STAT 472
Spring 2024
Test 2
April 3, 2024

1. (8 points) You are given:

   a. Z_{80} is the present value of a whole life insurance of 1 issued to (80) with a death benefit of 1 payable at the end of the year of death.

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   b. Z_{81} is the present value of a whole life insurance of 1 issued to (81) with a death benefit of 1 payable at the end of the year of death.

   c. $A_{80} = 0.80$

   d. $Var[Z_{80}] = 0.09$

   e. $A_{81} = 0.82$

   f. $q_{80} = 0.08$

   Calculate the $Var[1000Z_{81}]$ $= (1000)^2 \, var(Z_{81}) = \boxed{103,828}$

$var(Z_{81}) = {}^2A_{81} - (A_{81})^2 = 0.776228011 - (0.82)^2 = 0.103828$

need this

0.82

$var(Z_{80}) = {}^2A_{80} - (A_{80})^2$

$0.09 = {}^2A_{80} - (0.80)^2 \Rightarrow {}^2A_{80} = 0.73$

recursive 1 $\rightarrow$ solve for ${}^2A_{81}$

${}^2A_{80} = v^2 q_{80} + v^2 p_{80} ({}^2A_{81})$

$0.73 = v^2 [0.08 + 0.92({}^2A_{81})] \Rightarrow {}^2A_{81} = 0.776228011$

need this

recursive 2 $\rightarrow$ solve for v

$A_{80} = v q_{80} + v p_{80} A_{81}$

$0.80 = v[0.08 + 0.92(0.82)] \Rightarrow v = 0.95877277$

2. (8 points) A special whole life policy is issued to (25). The whole life policy pays a death benefit at the moment of death. The death benefit is 125,000 for the first 20 years. The death benefit for death between ages 45 and 65 is 500,000. If this person dies after age 65, the death benefit will be 330,000.

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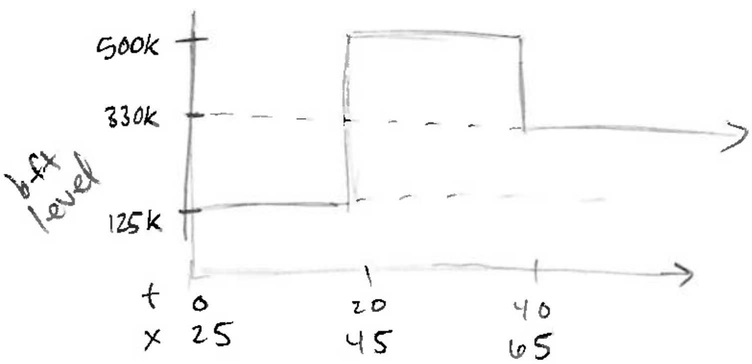
You are given:

i. Mortality follows the Standard Ultimate Life Table.
ii. Deaths are uniformly distributed between integral ages.
iii. $i = 0.05$

Calculate the Actuarial Present Value of this special whole life policy.



$$APV = 125,000\,\bar{A}_{25} + (500,000-125,000)({}_{20}E_{25})\,\bar{A}_{45} - (500,000-330,000)({}_{40}E_{25})\,\bar{A}_{65}$$

$$= 1000\left(\frac{i}{\delta}\right)\left[125\,A_{25} + 375\,A_{45}\,({}_{20}E_{25}) - 170\,A_{65}\,({}_{20}E_{25})({}_{20}E_{45})\right]$$

$$= 1000(1.0248)\left[125(0.06147) + 375(0.15161)(0.37373) - 170(0.35477)\underbrace{(0.37373)(0.35994)}_{0.1345203776}\right]$$

$$= \boxed{21,334.96}$$

3. (12 points) ABC Insurance Company sells a 20-year continuous term life annuity due to (50). The annuity makes payments at a rate of 100 per year.

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You are given:

i. Mortality follows the Standard Ultimate Life Table.
ii. $i = 0.05$
iii. Deaths are uniformly distributed between integral ages. ✓ UDD

Y is the present value random variable for this annuity.

Calculate the $Var[Y]$.

$$Y = 100\,\ddot{a}_{\overline{T_x}|}$$

$$Var(Y) = \left[\frac{{}^2\bar{A}_{50:\overline{20}|} - (\bar{A}_{50:\overline{20}|})^2}{\delta^2}\right](100)^2$$

$$\bar{A}_{50:\overline{20}|} = \frac{i}{\delta}\Big[\underbrace{A_{50:\overline{20}|} - {}_{20}E_{50}}_{A^{1}_{50:\overline{20}|}}\Big] + {}_{20}E_{50} = (1.0248)(0.38844 - 0.34824) + 0.34824$$

$$= 0.38943696$$

$${}^2\bar{A}_{50:\overline{20}|} = \left[\frac{(1+i)^2 - 1}{2\delta}\right]\Big[\underbrace{{}^2A_{50} - \overbrace{({}_{20}E_{50})(v^{20})}^{{}^2_{20}E_{50}}({}^2A_{70})}_{{}^2A^{1}_{50:\overline{20}|}}\Big] + {}^2_{20}E_{50}$$

$$= \left[\frac{(1.05)^2 - 1}{2\ln(1.05)}\right]\left[0.05108 - (0.34824)\left(\frac{1}{1.05}\right)^{20}(0.21467)\right] + 0.131248$$

$$= (1.050416634)\left[0.05108 - 0.028175007\right] + 0.131248$$

$$= 0.155307779$$

$$var(Y) = \left[\frac{0.155307779 - (0.38943696)^2}{(\ln 1.05)^2}\right](100)^2 = \boxed{15,318.898}$$

4 ~~3~~. For 3-year term insurance on (60), you are given:

i. Benefits are payable at the end of the year of death and vary as follows.

x	Death Benefit
60	3,000
61	2,300
62	4,600

ii. Z is the present value random variable for this insurance.

iii. $i = 0.05$

iv. The following is an excerpt from a select-and-ultimate mortality table:

x	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	x+2
60	0.019	0.024	0.030	62
61	0.021	0.026	0.032	63
62	0.024	0.031	0.040	64

a. (8 points) Calculate $E(Z)$.

scenario	Z	Probability
die yr. 1	$3000v$	0.019
die yr. 2	$2300v^2$	$(1-0.019)(0.024) = 0.023544$
die yr. 3	$4600v^3$	$(1-0.019)(1-0.024)(0.03) = 0.02872368$
live 3 yrs	0	$(1-0.019)(1-0.024)(1-0.03) = 0.92873232$

$\Sigma = 1$? yes

$E(Z) = (3000v)(0.019) + (2300v^2)(0.023544) + (4600v^3)(0.02872368) + 0(0.92873232)$

$= \boxed{217.5404}$

(Continued From Prior Page)

b. (8 points) Calculate $Var(Z)$.

$$E(Z^2) = (3000v)^2(0.019) + (2300v^2)^2(0.023544) + (4600v^3)^2(0.028723368)$$

$$= 711,112.3371$$

$$Var(Z) = 711,112.3371 - (217.5404)^2$$

$$= 663,788.5181$$

c. (8 points) XYZ Life Insurance has 500 identical policies sold to 500 independent lives. XYZ decides to hold 110,000 to cover the future death benefit payments on these policies. Calculate the probability that the present value of death benefits will be greater than 110,000.

$$E(\text{port}) = 500E(Z) = 500(217.5404) = 108,770.1924$$

$$Var(\text{port}) = 500Var(Z) = 500(663,788.5181) = 331,894,259$$

$$P\left(\text{PV death benefits} > 110,000\right)$$

$$= 1 - \Phi\left(\frac{110,000 - 108,770.1924}{\sqrt{331,894,259}}\right)$$

$$= 1 - \Phi(0.0675)$$

z-score

$$= 1 - 0.5279 = \boxed{0.4721}$$

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5 ~~6.~~ (8 points) Given the following:

a. $a_{60} = 11$ $\Longrightarrow$ $\ddot{a}_{60} = 12$

b. $a_{61} = 10.75$ $\Longrightarrow$ $\ddot{a}_{61} = 11.75$

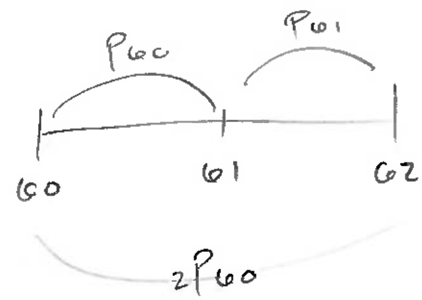
c. $a_{62} = 10.5$ $\Longrightarrow$ $\ddot{a}_{62} = 11.5$

d. $i = 0.06$

Calculate ${}_2p_{60}$.

$${}_2p_{60} = p_{60}\, p_{61}$$

$$= \boxed{0.983279926}$$



recursive

① $\ddot{a}_{60} = 1 + v\, p_{60}\, \ddot{a}_{61}$

$$12 = 1 + \left(\frac{1}{1.06}\right) p_{60}\,(11.75)$$

$$p_{60} = 0.992340426$$

② $\ddot{a}_{61} = 1 + v\, p_{61}\, \ddot{a}_{62}$

$$11.75 = 1 + \left(\frac{1}{1.06}\right) p_{61}\,(11.5)$$

$$p_{61} = 0.990869565$$

6 ~~7~~. (10 points) For a whole life annuity-due of 1 per year on (45), you are given:

i. $l_x = 10(110 - x),\ 0 \le x \le 110$

ii. $i = 0.02$

iii. $\ddot{a}_{45} = 25.3$

$$Y = \frac{1 - v^{K_x+1}}{d}$$

Calculate $\Pr(Y > E(Y))$.

3.5

$$E(Y) = \ddot{a}_{45} = 25.3$$

$$25.3 = \frac{1 - \left(\frac{1}{1.02}\right)^{K_x+1}}{\frac{0.02}{1.02}}$$

$$K_x + 1 = \frac{\ln(0.503921569)}{\ln\left(\frac{1}{1.02}\right)} = 34.608268$$

$$\therefore K_x = 33.608268$$

round up ↑ to 34

want $${}_{34}p_{45} = \frac{l_{79}}{l_{45}} = \frac{10(110-79)}{10(110-45)} = \boxed{0.476923}$$