COUPLING Term $\quad$ BIOT EQ'N $\quad$ ①

$$(\nabla\cdot q, w) + (q, \nabla w) = \langle q\cdot\nu, w\rangle \qquad (w = \beta\theta)$$

$$(\nabla(\beta\theta), v^s) + (\sigma_{ij}, \varepsilon_{ij}(v^s))$$

$$= (\sigma_{ij}, \varepsilon_{ij}(v^s)) - (\beta\theta^{n+1}, \nabla\cdot v^s) + \langle v^s\cdot\nu, \beta\theta^{n+1}\rangle$$

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$$-(\beta\theta^{n+1}, \nabla\cdot\varphi_k) = -\left(\beta\sum_{j=k-1}^{k+1}\theta_j^{n+1}\varphi_j, \frac{\partial\varphi_k}{\partial x}\right) + \langle\varphi_k\cdot\nu, \beta\theta^{n+1}\rangle_\Gamma$$

$$+\langle\underset{1}{\varphi_k}\cdot\underset{-1}{\nu}, \beta\theta^{n+1}\rangle_{\Gamma^L(x=0)} + \langle\varphi_k\cdot\underset{1}{\nu}, \beta\theta^{n+1}\rangle_{\Gamma^R(x=1)}$$

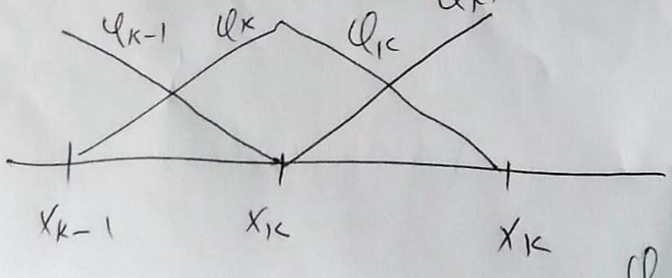
$$= \underbrace{-\left(\beta\sum_{j=k-1}^{k+1}\theta_j^{n+1}\varphi_j, \frac{\partial\varphi_k}{\partial x}\right)}_{I} - \beta\theta_1^{n+1}\delta_{k1} + \beta\theta_{N_x+1}^{n+1}\delta_{k,N_x+1}$$

$$I = -\int_{x_{k-1}}^{x_k}\beta\,\theta_{k-1}^{n+1}\varphi_{k-1}\frac{\partial\varphi_k}{\partial x} - \beta\int_{x_{k-1}}^{x_k}\theta_k^{n+1}\varphi_k\frac{\partial\varphi_k}{\partial x}$$

$$-\int_{x_k}^{x_{k+1}}\beta\,\theta_k^{n+1}\varphi_k\frac{\partial\varphi_k}{\partial x} - \int_{x_k}^{x_{k+1}}\beta\,\theta_{k+1}^{n+1}\varphi_{k+1}\frac{\partial\varphi_k}{\partial x}$$

$$= I_1 + I_2 + I_3 + I_4$$

$$\varphi_k = \begin{cases}\dfrac{x-x_{k-1}}{h}, & x_{k-1}\le x\le x_k\\[2mm] 1-\dfrac{x-x_k}{h}, & x_k\le x\le x_{k+1}\end{cases}$$



$$\varphi_{k-1}\begin{cases}1-\dfrac{x-x_{k-1}}{h}\\ x_{k-1}\le x\le x_k\end{cases}$$

$$\varphi_{k+1} = \begin{cases}\dfrac{x-x_k}{h}, & x_k\le x\le x_{k+1}\end{cases}$$

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$$I_1 = -\beta\,\theta_{k-1}^{n+1} \int_{x_{k-1}}^{x_k} \Big(1 - \underbrace{\frac{x-x_{k-1}}{h}}\Big)\Big(-\frac{1}{h}\Big)\,dx$$

$$u = \frac{x-x_{k-1}}{h} \qquad du = \frac{dx}{h}$$

$$I_1 = -\beta\,\theta_{k-1}^{n+1} \int_0^1 (1-u)\,du = -\beta\,\theta_{k-1}^{n+1}\Big(u - \frac{u^2}{2}\Big)\Big|_0^1$$

$$= -\beta\,\theta_{k-1}^{n+1}\,\frac{1}{2}$$

$$I_2 = -\beta \int_{x_{k-1}}^{x_k} \theta_k^{n+1}\Big(\underbrace{\frac{x-x_{k-1}}{h}}_{u}\Big)\frac{1}{h}\,dx =$$

$$= -\beta\Big(\int_0^1 u\,du\Big)\,\theta_k^{n+1} = -\frac{\beta}{2}\,\theta_k^{n+1}$$

$$I_3 = -\beta \int_{x_k}^{x_{k+1}} \theta_k^{n+1}\Big(1 - \underbrace{\frac{x-x_k}{h}}_{u}\Big)\Big(-\frac{1}{h}\Big)\,dx$$

$$= +\beta\,\theta_k^{n+1} \int_0^1 (1-u)\,du = +\frac{\beta}{2}\,\theta_k^{n+1}$$

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$$I_4 = -\beta \int_{x_K}^{x_{K+1}} \theta_{K+1}^{n+1} \left(\frac{x - x_K}{h}\right)\left(-\frac{1}{h}\right) dx$$

$$= \frac{\beta}{2} \int_0^1 \theta_{K+1}^{n+1}\, u\, du = \frac{\beta}{2}\, \theta_{K+1}^{n+1}$$

Then for $K = 2 .. N_x$

$$I = -\frac{\beta}{2}\theta_{K-1}^{n+1} - \frac{\beta}{2}\theta_K^{n+1} + \frac{\beta}{2}\theta_K^{n+1} + \frac{\beta}{2}\theta_{K+1}^{n+1}$$

$$= \frac{\beta}{2}\left(\theta_{K+1}^{n+1} - \theta_{K-1}^{n+1}\right)$$

For $K = 1$, $I_1 = I_2 = 0$

$$I = I_3 + I_4 = \frac{\beta}{2}\,\theta_1^{n+1} + \frac{\beta}{2}\,\theta_2 - \beta\,\theta_1$$

$$= \frac{\beta}{2}\left(\theta_2^{n+1} - \theta_1^{n+1}\right)$$

For $K = N_x + 1$ $I_3 = I_4 = 0$

$$I = I_1 + I_2 = -\frac{\beta}{2}\,\theta_{Nx}^{n+1} - \frac{\beta}{2}\,\theta_{Nx}^{n+1} + \beta\,\theta_{Nx+1}^{n+1}$$

$$= \frac{\beta}{2}\left(\theta_{Nx+1}^{n+1} - \theta_{Nx}^{n+1}\right)$$

SO CODE DOES NOT CHANGE

STABILITY ANALYSIS, NEW BRY TERM (4)

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$$\left|\sum_{n} \left\langle \tfrac{1}{2}(\theta^{n+1}+\theta^{n-1})\beta_m , \partial U^{s,n}\cdot\nu \right\rangle \Delta t\right|$$

$$\leq C \sum_{n} \|\theta^{n+1}\|_1 \, \|\partial U^{s,n}\|_1 \, \Delta t$$

$$\leq C \sum_{n} \left(\|\theta^{n+1}\|_1^2 + \|\partial U^{s,n}\|_1^2\right) \Delta t$$

$$\sum_{n} \left\langle \tfrac{1}{2}(\theta^{n+1}+\theta^{n-1})\beta_f , \partial U^{f,n}\cdot\nu \right\rangle \Delta t \Big|$$

$$\leq C \sum_{n} \|\theta^{n+1}\|_{1/2,\Gamma} \, \|\partial U^{f,n}\cdot\nu\|_{-1/2,\Gamma} \, \Delta t$$

$$\leq C \sum_{n} \|\theta^{n+1}\|_1 \, \|\partial U^{f,n}\|_{H(\mathrm{div},\Omega)} ,$$

since

$$H^1(\Omega) \xrightarrow{\ ?\ } H^{1/2}(\Gamma)$$ ~~VER SI ES CIERTO~~

$$v \longrightarrow v|_\Gamma$$

$$\rightarrow \ |v|_{1/2,\Gamma} \leq C\|v\|_1 \quad \checkmark$$

To be done: continuous case, uniqueness

ERROR ANALYSIS -