STAT 472

Test 1

Spring 2024

February 14, 2024

1. (15 points) You are given that $\mu_x = 0.05$ for $70 \le x < 90$.

   a. (7 points) Calculate ${}_{4|14}q_{70}$.

$${}_{4|14}q_{70} = ({}_4p_{70})({}_{14}q_{74}) \quad \text{or} \quad {}_4p_{70} - {}_{18}p_{70}$$

$${}_4p_{70} = e^{-\int_{70}^{74} 0.05\,dt} = e^{-0.05(74-70)} = e^{-0.2}$$

$${}_{14}q_{74} = 1 - {}_{14}p_{74} = 1 - e^{-\int_{74}^{88} 0.05\,dt} = 1 - e^{-0.05(14)}$$

$${}_{4|14}q_{70} = \underbrace{(e^{-0.2})}_{0.81873075}\underbrace{(1-e^{-0.7})}_{0.5034147} = \boxed{0.412161}$$

★ Given that $q_{90} = 1$, ★ therefore, $\omega = 91$

   b. (8 points) Approximate $\mathring{e}_{87}$.

$$\mathring{e}_{87} \quad e_{87} + \tfrac{1}{2}$$

$$\omega - x - 1 = 91 - 87 - 1 = 3$$

$$e_{87} = \sum_{k=1}^{3} {}_kp_{87} = p_{87} + {}_2p_{87} + {}_3p_{87}$$

$$= e^{-0.05} + e^{-0.05(2)} + e^{-0.05(3)}$$

$$= 2.7167748$$

$$\mathring{e}_{87} \approx e_{87} + \tfrac{1}{2}$$

$$\approx \boxed{3.2167748}$$

2. (10 points) You are given that mortality follows Makeham's Law with $A = 0.005$, $B = 0.0001$ and $c = 1.10$.

in SULT packet

You are also given:

a. $e_{40} = 25.1$

b. $e_{60} = 11.4$

Calculate $e_{40:\overline{20|}}$

$$e_{40} = e_{40:\overline{20|}} + {}_{20}p_{40}\, e_{60}$$

$$25.1 = e_{40:\overline{20|}} + \underline{{}_{20}p_{40}}\,(11.4)$$

$$\Downarrow$$

$$
\begin{aligned}
{}_{20}p_{40} &= \exp\left[-0.005(20) - \frac{0.0001}{\ln(1.10)}(1.1)^{40}\left[(1.1)^{20}-1\right]\right] \\
&= e^{-0.37197765} \\
&= 0.68936965
\end{aligned}
$$

$$
\begin{aligned}
e_{40:\overline{20|}} &= 25.1 - (0.68936965)(11.4) \\
&= \boxed{17.241186}
\end{aligned}
$$

3. (35 points) You are given the following select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
90	0.21	0.31	0.43	0.56	93
91	0.26	0.37	0.50	0.65	94
92	0.31	0.43	0.57	0.72	95
93	0.36	0.49	0.65	0.91	96
94	0.41	0.55	0.76	0.96	97
95	0.46	0.61	0.90	1.00	98

You are also given that $d = 0.07$. A person is underwritten at age 94 and is issued a whole life insurance policy with a death benefit of 10,000 payable at the end of the year of death.

a. (3 points) Write an expression for the present value random variable Z for this policy.

$$Z = 10{,}000\, v^{K_{[94]}+1}$$

b. (8 points) Calculate the actuarial present value for this policy.

$v = 1-d = 1-0.07 = 0.93$

$\omega = 99$ since $q_{98} = 1$

$$APV = E(Z) = 10{,}000\, A_{[94]}$$

	1000	590	265.50	63.72	2.5488	0
	410	324.5	201.78	61.1712	2.5488	
age	[94]	[94]+1	[94]+2	97	98	99
time	0	1	2	3	4	5

set $l_{[94]} = 1000$

$l_{[94]+1} = 1000(1-0.41) = 590$

$l_{[94]+2} = 590(1-0.55) = 265.50$

$l_{97} = 265.5(1-0.76) = 63.72$

$l_{98} = 63.72(1-0.96) = 2.5488$

$$1000 A_{[94]} = 410v + 324.5v^2 + 201.78v^3 + 61.1712v^4 + 2.5488v^5$$

$$1000\, A_{[94]} = 871.7956$$

$$A_{[94]} = 0.8717956$$

$$APV = 10{,}000\,(0.8717956)$$

$$= \boxed{8717.9562}$$

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c. **(8 points) Calculate $Var(Z)$ if Z is the present value random variable for a whole life insurance policy issued to a life underwritten at age 94.**

$$Var(Z) = var(10{,}000\, v^{K_{[94]}+1})$$

$$= (10{,}000)^2\, var(v^{K_{[94]}+1}) = (10{,}000)^2 ({}^2A_{[94]} - A_{[94]}^2)$$

$$1000({}^2A_{[94]}) = 410v^2 + 324.5v^4 + 201.78v^6 + 61.1712v^8 + 2.5488v^{10}$$

$${}^2A_{[94]} = 0.76336542$$

$$var(Z) = (10{,}000)^2 (0.76336542 - (0.8717956)^2)$$

$$= \boxed{333{,}785.182}$$

d. **(8 points) Given $l_{[94]+1} = 1000$, calculate $l_{[91]}$.**

first ultimate mortality in common is for age 97

① $l_{[91]} \underbrace{(p_{[91]})(p_{[91]+1})(p_{[91]+2})\, p_{94}\, p_{95}\, p_{96}}_{\checkmark\ 6\text{ years of experience}} = l_{97}$

② $l_{[94]+1} \underbrace{(p_{[94]+1})(p_{[94]+2})}_{\checkmark\ 2\text{ years of experience}} = l_{97}$

$$l_{[91]} = \frac{(1-0.55)(1-0.76)(1000)}{(1-0.26)(1-0.37)(1-0.50)(1-0.65)(1-0.72)(1-0.91)} = \boxed{52{,}530.66}$$

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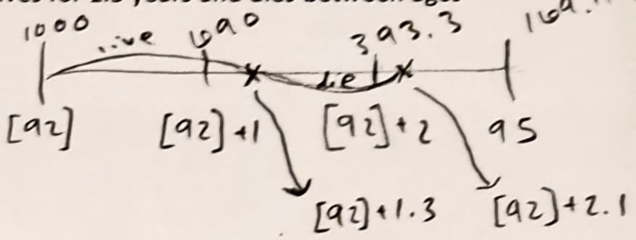
e. (8 points) Given deaths are uniformly distributed between integral ages, calculate the probability that a life underwritten at age 92 lives for 1.3 years and dies between ages 93.3 and 94.1.



Find ${}_{1.3|0.8}P_{[92]}$

choose $l_{[92]} = 1000$

$$l_{[92]+1} = 1000\,(1-0.31) = 690$$

$$l_{[92]+2} = 690\,(1-0.43) = 393.3$$

$$l_{[92]+3} = 393.3\,(1-0.57) = 169.119$$

$${}_{1.3|0.8}P_{[92]} = \frac{l_{[92]+1.3} - l_{[92]+2.1}}{l_{[92]}}$$

need $l_{[92]+1.3}$ & $l_{[92]+2.1}$

$$l_{[92]+1.3} = (l_{[92]+1})(0.7) + (l_{[92]+2})(0.3)$$

$$= (690)(0.7) + (393.3)(0.3) = 600.99$$

$$l_{[92]+2.1} = (l_{[92]+2})(0.9) + (l_{95})(0.1)$$

$$= (393.3)(0.9) + (169.119)(0.1) = 370.8819$$

$${}_{1.3|0.8}P_{[92]} = \frac{600.99 - 370.8819}{1000} = \boxed{0.2301081}$$

5. (10 points) You are given that $\mu_{80+t} = 0.04t$.

You are also given that L_{10} is the random variable representing the number of people who will be alive at age 90 if there are 5,000 people alive at age 80.

Calculate $Var[L_{10}]$.

Recognize Binomial

$L_t \sim$ Binomial

$$Var[L_{10}] = n({}_{10}p_{80})({}_{10}q_{80})$$

$n = 5000$

$${}_{10}p_{80} = e^{-\int_0^{10} 0.04t\,dt} = e^{-\frac{0.04t^2}{2}\Big|_0^{10}}$$

$$= e^{-0.02(100)} = e^{-2}$$

$${}_{10}q_{80} = 1 - {}_{10}p_{80} = 1 - e^{-2}$$

$$Var(L_{10}) = 5000\,(e^{-2})(1-e^{-2})$$

$$= \boxed{585.098}$$

6. (10 points) Since you're taking an exam on Valentine's Day, I want to provide an opportunity for free points (take that, cupid!). In baseball, players typically get to choose a song to play when it's their turn to go up to bat (a "walk up" song). Please list a song title and artist for a song that you would choose as your "walk up" song. I'm going to put together a class playlist with the results. Don't worry about keeping it G-rated for me.

Almost (Sweet Music) - Hozier