STAT 472

Spring 2024

Quiz ~~4~~ 5

March 26, 2024

1. (10 points) You are given the following mortality table:

x	q_x
95	0.06
96	0.12
97	0.24
98	0.50
99	1.00

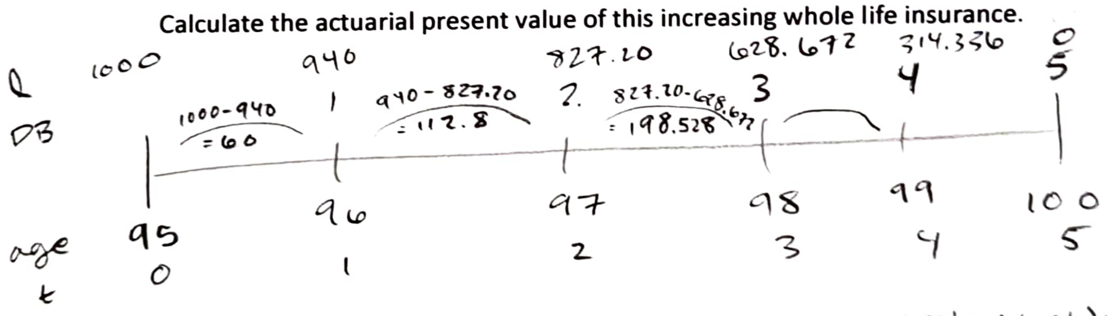
You are also given that $i = 10\%$.

An increasing whole life policy issued to (95) pays the death benefit at the end of the year of death.

The death benefit in the first year is 1. The death benefit in the second year is 2. The death benefit in the third year is 3, etc.

Calculate the actuarial present value of this increasing whole life insurance.

$v = \frac{1}{1.1}$



$l_{95} = 1000$

$l_{96} = 1000(0.94) = 940$

$l_{97} = 940(0.88) = 827.20$

$l_{98} = 827.20(0.76) = 628.672$

$l_{99} = 628.672(0.5) = 314.336$

$$(IA)_x = \frac{(1)(60)v + 2(112.8)v^2 + 3(198.528)v^3 + 4(314.336)v^4 + 5(314.336)v^5}{1000}$$

$$= \boxed{2.5231353}$$

or $APV = 1(0.06)v + 2(0.94)(0.12)v^2$
$+ 3(0.94)(0.88)(0.24)v^3$
$+ 4(0.94)(0.88)(0.76)(0.5)v^4$
$+ 5(0.94)(0.88)(0.76)(0.5)(1)v^5$
$= \boxed{2.5231353}$

keywords: normal, independent policies; this is portfolio problem

2. (10 points) XYZ Insurance Company sells a whole life insurance policy with a death benefit of 1000 to insureds who are age 50. The death benefit is paid at the end of the year of death.

The mortality for (50) follows the Standard Ultimate Life Table. You are also given that $i = 0.05$.

XYZ sells 625 policies. Further, XYZ sets aside 124,000 to cover future death benefits. Assuming the normal distribution and that policies are independent, calculate the probability that the 124,000 that XYZ set aside will be LESS than the present value of the benefits actually paid.

Individual Policy $\qquad i = 0.05$

$$E(Z) = 1000 \underbrace{A_{50}}_{0.18931} = 189.31$$

$$var(Z) = (1000)^2 \left(\underbrace{{}^2A_{50}}_{0.05108} - (\underbrace{A_{50}}_{0.18931})^2 \right) = 15{,}241.7239$$

Portfolio

$$E(Port) = 625\, E(Z) = 625(189.31) = 118{,}318.75$$

$$var(Port) = 625\, var(Z) = 625\,(15{,}241.7239) = 9{,}526{,}077.438$$

additive since independent (no covariance)

$$Prob\left[\text{Actual PV of bfts paid} > 124{,}000 \right] = 1 - \Phi\left(\frac{124{,}000 - 118{,}318.75}{\sqrt{9{,}526{,}077.438}} \right)$$

$$= 1 - \Phi\,(1.84072)$$

$$= 1 - 0.9671 = \boxed{0.0329}$$