\left(\frac{1 - \left(\frac{1}{1.06}\right)^4}{0.06} \right)}_{3.4651056} = \boxed{1732.55}$$

Retrospective Approach

$$X = 500 \left(\frac{1 - \left(\frac{1}{1.05}\right)^8}{0.05} \right) + \left(\frac{1}{1.05} \right)^8 \underbrace{(500) \left(\frac{1 - \left(\frac{1}{1.06}\right)^6}{0.06} \right)}_{2458.6662}$$

$$= 3231.60638 + 1664.119 = 4895.73$$

$$OLB_{10} = (4895.73)(1.05)^8(1.06)^2 - \underbrace{500 \left(\frac{(1.05)^8 - 1}{0.05} \right)}_{\text{pmts @ } t=1 \to t=8} (1.06)^2 - \underbrace{500(1.06)}_{\text{pmt @ } t=9} - \underbrace{500}_{\text{pmt @ } t=10}$$

$$= 8127.242 - 5364.6894 - 530 - 500$$

$$= \boxed{1732.55}$$

mc

47 min

Math 373
Test 2
Spring 2024
March 21, 2024

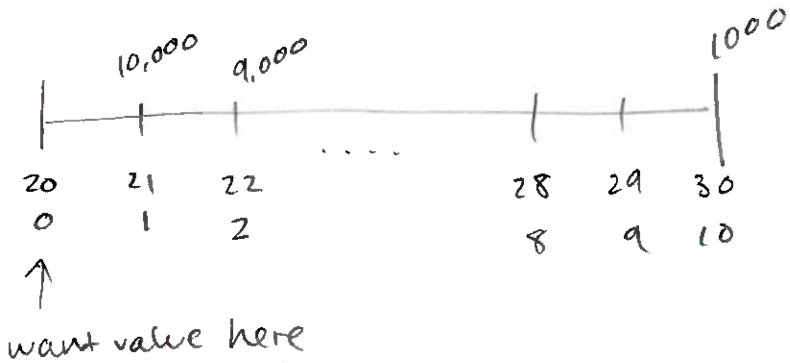
arithmetic annuity imm

5

1. You receive an annuity for your 20th birthday. The annuity pays 10,000 on your 21st birthday, 9,000 on your 22nd birthday, 8,000 on your 23rd birthday, etc. until your final payment of 1,000 on your 30th birthday.

3

Determine the present value of your annuity on your 20th birthday assuming the annual effective rate of interest is 3%.



$i = 0.03$

$P = 10{,}000$

$Q = -1000$

$n = 10$

$$PV = 10{,}000\, a_{\overline{10}|0.03} + \frac{(-1000)}{0.03}\left(a_{\overline{10}|0.03} - 10\left(\frac{1}{1.03}\right)^{10}\right)$$

7.440939

8.5302

$= 85{,}302.03 - 36{,}308.78959$

$= \boxed{48{,}993.2388}$

MC

★ See Q4 Problem #2

4.5

6 ~~1~~. You have the option to purchase one of the following:

a. A perpetuity where the first payment occurs at the beginning of the first year in the amount of 150. Each quarter thereafter, the payment increases by 1%.

due

geometric

b. An annuity with 20 annual payments. The first payment occurs at the end of the first year in the amount of 600. Each year thereafter, the payment increases by a constant, arithmetically Q.

The present value of these options is equivalent today using an annual effective interest rate of 8%.

Calculate Q.

$i = 0.08$

$$1.08 = \left(1+\frac{i^{(4)}}{4}\right)^4$$

$$\frac{i^{(4)}}{4} = 0.0194265$$

a)

quarterly

150   150(1.01)   150(1.01)

0   1   2

since int. rate greater than growth rate, approaches 0

$$PV = \frac{150 - 0}{1 - 1.01\left(\frac{1}{1.0194265}\right)} = 16{,}221.63$$

b)

600   600+Q   600+2Q

0   1   2   3   . . .   20

$$PV = 600\, a_{\overline{20|}0.08} + \frac{Q}{0.08}\left(a_{\overline{20|}} - \underbrace{20\left(\frac{1}{1.08}\right)^{20}}_{4.290964}\right)$$

9.818147

$$16{,}221.63 = 5890.888 + 69.08979\,Q$$

$$69.08979\,Q = 10{,}330.74$$

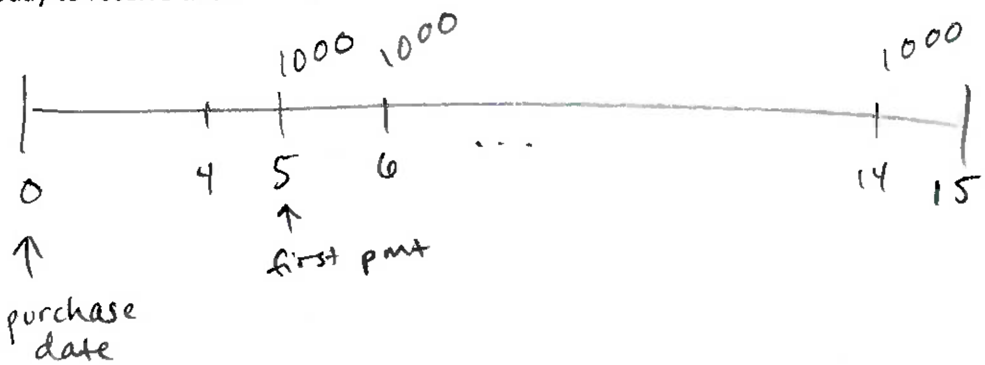
$$Q = \boxed{149.5264}$$

MC

x 1. You are purchasing an annuity today that begins making payments in 5 years. In 5 years, you will receive 1000 per year for 10 years, with the first payment exactly 5 years from today.

Assuming you can purchase at an annual effective interest rate of 4%, calculate what it will cost today to receive this benefit.

3



$r = 0.04$

need $PV = \left(\frac{1}{1.04}\right)^4 \left(1000\, a_{\overline{10}|0.04}\right) = \left(\frac{1}{1.04}\right)^4 (1000)\left(\frac{1-\left(\frac{1}{1.04}\right)^{10}}{0.04}\right)$

$= \boxed{6933.23}$

or

$= \left(\frac{1}{1.04}\right)^5 \left(1000\, \ddot{a}_{\overline{10}|0.04}\right)$

$= \left(\frac{1}{1.04}\right)^5 (1000)\left(\frac{1-\left(\frac{1}{1.04}\right)^{10}}{\frac{0.04}{1.04}}\right)$

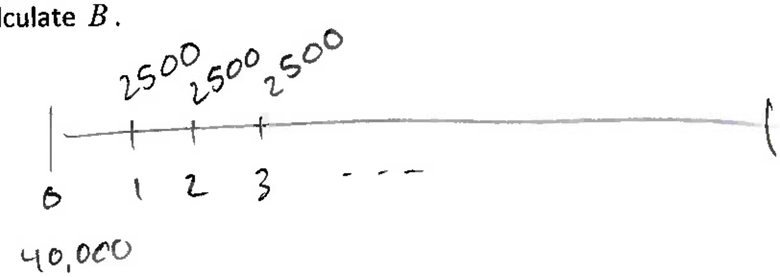
MC

3

8 ~~4~~. You borrow 40,000 today from the bank to purchase a car. You will make semiannual payments of 2,500 at an annual effective interest rate of 5% plus a balloon payment of B.

(clarify pmts 2x yr)

Calculate B.



$i = 0.05$

need $\frac{i^{(2)}}{2}$

$1.05 = \left(1 + \frac{i^{(2)}}{2}\right)^2$

$\frac{i^{(2)}}{2} = 0.024695$

on calculator

PV=40000

PMT=-2500

I/Y=2.4695077

FV=0

CPT N=20.60773

2ND AMORT P1=1

P2=20

BAL=1489.79

$40{,}000 = 2500\, a_{\overline{n}|}$

$\frac{40{,}000}{2500} = \frac{1 - v^n}{\frac{i^{(2)}}{2}}$

$\left(\frac{40{,}000}{2500}\right)\left(\frac{i^{(2)}}{2}\right) = 1 - v^n$

$-\left[\left(\frac{40{,}000}{2500}\right)\left(\frac{i^{(2)}}{2}\right) - 1\right] = +v^n$

$B = 2500 + 1489.79 = \boxed{3989.7923}$

$n =$

by hand

$$B = 40{,}000(1.024695)^{20} - 2500\underbrace{\left[\frac{(1.024695)^{19} - 1}{\frac{0.024695}{1.024695}}\right]}_{\ddot{s}_{\overline{19}|}}$$

$= 65{,}155.78507 - 61{,}165.9927 = 3989.7923$

✓match

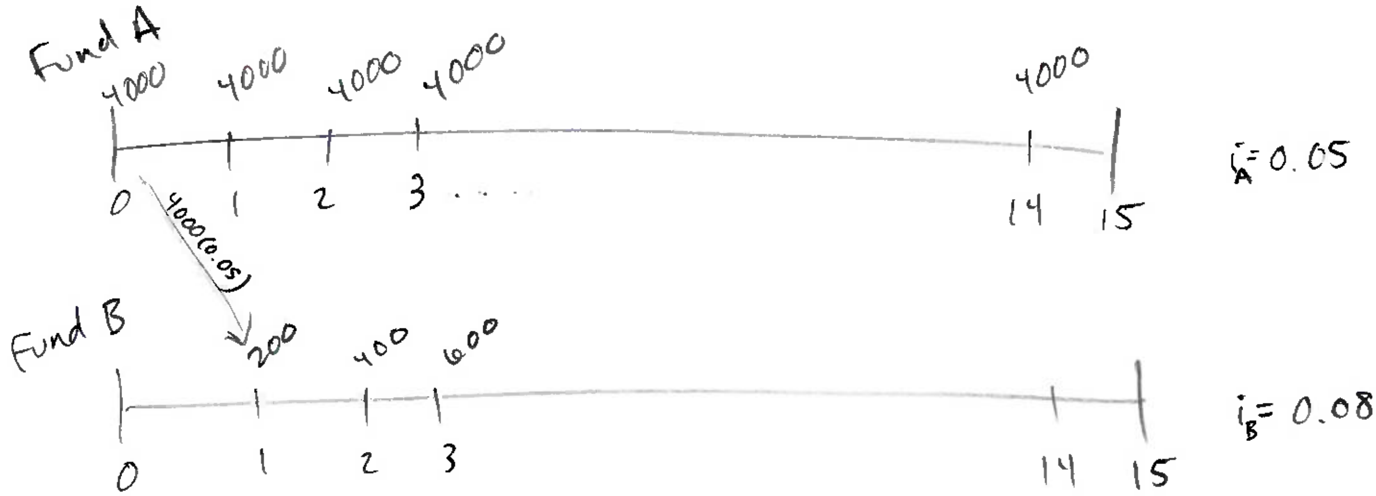
MC 9

6

9. You make payments of 4,000 into Fund A at the beginning of each year for 15 years. Fund A is invested at an annual effective rate of 5%.

The interest generated by Fund A is transferred at the end of each year to Fund B. Fund B earns an 8% annual effective interest rate.

Find the total accumulated value at the end of the 15 years of both Fund A and Fund B.



Fund A accumulated value = 4000 (15) = 60,000

Fund B arithmetic increasing ann. imm.

standard ~~notation~~ since P = Q = 200

$$FV = \left[200\,(Ia)_{\overline{15}|0.08}\right](1.08)^{15}$$

$$= 200\left(\frac{\ddot{a}_{\overline{15}|0.08} - 15\left(\frac{1}{1.08}\right)^{15}}{0.08}\right)(1.08)^{15}$$

total

$\boxed{95,810.71}$

$$= 200\left(\frac{\frac{1-\left(\frac{1}{1.08}\right)^{15}}{\frac{0.08}{1.08}} - 15\left(\frac{1}{1.08}\right)^{15}}{0.08}\right)(1.08)^{15}$$

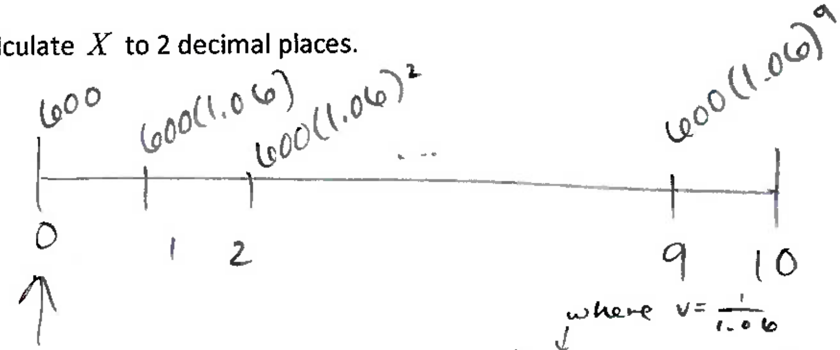
$$= \underbrace{200\left(\frac{9.244237 - 4.7286}{0.08}\right)}_{11,289.63}(1.08)^{15} = 35,810.71$$

MC

1.5 10 ~~8~~. An annuity makes annual payments at the beginning of each year at an annual effective interest rate of 6%. The first payment is 600. There are 10 total payments and each subsequent payment is 6% greater than the prior payment. The present value of the annuity is X.

due

Calculate X to 2 decimal places.



$i = 0.06$

where $v = \frac{1}{1.06}$

$X = PV = 600 + 600(1.06)v + 600(1.06)^2v^2 + \ldots + 600(1.06)^9v^9$

$= 600 + 600 + 600 + \ldots + 600$

$= 600(10) = \boxed{6000}$

MC

7

1) 1. You borrow 40,000 to buy a car. You will repay the loan with 120 monthly payments. The interest rate on the loan is 15% compounded monthly.

imm.

(10 years)

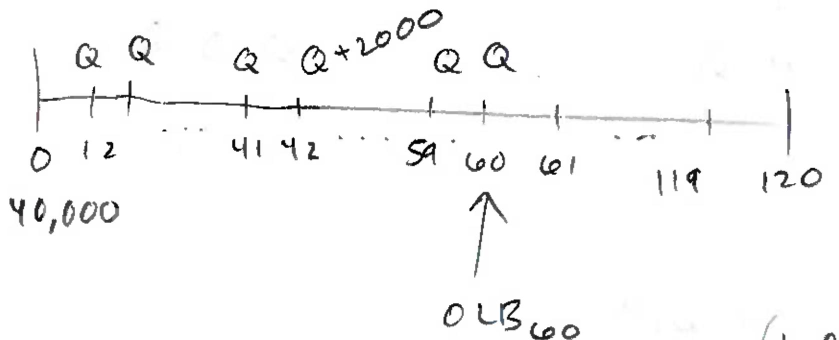
$i^{(12)} = 0.15$

You make the first 60 payments on time. Additionally, at the end of the 42^{nd} month, you make an additional payment of 2,000 over and above the normal payment.

Q+2000

Determine OLB_{60}.

$$\frac{i^{(12)}}{12} = 0.0125$$



*use retrospective

$$s_{\overline{60}|} = \frac{(1.0125)^{60}-1}{0.0125} = 88.5745$$

$$OLB_{60} = \underbrace{\left[40{,}000\,(1.0125)^{60} - Q\,s_{\overline{60}|0.0125}\right]}_{\text{normal}} - \underbrace{2000(1.0125)^{60-42}}_{\text{overpmt, no longer owe}}$$

original Q

N=120

I/Y = 1.25

PV= 40,000

FV=0

CPT PMT = 645.3398

or $40{,}000 = Q\,a_{\overline{120}|0.0125}$

$Q = 645.3398$

$$OLB_{60} = 84{,}287.25 - 645.3398(88.5745) - 2000(1.0125)^{18}$$

$$= 27{,}126.59625 - 2501.1548$$

$$= \boxed{24{,}625.44146}$$

MC

ann. imm.

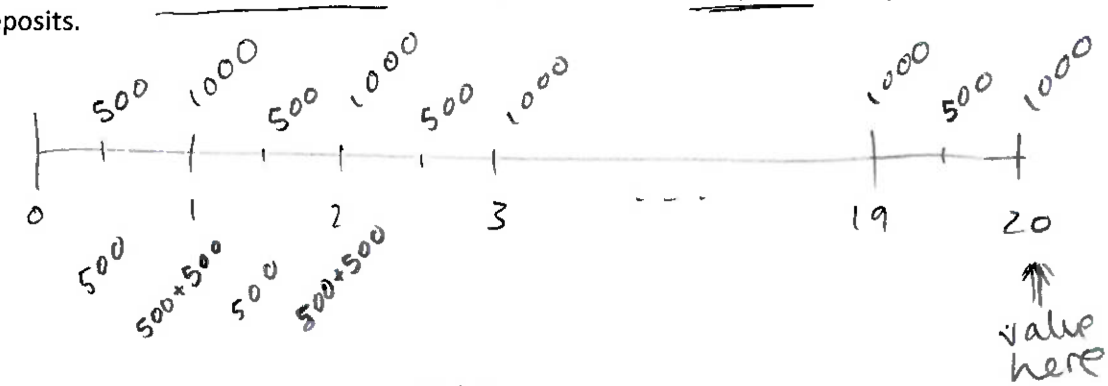
12 ~~11~~. You make payments of 1,000 into a savings account at the end of each year starting with the first year. You also make payments of 500 into the same account at the end of each 6-month period, starting with the first 6-month period. In other words, the first deposit into this account occurs 6 months from today in the amount of 500, the second deposit occurs 1 year from today in the amount of 1000, the third deposit occurs 1.5 years from today in the amount of 500, the fourth deposit occurs 2 years from today in the amount of 1000, etc.

5.5

$i = 0.01$

The savings account has an annual effective interest rate of 1%. (Note: The low rate reflects a reasonable annual return for a savings account, so don't be thrown off by this small of a rate.)

Calculate the accumulated value of the savings account after 20 years of this pattern of deposits.



find PV, then accumulate

→ need $\frac{i^{(2)}}{2}$ for 500 deposit

$$1.01 = \left(1 + \frac{i^{(2)}}{2}\right)^2 \qquad \frac{i^{(2)}}{2} = 0.00498756$$

$$FV = (27{,}113.33)(1.01)^{20}$$
$$= \boxed{33{,}083.41657}$$

$$PV = \underbrace{500\, a_{\overline{20}|0.01}}_{\text{addt'l 500 @ end of yr}} + \underbrace{500\, a_{\overline{40}|\frac{i^{(2)}}{2}}}_{\text{500 every 6-month period}}$$

$$= 500\left(\frac{1 - \left(\frac{1}{1.01}\right)^{20}}{0.01}\right) + 500\left(\frac{1 - \left(\frac{1}{1.00498756}\right)^{40}}{0.00498756}\right)$$

$$= 500(18.04555) + 500(36.181109)$$

$$= 9022.77648 + 18{,}090.5546$$
$$= 27{,}113.33$$