STAT 472
Spring 2024
Quiz 4
February 29, 2024

1. (~~8~~ 7 points) For ~~a 20-year~~ 15-year endowment insurance policy on (30), you are given:

   i. Death benefits are payable at the end of the quarter of death.
   ii. The death benefit amount is 1000.
   iii. Mortality follows the Standard Ultimate Life Table → given UDD
   iv. $i = 0.05$

   Calculate the Actuarial Present Value of this policy.

$$APV = 1000\, A^{(4)}_{30:\overline{15}|} = 1000\left[\left(\frac{i}{i^{(4)}}\right) A'_{30:\overline{15}|} + {}_{15}E_{30}\right]$$

($\frac{i}{i^{(4)}} = 1.01856$)

$$A'_{30:\overline{15}|} = A_{30} - {}_{15}E_{30}\, A_{45} = 0.00455960 1$$

($A_{30} = 0.07698$, $A_{45} = 0.15161$)

$${}_{15}E_{30} = \left({}_{5}E_{30}\right)\left({}_{10}E_{35}\right) = 0.4776756$$

($ {}_{5}E_{30} = 0.78219$, $ {}_{10}E_{35} = 0.61069$)

$$APV = 1000\left[(1.01856)(0.00455960 1) + 0.4776756\right]$$

$$= \boxed{482.3198}$$

* careful w/ rounding

2. (12 points) You are (20) and would like to purchase term insurance that pays a death benefit of 150,000 at the end of the year of death if you die in the next 10 years.

   You are given that $i = 0.05$ and mortality follows the Standard Ultimate Life Table.

   a. (8 points) Calculate the variance for this policy.

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$$APV = 150{,}000\, A^{1}_{20:\overline{10}|}$$

$$= A_{20:\overline{10}|} - {}_{10}E_{20}$$

$$A^{1}_{20:\overline{10}|} = 0.61433 - 0.61224 = 0.00209$$

$$Var(Z) = \left[{}^{2}A^{1}_{20:\overline{10}|} - \left(A^{1}_{20:\overline{10}|}\right)^{2} \right] (150{,}000)^{2}$$

$${}^{2}A^{1}_{20:\overline{10}|} = {}^{2}A_{20} - {}^{2}_{10}E_{20}\,({}^{2}A_{30})$$

$$= {}^{2}A_{20} - v^{10}({}_{10}E_{20})({}^{2}A_{30})$$

$$= 0.0058 - \left(\frac{1}{1.05}\right)^{10}(0.61224)(0.01109)$$

$$= 0.00163168$$

$$Var(Z) = (150{,}000)^{2}\left(0.00163168 - (0.00209)^{2}\right)$$

$$= \boxed{36{,}614{,}689.73}$$

   b. (4 points) If instead you would like to purchase an endowment policy of the same term, how much does this increase the price? In other words, how much does this add to the actuarial present value?

0.61433

end. $APV = 150{,}000\, A_{20:\overline{10}|} = 92{,}149.50$

term $APV = 150{,}000(0.00209) = 313.50$

$\boxed{91{,}836}$