STAT 472
Spring 2024
Quiz 3
February 6, 2024

1. You are given:
   a. $p_{80} = 0.94$
   b. $p_{81} = 0.91$
   c. $e_{80:3} = 9$
   d. The force of mortality is constant between ages 80 and 81.
   e. Deaths are uniformly distributed between 81 and 82.

   Calculate $e_{81:3}$.

$$e_{80:3} = p_{80:3}(1 + e_{81:3})$$

$$p_{80:3} = \frac{\ell_{81:3}}{\ell_{80:3}}$$

Set $\ell_{80} = 100$

$\ell_{81} = 100(0.94) = 94$

$\ell_{82} = 94(0.91) = 85.54$

LFM $\ell_{80:3} = (\ell_{80})^{0.7}(\ell_{81})^{0.3} = 98.16$

UDD $\ell_{81:3} = (\ell_{81})(0.7) + (\ell_{82})(0.3) = 91.462$

$$p_{80:3} = \frac{\ell_{81:3}}{\ell_{80:3}} = \frac{91.462}{98.16} = 0.931764$$

$$9 = (0.931764)(1 + e_{81:3})$$

$$e_{81:3} = \boxed{8.65918}$$

2. (10 points) You are given two groups of people. Group 1 contains 100,000 people, each aged 50 whose mortality is assumed to follow the Standard Ultimate Life Table.

Group 2 also contains 100,000 people, each aged 50. Their mortality is described in the following table.

t	${}_{t\|}q_{50}$
0	0.00148
1	0.00150
2	0.00164
3	0.00177

Calculate how many more people from Group 1 survive to age 54 than do from Group 2.

Group 1

$$100{,}000\ ({}_4p_{50}) = 100{,}000\left(\frac{l_{54}}{l_{50}}\right) = 100{,}000\left(\frac{98{,}022.40}{98{,}576.40}\right)$$

$$= 99{,}437.999$$

$\Longrightarrow$ Lose $\approx$ 562

Group 2

${}_{t|}q_{50}$ = "chance of death in year $(50+t)$ to $(50+t+1)$ provided the life has survived to $(50+t)$"

	# die
year 1 $\longrightarrow$	$100{,}000\,(0.00148) = 148$
" 2 $\longrightarrow$	150
" 3 $\longrightarrow$	164
" 4 $\longrightarrow$	177
	639 die

$\therefore\ 639 - 562 = \boxed{77}$