P-wave attenuation in thin-layered non-isothermal fluid-saturated poroelastic solids

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Waves in non-isothermal poroelastic media. I

- This work uses a model that combines the Biot (J.App.Phys., 1957) and Lord-Shulman (J.Mech.Phys.Sol., 1967) theories to study mesoscopic P-wave attenuation in a thin-layered non-isothermal poroelastic medium alternately saturated by gas and water.
- The model predicts the existence of four waves: two compressional P waves, one fast (P1) and one slow (diffusive) (P2), a slow (diffusive) thermal (T) wave, and a shear (S) wave.
- The T wave is coupled with both P-waves (Sharma, J.Earth.Sys.Sci, 2008; Carcione et al., J.Geophys.Res.,2019). S-waves are uncoupled with T-waves.
- The model assumes that the temperature in the solid and fluid is the same.

Waves in non-isothermal poroelastic media. II

- The mesoscopic loss mechanism is due to mode conversion at the gas-water interfaces.
- The objective is to study P-wave attenuation associated with the presence of T-waves and its relation with the wave induced fluid flow (WIFF) mechanism.
- A Finite Element (FE) procedure is used to solve an initial boundary value problem (IBVP) for the Biot/Lord-Shulman equations in layered media .
- The solution of the IBVP yields the seismic response in terms of displacements of the solid and fluid phases and temperature, which are recorded to observe and quantify attenuation effects of P-waves.
- The spectral ratio and frequency shift methods were used to estimate the quality factor Q.

Mathematical model. I

Consider a fluid-saturated poroelastic medium, and asume that whole aggregate is isotropic.

$\boldsymbol{\theta}$: increment of temperature above a reference absolute temperature T_0 for the state of zero stress and strain.

$\boldsymbol{u}^s = (\boldsymbol{u}_i^s),\ \boldsymbol{u}^f = \left(\boldsymbol{u}_i^f\right)$: average particle displacement vectors of the solid and relative fluid phases, respectively, $\boldsymbol{u} = \left(\boldsymbol{u}^s, \boldsymbol{u}^f\right)$

$\boldsymbol{\varepsilon}(\boldsymbol{u}^s) = \left(\boldsymbol{\varepsilon}_{ij}(\boldsymbol{u}^s)\right),\ \boldsymbol{\sigma}(\boldsymbol{u}, \boldsymbol{\Theta}) = \left(\boldsymbol{\sigma}_{ij}(\boldsymbol{u}, \boldsymbol{\Theta})\right)$: strain and stress tensors of the solid and bulk material, respectively

$\boldsymbol{p}_f = \boldsymbol{p}_f(\boldsymbol{u}, \boldsymbol{\Theta})$: fluid pressure

Dynamical equations in thermo-poroelastic media

$$\rho_b \mathbf{u}^s + \rho_f \mathbf{u}^f - \nabla \cdot \sigma(\mathbf{u}, \theta) = \mathbf{f}^s,$$
$$\rho_f \mathbf{u}^s + g \mathbf{u}^f + \frac{\eta}{\kappa} \mathbf{u}^f + \nabla p_f(\mathbf{u}, \theta) = \mathbf{f}^f.$$

ρ_b, ρ_f: mass density of the bulk material and the fluid

η: fluid viscosity κ:permeability

g: mass coupling parameter.

To represent thermal effects, Biot's original constitutive equations are modified by adding positive coupling coefficients β and β_f of the bulk material and fluid, respectively.

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Constitutive relations

$$\sigma_{ij}(\mathbf{u},\theta) = 2\mu\,\varepsilon_{ij}(\mathbf{u}^s) + \delta_{ij}(\lambda_u \nabla\cdot\mathbf{u}^s + B\nabla\cdot\mathbf{u}^f - \beta\,\theta),$$
$$-p_f(\mathbf{u},\theta) = B\,\nabla\cdot\mathbf{u}^s + M\nabla\cdot\mathbf{u}^f - \beta_f\theta.$$

μ: dry-material shear modulus,

$$M = \left(\frac{\alpha-\phi}{K_s} + \frac{\phi}{K_f}\right)^{-1}, \quad \phi\text{: porosity} \quad \alpha = 1 - K_m/K_s$$

$$B = \alpha\, M, \quad \lambda_u = \lambda + \alpha^2 M$$

K_s, K_m and K_f: bulk moduli of the grains, solid frame and fluid, respectively.

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Generalized heat equation

$$\tau\, c\, \ddot{\theta} + c\, \dot{\theta} - \nabla \cdot (\gamma \nabla \theta) + \beta T_0 \nabla \cdot \dot{\mathbf{u}}^s$$
$$+\beta T_0 \nabla \cdot \dot{\mathbf{u}}^f + \tau \beta T_0 \nabla \cdot \ddot{\mathbf{u}}^s + \tau \beta T_0 \nabla \cdot \ddot{\mathbf{u}}^f = -q.$$

γ: thermal conductivity

c: bulk specific heat of the unit volume in the absence of deformation

τ: relaxation time, q: heat source.
These equations assume that the temperature in both phases is the same.

β, β_f, γ and c are considered as parameters, obtained from experiments or from a specific theoretical model.

Numerical experiments. Quality factor Q.

Attenuation is defined as the decrease in amplitude of a wave as it propagates over a distance.

The quality factor Q estimates the material dissipation and quantifies the attenuation.

Estimation of the quality factor *Q*. Spectral-ratio (SR) method

The Spectral-ratio method estimates ***Q*** from the relation:

$$\ln\left[\frac{A(f,r_s)}{A(f,r_t)}\right] = \frac{\pi(d_t - d_s)}{c_p Q} f \qquad (1)$$

$A(f, r_s)$ and $A(f, r_t)$: amplitude spectrum of receivers r_s, r_t ,
r_s considered as a source for the receiver r_t

d_s , d_t : the distances of r_s and r_t from the source

c_p : average compressional phase velocity in a region containing the receivers r_s and r_t estimated from the corresponding **arrival times**.

Q is computed from the slope of the semi-log relationship (1)

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Estimation of the quality factor Q. Frequency-shift (FS) method

The Frequency-shift method relates Q with the **centroid frequencies f_s and f_t of r_s and r_t** using the relation:

$$\pi \frac{(d_t - d_s)}{c_p Q} = \frac{(f_s - f_t)}{\sigma_s^2}$$

$$f_j = \frac{\int_0^\infty f A(f, r_j) df}{\int_0^\infty A(f, r_j) df}, \qquad j = s, t$$

A wave propagating through this layered medium loses high frequencies, thus the centroid decreases. This effect is measured by the downshift $\Delta f = f_s - f_t$.

σ_s : obtained by fitting a Gaussian curve to the amplitude spectrum $A(f, r_s)$.

Numerical experiments. White Model.

To analyze the amplitude damping of a wave propagating in a thin-layered non-isothermal poroelastic medium alternatively saturated by gas or water, the results are compared with those of **White's isothermal theory**.

White's isothermal theory describes dissipation in this type of medium, which is induced by the interaction generated by the presence of two fluids with different compressibilities.

Values of White's model are obtained using the material properties in Table 1.

Numerical experiments. Material properties.

In the examples we consider a periodic layered medium of material properties as in Table 1 of layer thicknesses 15, 20 and 30 cm, alternatively saturated with either gas or water.

Table 1. Material Properties

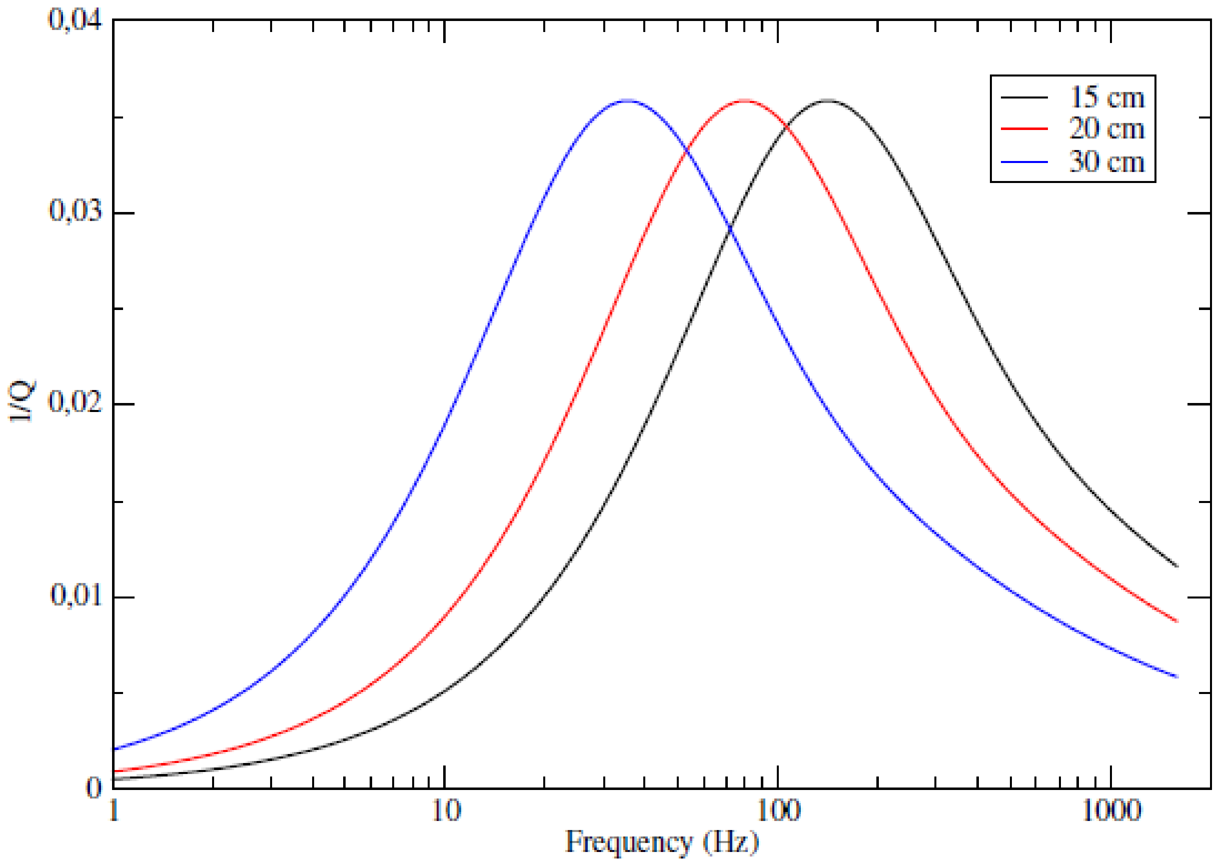
Grain bulk modulus, K_s	37 GPa
density, ρ_s	2650 kg/m^3
Frame bulk modulus, K_m	8 GPa
shear modulus, μ_m	9.5 GPa
porosity, ϕ	0.3
permeability, κ	1 darcy
Water bulk modulus, K_w	2.25 GPa
Water density, ρ_w	1040 kg/m^3
Water viscosity, η_w	0.003 Pa $\cdot$ s
Gas bulk modulus, K_g	0.012 GPa
Gas density, ρ_g	78 kg/m^3
Gas viscosity, η_g	0.000015 Pa $\cdot$ s
Bulk specific heat, c	820 kg/(m s^2 K)
thermoelasticity coefficient, β	90000 kg/(m s^2 K)
thermoelasticity coefficient, β_f	50000 kg/(m s^2 K)
absolute temperature, T_0	300 K
thermal conductivity, γ	4.5 $\times 10^6$ kg/m^3
relaxation time, τ	1.5 $\times 10^{-2}$ s

Numerical experiments. White Model.

Q^{-1} determined by using the White model as a function of frequency for a periodic layered medium of thicknesses 15 cm, 20 cm and 30 cm. The minimum value of the quality factor for all thicknesses is $\cong$ 28.

This minimum value of Q is attained at f_0 = 140 Hz, f_0 = 77 Hz and f_0 = 34 Hz for **layer thickness 15 cm, 20 cm and 30 cm**, respectively. These frequencies are used as dominant frequency of the external source in the next experiments.

Figure 1: Q^{-1}, periodic layered material alternatively saturated with either gas or water.
Layer thicknesses: 15, 20, 30 cm



Numerical experiments. The IBVP.

The seismic response (time histories) of a periodic seismic non-isothermal poroelastic sequence Ω is obtained by exciting the medium with a point source.

Thus, an **IBVP** for the Biot/Lord-Shulman equations is formulated and solved by using an explicit conditionally stable (i.e., satisfying a CFL stablity constrain) Finite Element Method (FEM) in an open interval $\Omega = (0, 400 \text{ m})$.

The FEM uses a uniform partition $T^h(\Omega)$ of Ω into subintervals Ω_j of size h .

The solid and fluid phases and the temperature are represented by using globally continuous piecewise-linear polynomials over $T^h(\Omega)$.

Numerical experiments. The IBVP. Layer thickness 20 cm

The medium Ω_j, initially at rest, is excited with a **dilatational point source** located at $x_s = 4$ m, of time history,

$$g(t) = -16\, f_0^2 (t - t_0) e^{-8 f_o^2 (t - t_0)^2}$$

with $t_0 = 1.25/f_0$, f_0 = 77 Hz being the dominant frequency.

This source generates P-waves with wavelengths much larger than the layer thickness.

Time histories of the frame particle velocities are recorded at equally spaced receivers r1, r2, r3 and r4, located at x1 = 70 m, x2 = 100 m, x3 = 130 m and x4 = 160 m.

The time histories amplitudes and their amplitude spectrums are normalized to the maximum amplitude of the signal at r1 in the uncoupled case.

This normalization provides a better representation of the relative changes in wave propagation and attenuation between the uncoupled and coupled cases.

Numerical experiments. Time histories at receivers r1 and r4. Layer thickness 20 cm.

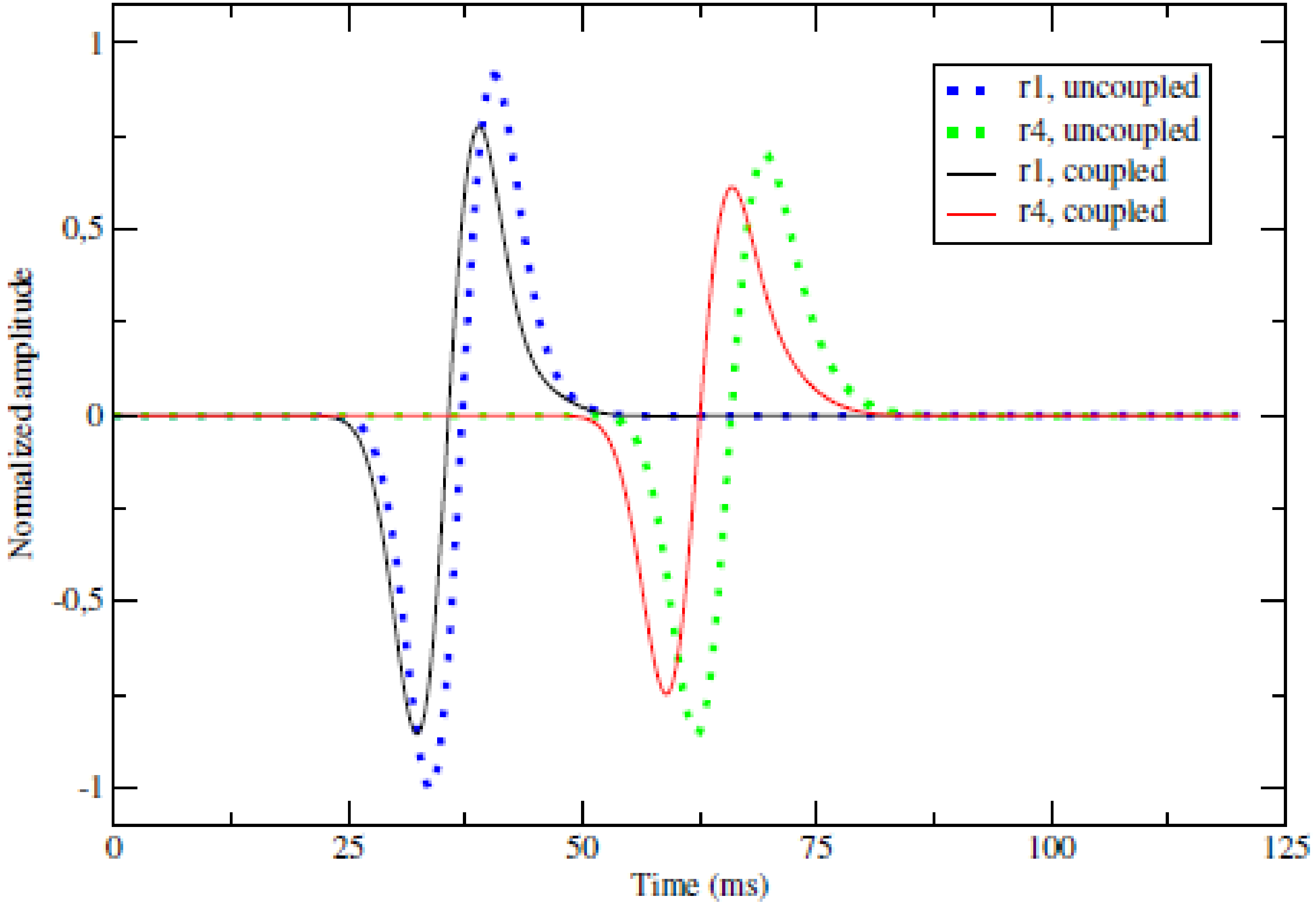


Figure 2: Time histories of the frame particle velocity at r1 and r4.

P-Waves **arrive faster** and have lower amplitude in the **coupled case than** in the uncoupled one, with **wave amplitudes being reduced** as source-receiver distance increases

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Numerical experiments. Amplitude spectrums at r1 and r4. Layer thickness 20 cm.

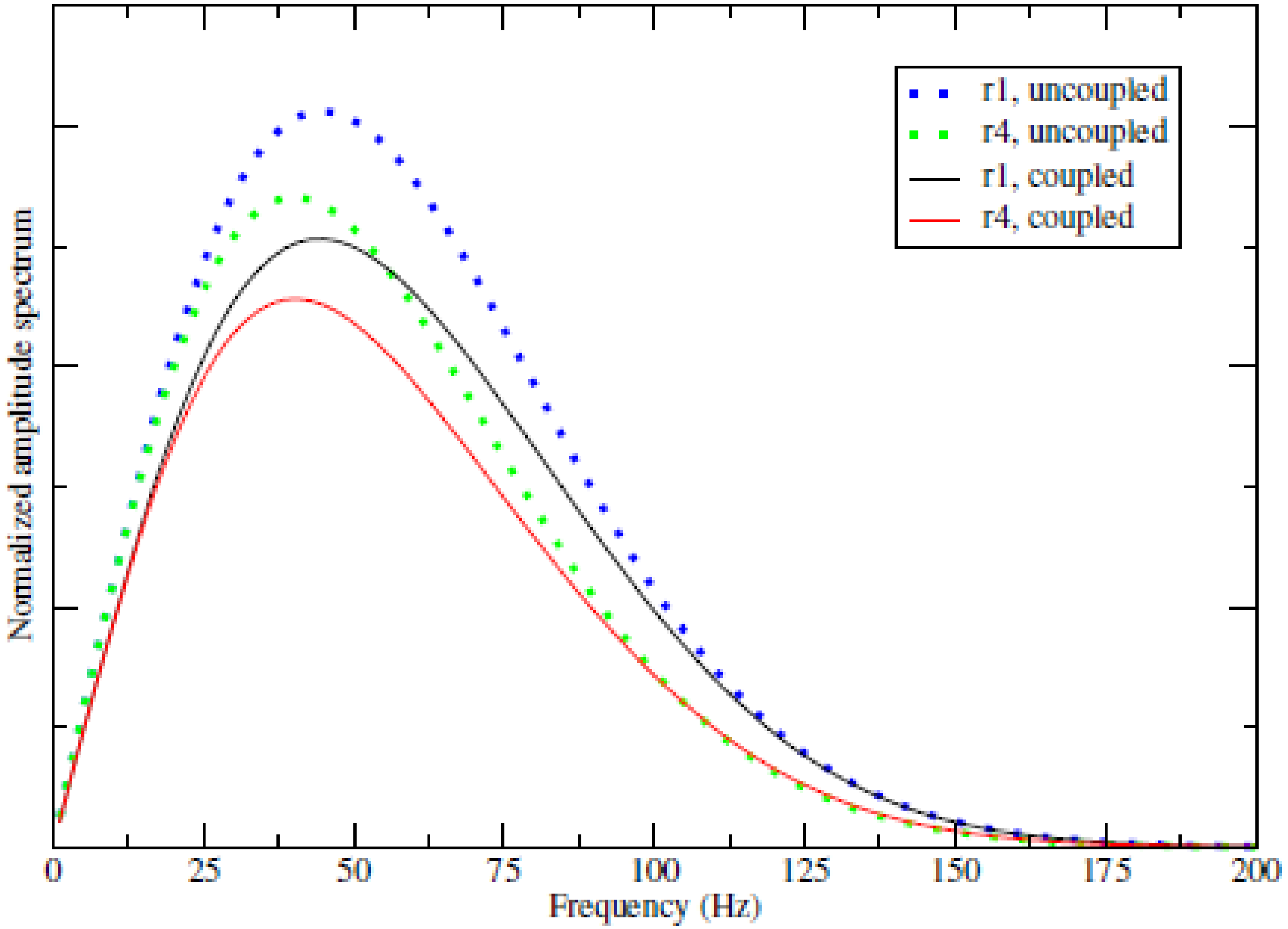

Figure 3: amplitude spectrums of the time histories at receivers r1 and r4.

Numerical experiments. Amplitude spectrums for receivers r1, r2, r3, r4. Coupled case. Layer thickness 20 cm.

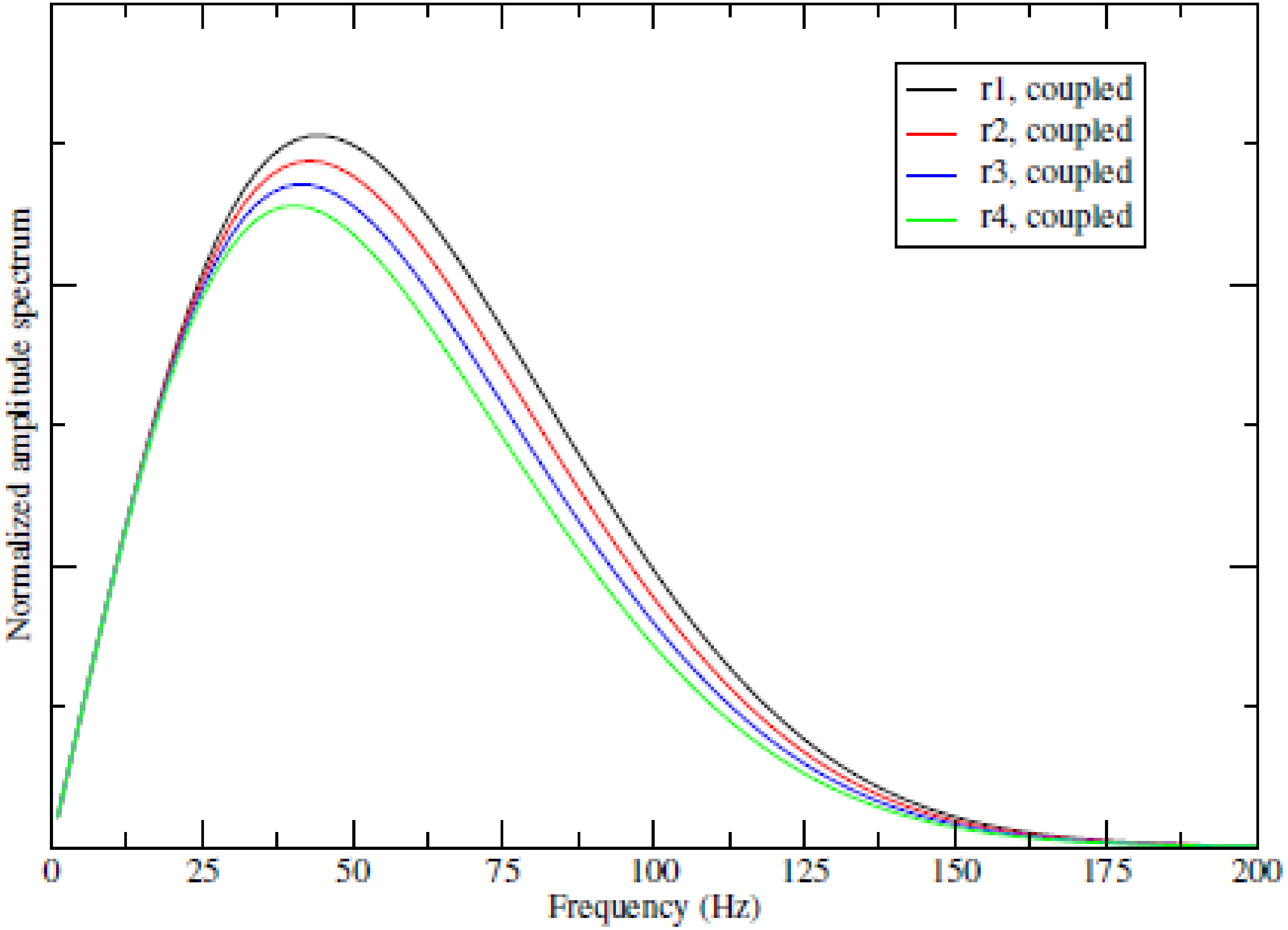

Figure 4: amplitude spectrums of time histories of the receivers r1, r2, r3 and r4. Coupled case

Numerical experiments. *Q* estimates. Layer thickness 20 cm.

Table 2. Estimated ***Q*** computed with **SR** and **FS** methods. Coupled and uncoupled cases

source	receiver	SR coupled	FS coupled	SR uncoupled	FS uncoupled
r1	r2	24.53	23.9	29.17	27.47
r1	r3	24.51	24.12	28.48	27.1
r1	r4	24.07	23.91	28.72	27.68
r2	r3	24.53	24.35	27.79	26.72
r2	r4	23.84	23.91	28.49	27.78
r3	r4	23.15	24.46	29.22	28.85

As expected, *Q*-values in the uncoupled case are close to that given by White's theory (Q = 28) and higher than the estimates for the coupled case.

In the **coupled case attenuation is higher**,
thus ***Q*-values decrease.**

Second example. Comparison of Q-estimates for different layer thickness. Coupled case.

The second example considers an interval $\Omega = (0, 400 \text{ m})$ and two sequences of periodic layers of layer thickness 15 and 30 cm, respectively.

The source dominant frequency is $f_0 = 34$ Hz for layer thickness 30 cm and $f_0 = 140$ Hz for layer thickness 15 cm, respectively.

The frame particle velocities are recorded at receivers R1 and R2 located at $x1 = 200$ m, $x2 = 250$ m, respectively.

Amplitude spectrums for receiver R2 and two layer thickness. Coupled case.

The higher amplitude spectrum at receiver R2 for layer thickness 30 cm as compared with that of 15 cm is consequence of less mode conversion and consequently, larger amplitude spectrum and lower attenuation.

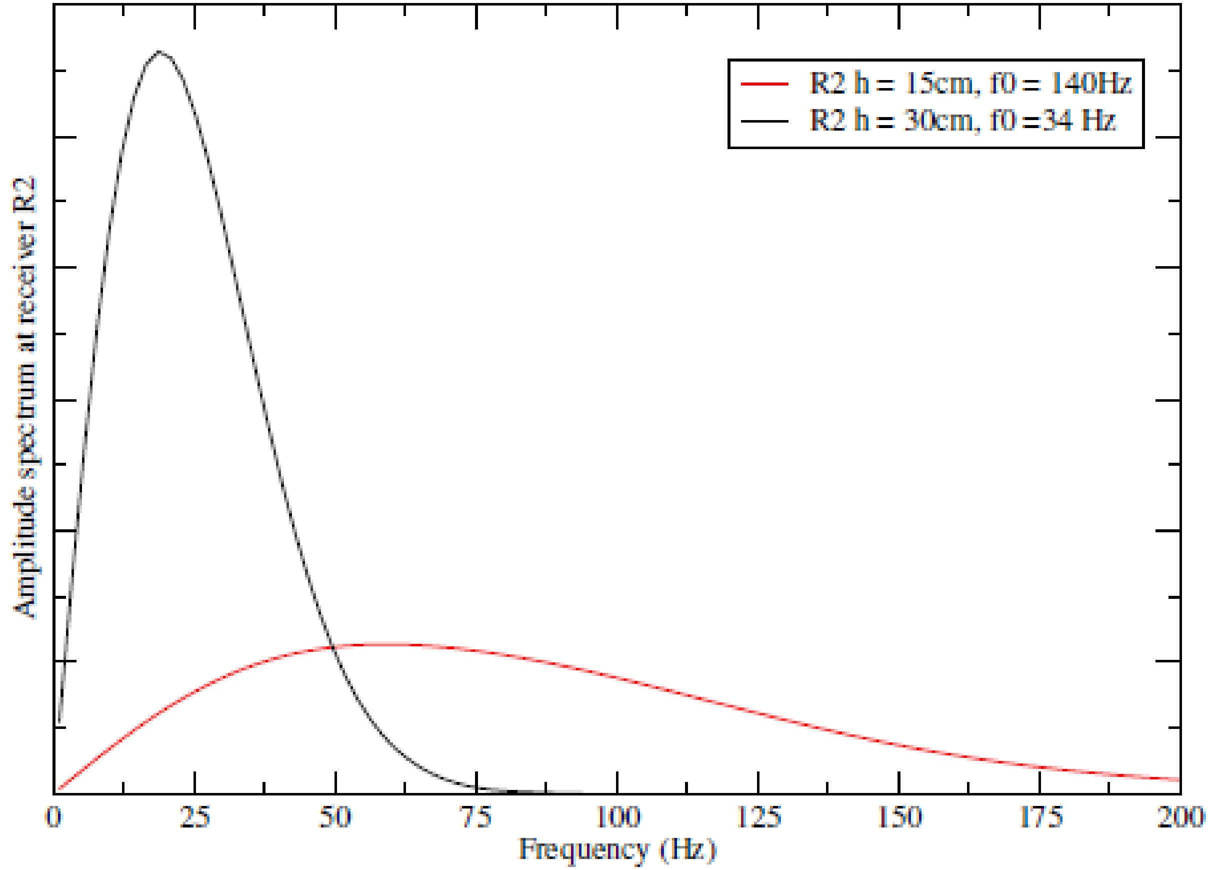

Figure 5: Amplitude spectrums corresponding to the time history of receiver R2.
Layer thickness 15 cm and 30 cm

Q estimates for different layer thickness. Coupled case.

Table 3. Estimated ***Q*** for layer thickness 15 cm and 30 cm computed with the **SR** and **FS** methods. Coupled case.

15 cm layer thickness			
source	receiver	SR	FS
R1	R2	24.16	23.56
30 cm layer thickness			
R1	R2	25.23	24.98

Q-values for a layer thickness of 30 cm are higher than those of 15 cm, since there is less mode conversion.

Thus **for 30 cm we have larger amplitude spectrum and less attenuation than for 15 cm,**

Conclusions

- We study attenuation and dispersion effects of P-waves of wavelenghts much larger than the layer thickness in non-isothermal poroelastic layered media alternatively saturated with either gas or water.
- Time histories of frame particle displacements were recorded at equally spaced receivers.
- P-waves attenuation was measured by estimating the quality factor Q using two different procedures.
- The experiments clearly show the additional energy losses in non-isothermal media, associated with the presence of T-waves, in addition to WIFF loss effects present in the isothermal case.

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THANKS FOR YOUR ATTENTION !!!!!

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