Estimation of third-order elasticity parameters from local seismic anisotropy measurements and geomechanical modeling

Ran Bachrach and Andrey Bakulin, WesternGeco/Schlumberger

Summary

We present a method to estimate a profile of stress-sensitivity parameters using local measurements of seismic anisotropy and geomechanical modeling. The method assumes that anomalous stresses at a location of interest create a measurable perturbation in local anisotropic velocity field. If velocity field is known for "normal" or baseline stress conditions then measured perturbation can be inverted for stress sensitivity parameters provided that geomechanical estimates of anomalous stresses are available from either modeling or measurement. The method can be used for example to derive depth-dependent stress sensitivity parameters along a well. We present simple synthetic and then more realistic example illustrating the method and associated errors.

Introduction

Stress sensitivity of rock is an important rock property that connects seismic and geomechanics. For a long time seismically derive velocities are routinely used for pore pressure prediction which requires some kind of transform between vertical velocity and vertical effective stress. Recently geophysical measurements were applied to capture the effects of an overburden changes in a 3D stress state (Hatchell and Bourne, 2005; Herwanger et al., 2007; Bachrach and Sengupta, 2008; Fuck et al., 2009). In time-lapse seismic, depletion-induced 3D stress changes cause offset-dependent time-shifts in the entire overburden (Herwanger et al, 2007; Fuck et al, 2009), whereas in near-salt exploration salt-induced stresses change velocity and anisotropy of surrounding shales (Bachrach and Sengupta, 2008). To better interpret depletion signatures or to conduct near-salt exploration one needs to connect effects of 3D stress and anisotropic velocity. Sarkar et al (2003) and Prioul et al. (2004) showed that third-order elasticity (TOE) theory can provide required rock physics transform mapping 3D stress to anisotropic velocity. In general, TOE parameters can be derived from velocity measurements at different angles under different states of stress. Unfortunately, most of the data comes from laboratory measurements, which if available only sample very few depth locations. A method to estimate TOE parameters in situ will greatly facilitate the ability to map velocity to 3D stress. Recently, Bachrach (2008) proposed a method that uses estimates of vertical effective stress and rock model to derive TOE parameters from well logs. In this study we discuss a method that uses geomechanical modeling and local estimates of anisotropy to estimate the TOE parameters. Local anisotropy measurement may come from seismic, borehole or well log data in either time-lapse or explorations settings. We analyze the method and specifically the sensitivity of the method to the assumptions and parameters.

Theory

Following Prioul et al. (2004), we assume a formation that is vertically transversely isotropic (VTI) in a normal stress regime. We assume that perturbed or anomalous stress regime is characterized by an excess strain tensor ΔE that has vertical symmetry axis and is of bi-axial nature for the purpose of this study ($\Delta E_{33} \neq 0, \Delta E_{11} = \Delta E_{22}, \Delta E_{12} = \Delta E_{13} = \Delta E_{23} = 0$). Under these assumptions perturbed material preserves VTI symmetry but its stiffnesses (C_{ij}) are altered and related to normal-regime stiffnesses (C_{ij}^0) via following equations

$$\begin{aligned}
C_{11} &\cong C_{11}^0 + c_{111}\Delta E_{11} + c_{112}(\Delta E_{11} + \Delta E_{33}),\\
C_{33} &\cong C_{33}^0 + c_{111}\Delta E_{33} + 2c_{112}\Delta E_{11},\\
C_{13} &\cong C_{13}^0 + c_{123}\Delta E_{11} + c_{112}(\Delta E_{11} + \Delta E_{33}),\\
C_{12} &\cong C_{12}^0 + c_{123}\Delta E_{33} + 2c_{112}\Delta E_{11},\\
C_{66} &\cong C_{66}^0 + c_{144}\Delta E_{33} + 2c_{155}\Delta E_{11},\\
C_{44} &\cong C_{44}^0 + c_{144}\Delta E_{11} + c_{155}(\Delta E_{11} + \Delta E_{33}),
\end{aligned} \tag{1}$$

where c_{111}, c_{112}, and c_{123} are three independent TOE parameters describing simplest isotropic stress-sensitivity tensor, whereas $c_{144} = 0.5(c_{112} - c_{123})$, $c_{155} = 0.25(c_{111} - c_{112})$ are additional TOE parameter combinations. Due to VTI constraint $C_{66} = 0.5(C_{11} - C_{12})$, only five unstressed stiffnesses are independent both for perturbed and unperturbed states.

Introducing stiffness ($[C^0]$) and compliance ($[S^0]$) matrices the stress-strain relations at a given reference is written as

$$\begin{aligned}
[\Delta T] &= [C^0]\cdot[\Delta E],\\
[\Delta E] &= [S^0]\cdot[\Delta T].
\end{aligned} \tag{2}$$

In our examples we will focus on analyzing anisotropic *P*-wave velocity field, therefore we pay primary attention to evaluating first three stiffnesses from equations (1). Combining equations (1) and (2) we can obtain the change in stiffnesses in terms of stress perturbations as

$$\begin{aligned}
&\Delta C_{11} \cong c_{111}\left[(S_{11}^0 + S_{12}^0)\Delta T_{11} + S_{13}^0\Delta T_{33}\right] +\\
&c_{112}\left[(S_{12}^0 + S_{11}^0 + 2S_{13}^0)\Delta T_{11} + (S_{33}^0 + S_{13}^0)\Delta T_{33}\right],\\
&\Delta C_{33} \cong c_{111}(2S_{13}^0\Delta T_{11} + S_{33}^0\Delta T_{33}) +\\
&c_{112}\left[2(S_{12}^0 + S_{11}^0)\Delta T_{11} + 2S_{13}^0\Delta T_{33}\right],\\
&\Delta C_{13} \cong c_{123}\left[(S_{11}^0 + S_{12}^0)\Delta T_{11} + S_{13}^0\Delta T_{33}\right] +\\
&c_{112}\left[(S_{11}^0 + S_{12}^0 + 2S_{13}^0)\Delta T_{11} + (S_{33}^0 + S_{13}^0)\Delta T_{33}\right],
\end{aligned} \tag{3}$$

which can be written in matrix notation as

$$[\Delta C]=[B][c], \tag{4}$$

with vectors $[\Delta C]=[\Delta C_{11},\Delta C_{33},\Delta C_{13}]$ and $[c]=[c_{111},c_{112},c_{123}]$. Thus, if the matrix $[B]$ is invertible and we can estimate changes in the stiffness $[\Delta C]$ and changes in the bi-axial state of stress $(\Delta T_{11},\Delta T_{33})$ we can find the three TOE parameters needed to characterize the response of the rock. Generally non-hydrostatic stress perturbations $(\Delta T_{11}\neq\Delta T_{33})$ are required to make matrix $[B]$ invertible which is the case for depletion-induced or salt-induced changes in the overburden sediments.

Estimating TOE parameters from changes in anisotropic *P*-wave field

Seismic signatures are more conveniently expressed using Thomsen parameters, that are related to VTI stiffnesses as

$$V_{P0}=\sqrt{\frac{C_{33}}{\rho}},V_{S0}=\sqrt{\frac{C_{44}}{\rho}},\varepsilon=\frac{C_{11}-C_{33}}{2C_{33}}, \tag{5}$$
$$\gamma=\frac{C_{66}-C_{44}}{2c_{44}},\delta=\frac{(C_{13}+C_{44})^2-(C_{33}-C_{44})^2}{2C_{33}(C_{33}-C_{44})}.$$

It is well-known that *P*-wave velocity field is controlled by only three parameters: V_{P0}, ε and δ. We assume that at the location of interest with a perturbed stresses, for example, around a well, *P*-wave seismic and/or borehole data are inverted and a profiles of V_{P0}, ε and δ are recovered as a function of depth. We further assume that profiles of unperturbed parameters $V^0_{P0},\varepsilon^0,\delta^0$ are also known with a certain accuracy as well as estimates of stress changes $(\Delta T_{11},\Delta T_{33})$ for each depth. In a time-lapse scenario this information may come from a previous baseline measurements, whereas in an exploration scenario it may come from an offset well or basin knowledge. Changes in V_{P0}, ε and δ can be converted to a desired vector $[\Delta C]=[\Delta C_{11},\Delta C_{33},\Delta C_{13}]$ if we assume that the vertical shear-wave velocity can be estimated through a general V_{P0}/V_{S0} ratio for the basin and that γ is related to ε through a certain correlation (Sayers, 2005). We will discuss the sensitivity of these assumptions in the following section.

A simple example and sensitivity analysis

In Figure 1 we present a hypothetical Gulf of Mexico profile for $V^0_{P0},\varepsilon^0,\delta^0$ and density. The vertical stress is calculated by integrating the density profile. We assume that the horizontal stress is lower than the vertical stress, which is an assumption typical for extensional basins (Finkbiner, 1998; Fredreich et al, 2003). We impose a negative perturbation of up to -10 MPa on the horizontal stress while keeping $\Delta T_{33}=0$. Due to usual convention that compressive stresses are negative, this perturbation implies increase in the magnitude of horizontal stress. In Figure 2 we present the hypothetical TOE profile and in Figure 3 we present the change in vertical *P*-wave velocity, vertical *S*-wave velocity, and anisotropy, associated with the stress perturbation and computed using given TOE parameters.

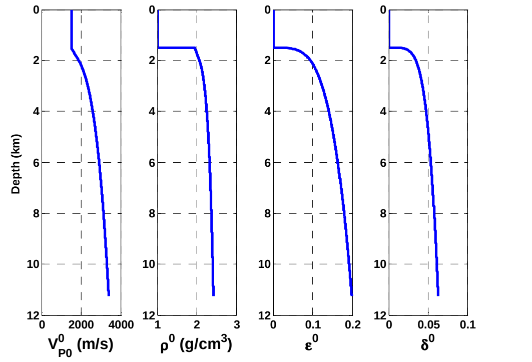

Figure 1: Velocity, density and anisotropy profiles for undisturbed sediment.

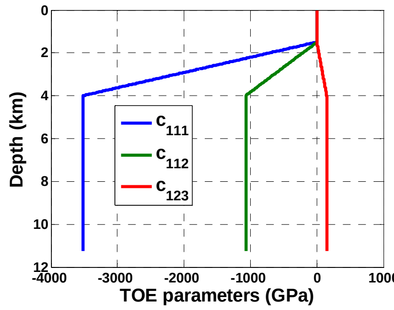

Figure 2. Profile of assumed TOE parameters.

If we have the error-free measurement and know stress perturbations we can directly invert them for three TOE parameters. To analyze the sensitivity of these equations to the assumptions and errors we proceed as follows. From equation 4 we can directly see that the TOE parameters derived by the inversion as $[c]=[B(S^0,\Delta T)]^{-1}[\Delta C]$. Mathematically, the sensitivity of the TOE estimate can be written as

$$c=c(S^0,\Delta T,\Delta C),$$
$$\delta c=\frac{\partial c}{\partial S^0}\delta S^0+\frac{\partial c}{\partial \Delta T}\delta\Delta T+\frac{\partial c}{\partial \Delta C}\delta\Delta C, \tag{6}$$

and the three sources of errors in the estimates can be identified as follows:

a. The error in the background elastic parameters $(\frac{\partial[c]}{\partial S^0}\delta S^0)$. This error accounts for inaccuracies in the unperturbed parameters and relationships between V_{P0} to V_{S0} and between γ and ε.

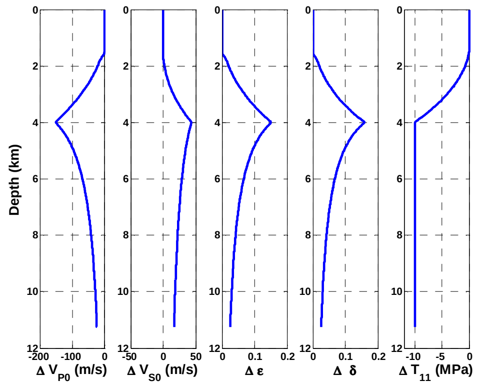

Figure 3: Perturbations in elastic properties and stress shown as difference between perturbed and original parameters. Observe increase in anisotropy due to increase in the magnitude of compressive horizontal stress.

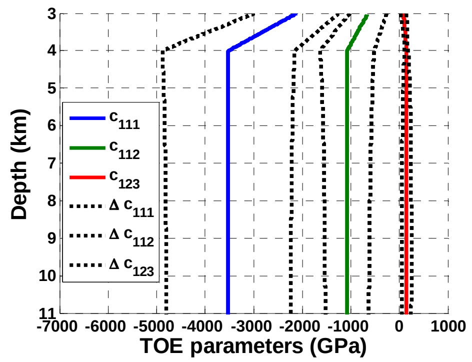

Figure 4: Inverted profiles of TOE parameters assuming errors of 10% in C^0_{33} and absolute accuracy of ε and δ estimates being ±0.05.

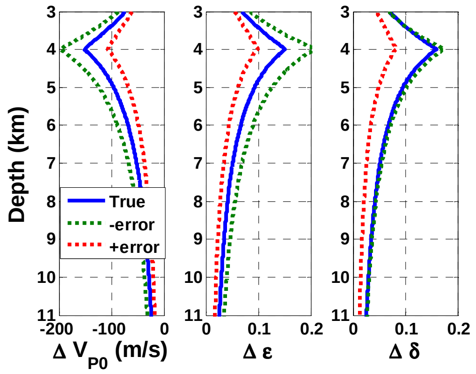

Figure 5: Calculated $\Delta V_{P0}, \Delta\varepsilon, \Delta\delta$ using true and erroneous TOE coefficients presented in Figure 4.

b. The error in the stress estimate ($\frac{\partial[c]}{\partial \Delta T}\delta\Delta T$). This error accounts for imperfection in the stress perturbations.

c. The error in the measured changes of velocity and Thomsen parameters ($\frac{\partial[c]}{\partial \Delta C}\delta\Delta C = \frac{\partial[c]}{\partial \Delta C}\frac{\partial \Delta C}{\partial\Delta[V_{P0},\varepsilon,\delta]}\delta\Delta[V_{P0},\varepsilon,\delta]$). This error accounts for the inaccuracies of our local estimation of changes $\Delta V_{P0}, \Delta\varepsilon, \Delta\delta$.

In this example we assume that the background stiffness matrix and the stress perturbations are known with the accuracy of 10% of their correct values. The in-situ observations are providing vertical velocity within 10% accuracy, whereas anisotropic parameters ε and δ are measured with an absolute error of 0.05 (which is between 40% to 100% relative error). These values represent possible accuracy achievable with seismic and VSP measurements. The TOE best estimate and the uncertainty associated with contributions of the three error terms (equation 6) are shown in Figure 4. In Figure 5 we present perturbations $\Delta V_{P0}, \Delta\varepsilon, \Delta\delta$ obtained using exact non-linear equations (1) and (5) with true and erroneous estimates of TOE parameters plotted in Figure 4. We note that with minor exceptions our predictions remain within the ±0.05 corridor around the true anisotropy profiles.

Gulf of Mexico example

In this example we are interested in stress perturbations generated by an extensive salt body. We calculate stress field from geomechanical modeling for two sediment models with salt and without the salt (Bachrach and Sengupta, 2008). The salt-induced stress perturbations are obtained by subtracting the two geomechanical solutions (Figure 6). We map the salt-induced stress changes into changes in Thomsen's parameters ε and δ using TOE profile presented in Figure 2. In this particular example we assume that sediment is isotropic in the absence of anomalous salt stresses. After applying rock physics transform based on non-linear elasticity we obtain volumes of Thomsen's parameters shown in Figure 7a,b. In further analysis we concentrate at a location of interest that corresponds to a vertical well shown in Figure 7a,b. Figure 7c,d,e displays profiles of stress perturbations and anisotropy perturbations correspondingly. Decrease in the magnitude of the vertical compressive stress as well as increase in the magnitude of the horizontal stress both lead to lower vertical and higher horizontal velocities, thus creating positive anomalies in Thomsen parameters. Let us examine whether we will be able to recover estimates of TOE parameters from measurements of seismic anisotropy and changes in vertical velocity assuming that the stress perturbations are known from geomechanical modeling.

Figure 8 presents the inverted TOE parameters obtained using error-free observations, exact material properties, and stresses and compares them with the estimates recovered from input with errors. As in synthetic example, we assume both positive and negative errors in background stiffness (10%), stress (10%) and local anisotropy measurements (absolute error of ±0.05). As seen in Figure 8, the TOE parameters with errors in the input are still reasonably representing the subsurface properties. We note that our inversion for TOE parameters is stable as long as the observed perturbations in ε and δ are larger than 0.015.

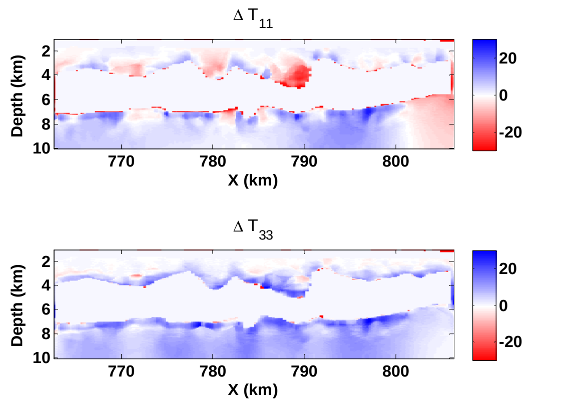

Figure 6: Salt-induced stress perturbations (MPa) in horizontal (top) and vertical (bottom) stresses in the Gulf of Mexico model (salt body in the middle of the section is shown by white color). Model with salt is subtracted from model without salt.

Summary and conclusions

We demonstrated that stress modeling and local observation of anisotropy can be used to estimate the depth-dependent variation of subsurface stress sensitivity expressed as third-order elasticity (TOE) coefficients. Seismic, VSP or acoustic measurement of stress-induced anisotropy along a well profile can provide a way to characterize depth-dependent stress sensitivity of rocks provided that anomalous stress field is quantified with modeling or borehole measurements. Depth-dependent stress sensitivity is required to interpret geomechanical effects in 4D seismic. They are also required to aid anisotropic velocity model building process in complex areas with anomalous stress fields such basins with a salt tectonics. We have shown that even with a very crude assumptions and large errors in measurements we can obtain reasonable estimates of TOE parameters that are still useful for applications requiring a link between seismic and 3D geomechanics. Considering sparseness of core sampling and potential pitfalls associated with the laboratory measurements of stress sensitivity parameters, the in-situ technique utilizing local anisotropy measurements may represent a more robust practical alternative capturing depth variation of the stress sensitivity. Local anisotropy measurements can be obtained using localized seismic tomography, VSP inversion or acoustic logging measurements.

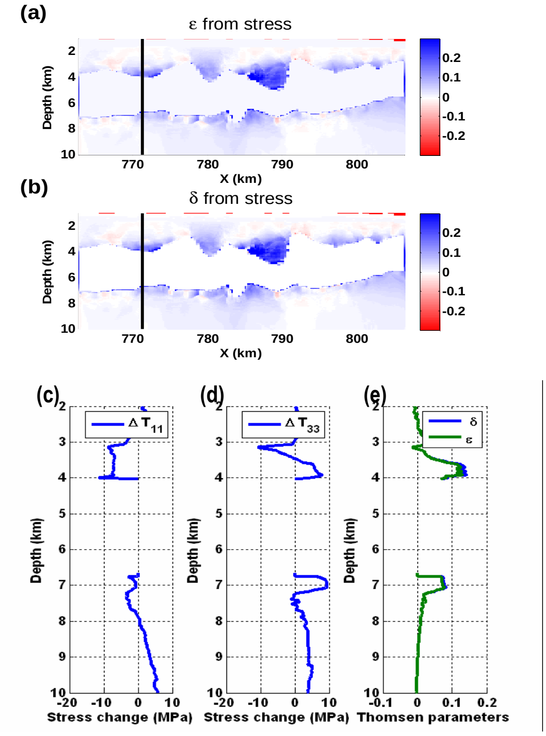

Figure 7: Stress-induced perturbation in Thomsen parameters ε (a) and δ (b) predicted from geomechanical modeling. Black line shows vertical well at the location of interest. c) Profile of horizontal stress perturbation along the vertical well. d) Profile of vertical stress perturbation along the vertical well. e) Profile of perturbations in Thomsen's parameters along the vertical well. Middle section with zero values represents salt body where no inversion is done.

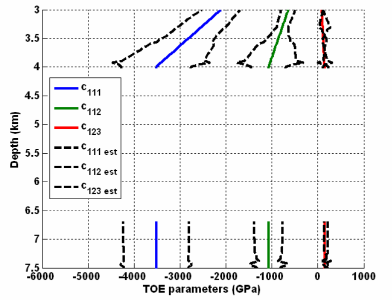

Figure 8: TOE parameters from inversion using error-free measurement and correct parameters (solid line) and with errors in stress, material properties and anisotropy measurements (± 0.05 absolute error).

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2009 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Bachrach, R., 2008, In situ estimates of third-order elastic parameters: Theory and example from the Gulf of Mexico: 70th Conference and Exhibition, EAGE, Extended Abstracts.

Bachrach, R., and M. Sengupta, 2008, Using geomechanical modeling and wide-azimuth data to quantify stress effects and anisotropy near salt bodies in the Gulf of Mexico: 78th Annual International Meeting, SEG, Expanded Abstracts, 212–216.

Finkbeiner, T., 1998, In-situ stress, pore pressure and hydrocarbon migration & accumulation in sedimentary basins: Ph.D. thesis, Stanford University.

Fredrich, J. T., D. Coblentz, A. F. Fossum, and B. J. Thorne, 2003, Stress perturbation adjacent to salt bodies in deepwater Gulf of Mexico: 78th Annual Technical Conference & Exhibition, SPE, 84554.

Fuck, R. F., A. Bakulin, and I. Tsvankin, 2009, Theory of traveltime shifts around compacting reservoirs: 3D solutions for heterogeneous anisotropic media: Geophysics, **74**, no. 1, D25–D36.

Hatchell, P., and S. Bourne, 2005, Rocks under strain: Strain-induced time-lapse time shifts are observed for depleting reservoirs: The Leading Edge, **24**, 1222–1225.

Herwanger, J., E. Palmer, and C. R. Schiøtt, 2007, Anisotropic velocity changes in seismic time-lapse data: 77th Annual International Meeting, SEG, Expanded Abstracts, 2883–2887.

Prioul, R., A. Bakulin, and V. Bakulin, 2004, Non-linear rock physics model for estimation of 3D subsurface stress in anisotropic formations: Theory and laboratory verification: Geophysics, **69**, 415–425.

Sarkar, D., A. Bakulin, and R. Krantz, 2003, Anisotropic inversion of seismic data for stressed medium: Theory and physical modeling study on Barea sandstone: Geophysics, **68**, 690–704.

Sayers, C. M., 2005, Seismic anisotropy of shales: Geophysical Prospecting, **53**, 667–676.