Student Name:______________________

Purdue ID:__________________________

**STAT 472 – Spring 2025**
**Optional Quiz 7**

**MTHW 304 12:50 – 1:15 PM**
**Tuesday, April 29, 2025**

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**INSTRUCTIONS**

- Do not open this quiz until you are told to do so.
- There are 20 points possible on this quiz.
- You have 25 minutes to complete this quiz.
- Be sure you have filled in your name and Purdue ID in the slots at the top of the page.
- Show all work to maximize partial credit.
- Be sure all cell phones are silenced and put away out of view. This policy applies to smart watches as well.
- Headphones are not permitted unless prior approval was granted by your instructor.
- Formula sheets are not permitted.
- You are only permitted to use calculator(s) from the following list:
  - BA II Plus
  - BA II Plus Professional
  - BA-35
  - TI–30Xa or TI–30XA (same model just different casing)
  - TI-30X II (IIS solar or IIB battery)
  - TI-30XS MultiView (or XB battery)
- When time expires, put your pencil down and close your exam. Failure to do so will result in automatic disqualification from obtaining University-Earned Credit.

**PURDUE HONORS PLEDGE**

"As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together - we are Purdue."

**STUDENT AGREEMENT**

By signing below,

- I agree with the Purdue Honors Pledge stated above.
- I will not give or receive any assistance on this exam, and I will report any infractions of the honors pledge.
- I acknowledge that I only used calculator(s) from the above list.
- I am claiming all work in this exam as my own.

X__________________________________________________

1. (5 points) You are given that $S_0(x) = \dfrac{7921 - x^2}{7921}$ for $0 \le x \le 89$ .

Calculate ${}_{3|6}q_{55}$ .

<u>Solution:</u>

$$ {}_t p_{55} = \frac{S_0(55+t)}{S_0(55)} = \frac{7921-(55+t)^2}{7921-55^2} = \frac{4896-110t-t^2}{4896} $$

$$ {}_{3|6} q_{55} = {}_3 p_{55} - {}_9 p_{55} = \frac{4896-110(3)-3^2}{4896} - \frac{4896-110(9)-9^2}{4896} $$

$$ = 0.930759804 - 0.78125 = \boxed{0.149509804} $$

2. (5 points) You are given:

   i. Mortality follows the Standard Ultimate Life Table
   ii. $i = 0.05$

   A 30-year term insurance policy that pays 150,000 at the end of the year of death is issued to (36).

   Calculate the actuarial present value of this policy to two decimal places.

   Solution:

$$APV = 150{,}000 \cdot A^{1}_{36:\,\overline{30|}}$$
$$= 150{,}000\,[A_{36} - {}_{30}E_{36} \cdot A_{66}]$$
$$= 150{,}000\,[A_{36} - {}_{20}E_{36} \cdot {}_{10}E_{56} \cdot A_{66}]$$
$$= 150{,}000\,[0.10101 - (0.36982)(0.59109)(0.36878)]$$
$$\boxed{= 3059.375072}$$

**---OR---**

$$APV = 150{,}000 \cdot A^{1}_{36:\,\overline{30|}}$$
$$= 150{,}000\,[A_{36} - {}_{30}E_{36} \cdot A_{66}]$$
$$= 150{,}000\left[A_{36} - v^{30}\left(\frac{l_{66}}{l_{36}}\right) \cdot A_{66}\right]$$
$$= 150{,}000\left[0.10101 - \left(\frac{1}{1.05}\right)^{30}\left(\frac{94{,}020.3}{99{,}517.8}\right)(0.36878)\right]$$
$$\boxed{= 3059.433891}$$

3. (10 points) You (50) won the lottery. You will receive <u>both</u> of the following:

   i. (5 points) A deferred term life annuity due that makes monthly payments of 800 for 10 years, starting at age 65.

   ii. (5 points) An annuity due with 15 years of guaranteed monthly payments of 400, starting now (age 50).

You are given that $i = 0.05$, deaths are uniformly distributed between integral ages, and mortality follows the Standard Ultimate Life Table.

Calculate the actuarial present value of the payments to two decimal places.

<u>Solution:</u>

**i)**

$$APV = (800)(12)({}_{15}E_{50}) \cdot \ddot{a}^{(12)}_{65:\overline{10|}} = (800)(12)({}_{10}E_{50})({}_{5}E_{60}) \cdot \ddot{a}^{(12)}_{65:\overline{10|}}$$

$$\ddot{a}^{(12)}_{65:\overline{10|}} = \ddot{a}^{(12)}_{65} - {}_{10}E_{65} \cdot \ddot{a}^{(12)}_{75}$$

$$\ddot{a}^{(12)}_{65} = \alpha(12) \cdot \ddot{a}_{65} - \beta(12) = (1.0002)(13.5498) - 0.46651 = 13.08599996$$

$$\ddot{a}^{(12)}_{75} = \alpha(12) \cdot \ddot{a}_{75} - \beta(12) = (1.0002)(10.3178) - 0.46651 = 9.85335356$$

$$APV = (800)(12)(0.60182)(0.76687) \cdot [13.08599996 - (0.55305)(9.85335356)]$$

$$= 4430.569953\,(7.636602774) = \boxed{33{,}834.50279}$$

**---OR---** **(if ${}_{t}E_{x}$ calculated directly)**

$$APV = (800)(12) \cdot v^{15}\left(\frac{l_{65}}{l_{50}}\right) \cdot \ddot{a}^{(12)}_{65:\overline{10|}}$$

$$= 9600 \cdot \left(\frac{1}{1.05}\right)^{15} \cdot \left(\frac{94{,}579.7}{98{,}576.4}\right) \cdot [7.636602774] = \boxed{33{,}834.27901}$$

**ii)**

$$APV = (400)(12) \cdot \ddot{a}^{(12)}_{\overline{15|}} = (400)(12)\left(\frac{1 - \left(\frac{1}{1.05}\right)^{15}}{d^{(12)}}\right)$$

$$= 4800\left(\frac{1 - \left(\frac{1}{1.05}\right)^{15}}{0.04869}\right) = \boxed{51{,}162.82459}$$