Effective viscoelastic medium from fractured fluid-saturated poroviscoelastic media. A finite element approach.

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Abstract

An effective viscoelastic medium is derived using harmonic FE numerical experiments in a poroviscoelastic fluid-saturated medium containing a dense set of horizontal fractures.

Key words: , Poroviscoelasticity, Finite element methods, Effective anisotropic media

1 Introduction

to be written

2 A fractured Biot's medium

We consider a porous solid saturated by a single phase, compressible viscous fluid and assume that the whole aggregate is isotropic. Let $u_s = (u_{s,i})$ and $\widetilde{u}_f = (\widetilde{u}_{f,i})$, $i = 1, \cdots, E$ denote the averaged displacement vectors of the solid and fluid phases, respectively, where E denotes the Euclidean dimension. Also let

$$u_f = \phi(\widetilde{u}_f - u_s),$$

be the average relative fluid displacement per unit volume of bulk material, with ϕ denoting the effective porosity. Set $u = (u_s, u_f)$ and note that

$$\xi = -\nabla \cdot u_f,$$

represents the change in fluid content.

Let $\varepsilon_{ij}(u_s)$ be the strain tensor of the solid. Also, let $\sigma_{ij},\ i,j = 1, \cdots, E$, and p_f denote the stress tensor of the bulk material and the fluid pressure, respectively. Following [3], the stress-strain relations can be written in the form:

$$\sigma_{ij}(u) = 2\mu\, \varepsilon_{ij}(u_s) + \delta_{ij}(\lambda_c\, \nabla \cdot u_s - \alpha\ K_{av}\, \xi), \tag{1a}$$

$$p_f(u) = -\alpha\ K_{av}\, \nabla \cdot u_s + K_{av}\xi. \tag{1b}$$

The coefficient μ is equal to the shear modulus of the bulk material, considered to be equal to the shear modulus of the dry matrix. Also

$$\lambda_c = K_c - \frac{2}{E}\mu, \tag{2}$$

with K_c being the bulk modulus of the saturated material. Following [22] [12] the coefficients in (1) can be obtained from the relations

$$\alpha = 1 - \frac{K_m}{K_s}, \quad K_{av} = \left(\frac{\alpha - \phi}{K_s} + \frac{\phi}{K_f}\right)^{-1} \quad K_c = K_m + \alpha^2 K_{av}, \tag{3}$$

where K_s, K_m and K_f denote the bulk modulus of the solid grains composing the solid matrix, the dry matrix and the the saturant fluid, respectively. The coefficient α is known as the effective stress coefficient of the bulk material.

2.1 The equations of motion

Let ρ_s and ρ_f denote the mass densities of the solid grains and the fluid and let

$$\rho_b = (1-\phi)\rho_s + \phi\rho_f$$

denote the mass density of the bulk material. Let the positive definite matrix $\mathcal{P}$ and the nonnegative matrix $\mathcal{B}$ be defined by

$$\mathcal{P} = \begin{pmatrix} \rho_b I & \rho_f I \\ \rho_f I & mI \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} 0I & 0I \\ 0I & bI \end{pmatrix}.$$

Here I denoted the identity matrix in $R^{E\times E}$. The mass coupling coefficient m represents the inertial effects associated with dynamic interactions between the solid and fluid phases, while the coefficient b includes the viscous coupling effects between such phases. They are given by the relations

$$b = \frac{\eta}{k}, \qquad m = \frac{S\rho_f}{\phi}, \qquad S = \frac{1}{2}\left(1 + \frac{1}{\phi}\right), \tag{4}$$

where η is the fluid viscosity and k the absolute permeability. S is known as the structure or tortuosity factor. Next, let $\mathcal{L}(u)$ be the second order differential operator defined by

$$\mathcal{L}(u) = (\nabla\cdot\sigma(u), -\nabla p_f(u)).$$

Then if $\omega = 2\pi f$ is the angular frequency, in the absence of body forces Biot's equations of motion, stated in the space-frequency domain are [1] [2]

$$-\omega^2 \mathcal{P} u(x,\omega) + i\omega \mathcal{B} u(x,\omega) - \mathcal{L}(u(x,\omega)) = 0. \tag{5}$$

2.2 The bounday conditions at a fracture

Consider a square domain $\Omega = (0, D)^2$ with boundary Γ in the (x_1, x_3)-plane and that we have one horizontal fractures $\Gamma^{(f,1)}$ of length D in our domain Ω. This fracture divides our domain in two nonoverlapping rectangles $R^{(1)}, R^{(2)}$ having as a common side $\Gamma^{(f,1)}$, so that

$$\Omega = \cup_{l=1}^{2} R^{(l)}.$$

Let $\nu_{1,2}, \chi_{1,2}$ be the unit outer normal and a unit tangent (oriented counterclockwise) on $\Gamma^{(f,1)}$ from $R^{(1)}$ to $R^{(2)}$, such that $\{\nu_{1,2}, \chi_{1,2}\}$ is an orthonormal system on $\Gamma^{(f,1)}$.

Let $[u_s], [u_f]$ denote the jumps of the solid and fluid displacement vectors at $\Gamma^{(f,1)}$ i.e.,

$$[u_s] = \left(u_s^{(2)} - u_s^{(1)} \right) |_{\Gamma^{(f,1)}}$$

$u_s^{(1)}$ denoting the displacement vector values restricted to Ω^1, and simularly for $u_s^{(2)}$.

Following Nakawa and Schoemberg, their equation (53) , JASA 2007, we impose the following boundary conditions on $\Gamma^{(f,1)}$ (here we use Z_N and Z_T for η_N and η_T in Nakawa and Schoemberg JASA07)

$$[u_s] \cdot \nu_{1,2} = Z_N \sigma(u) \nu_{12} \cdot \nu_{12} + \alpha Z_N p_f(u), \quad \Gamma^{(f,1)} \tag{6}$$
$$[u_s] \cdot \chi_{1,2} = Z_T \sigma(u) \nu_{12} \cdot \chi_{12} \quad \Gamma^{(f,1)} \tag{7}$$
$$[u_f] \cdot \nu_{1,2} = -\alpha Z_N \sigma(u) \nu_{12} \cdot \nu_{12} - \frac{\alpha Z_N}{\widetilde{B}} p_f(u) \quad \Gamma^{(f,1)}. \tag{8}$$

Now from (6) and (8) we obtain

$$\sigma(u) \nu_{12} \cdot \nu_{12} = d_{11} [u_s] \cdot \nu_{1,2} + d_{12} [u_f] \cdot \nu_{1,2} \quad \Gamma^{(f,1)} \tag{9}$$
$$-p_f(u) = d_{12} [u_s] \cdot \nu_{1,2} + d_{22} [u_f] \cdot \nu_{1,2} \quad \Gamma^{(f,1)}, \tag{10}$$

where

$$f_{11} = \frac{\alpha Z_N}{\widetilde{B}} / \Theta, \quad f_{12} = \alpha Z_N / \Theta, \quad f_{22} = Z_N / \Theta, \tag{11}$$
$$\Theta = \alpha Z_N^2 \left(\frac{1}{\widetilde{B}} - \alpha \right).$$

Set

$$\mathbf{F} = \begin{pmatrix} f_{11} & 0 & f_{12} \\ 0 & \dfrac{1}{Z_T} & 0 \\ f_{12} & 0 & f_{22} \end{pmatrix} \tag{12}$$

Then we can write the boundary conditions - as follows:

$$\begin{aligned} &(\sigma(u)\nu_{12} \cdot \nu_{12}, \sigma(u)\nu_{12} \cdot \chi_{12}, -p_f(u)) \\ &\qquad = \mathbf{F}\left([u_s] \cdot \nu_{1,2}, [u_s] \cdot \chi_{1,2}, [u_f] \cdot \nu_{1,2}\right)^T, \end{aligned} \tag{13}$$

where ^{T} indicates the traspose.

Set $\Gamma = \Gamma^L \cup \Gamma^B \cup \Gamma^R \cup \Gamma^T$, where

$$\begin{aligned} \Gamma^L &= \{(x_1, x_3) \in \Gamma : x_1 = 0\}, \quad \Gamma^R = \{(x_1, x_3) \in \Gamma : x_1 = D\}, \\ \Gamma^B &= \{(x_1, x_3) \in \Gamma : x_3 = 0\}, \quad \Gamma^T = \{(x_1, x_3) \in \Gamma : x_3 = D\}. \end{aligned}$$

Denote by ν the unit outer normal on Γ and let χ be a unit tangent on Γ so that $\{\nu, \chi\}$ is an orthonormal system on Γ.

For obtaining the complex coefficient p_{33} let us consider the solution of (5) with the boundary condition (13) together with the following boundary conditions

$$\sigma(u)\nu \cdot \nu = -\Delta P, \quad (x, y) \in \Gamma^T, \tag{14}$$
$$\sigma(u)\nu \cdot \chi = 0, \quad (x, y) \in \Gamma^T, \tag{15}$$
$$\sigma(u)\nu \cdot \chi = 0, \quad (x, y) \in \Gamma^L \cup \Gamma^R, \tag{16}$$
$$u^s \cdot \nu = 0, \quad (x, y) \in \Gamma^L \cup \Gamma^R, \tag{17}$$
$$u^s = 0, \quad (x, y) \in \Gamma^B, \tag{18}$$
$$u^f \cdot \nu = 0, \quad (x, y) \in \Gamma. \tag{19}$$

Denoting by V the original volume of the sample, its (complex) oscillatory volume change, $\Delta V(\omega)$, allows us to define p_{33} by using the relation

$$\frac{\Delta V(\omega)}{V} = -\frac{\Delta P}{p_{33}(\omega)}, \tag{20}$$

valid for a viscoelastic homogeneous medium in the quasistatic case.

After solving (5) with the boundary conditions (14)-(19) and (13), the vertical displacements $u_y^s(x, L, \omega)$ on Γ^T allow us to obtain an average vertical displacement $u_y^{s,T}(\omega)$ suffered by the boundary Γ^T. Then, for each frequency ω, the volume change produced by the compressibility test can be approximated by $\Delta V(\omega) \approx L u_y^{s,T}(\omega)$, which enable us to compute $p_{33}(\omega)$ by using the relation (20). The corresponding complex compressional velocity is

$$V_{pc}(\omega) = \sqrt{\frac{p_{33}(\omega)}{\overline{\rho}_b}}, \tag{21}$$

where $\overline{\rho}_b$ is the average bulk density of the sample.

The following relations allow us to estimate the *equivalent* compressional phase velocity $V_p(\omega)$ and quality factor $Q_p(\omega)$ in the form [21]:

$$V_p(\omega) = \left[Re\left(\frac{1}{V_{pc}(\omega)}\right)\right]^{-1}, \qquad \frac{1}{Q_p(\omega)} = \frac{\mathrm{Im}(V_{pc}(\omega)^2)}{\mathrm{Re}(V_{pc}(\omega)^2)} \tag{22}$$

For obtaining the *equivalent* complex shear modulus of our fluid-saturated porous medium, let us consider the solution of (5) with (13) and the following boundary conditions

$$-\sigma(u)\nu = g, \quad (x, y) \in \Gamma^T \cup \Gamma^L \cup \Gamma^R, \tag{23}$$
$$u^s = 0, \quad (x, y) \in \Gamma^B, \tag{24}$$
$$u^f \cdot \nu = 0, \quad (x, y) \in \Gamma, \tag{25}$$

where

$$g = \begin{cases} (0, \Delta p), & (x, y) \in \Gamma^L, \\ (0, -\Delta p), & (x, y) \in \Gamma^R, \\ (-\Delta p, 0), & (x, y) \in \Gamma^T. \end{cases}$$

The change in shape of the rock sample allows to recover its *equivalent* complex shear modulus $\overline{\mu}_c(\omega)$ by using the relation

$$tg(\beta\omega)) = \frac{\Delta p}{\overline{\mu}_c(\omega)}, \tag{26}$$

where $\beta(\omega)$ is the departure angle between the original positions of the lateral boundaries and those after applying the shear stresses (see, for example, [15]). Equation (26) holds for this experiment in a viscoelastic homogeneous media in the quasistatic approximation.

The horizontal displacements $u_x^s(x, L, \omega)$ at the top boundary Γ^T allow us to obtain, for each frequency, an average horizontal displacement $u_x^{s,T}(\omega)$ suffered by the boundary Γ^T. This average value allows us to approximate the change in shape suffered by the sample, given by $tg(\beta(\omega)) \approx u_x^{s,T}(\omega)/L$, which from (26) let us estimate $\overline{\mu}_c(\omega)$.

The complex shear velocity is given by

$$V_{sc}(\omega) = \sqrt{\frac{\overline{\mu}_c(\omega)}{\overline{\rho_b}}} \tag{27}$$

and the *equivalent* shear phase velocity $V_s(\omega)$ and (inverse) quality factor $Q_s(\omega)$ are estimated using the relations

$$V_s(\omega) = \left[\mathrm{Re}\left(\frac{1}{V_{sc}(\omega)} \right) \right]^{-1}, \qquad \frac{1}{Q_s(\omega)} = \frac{\mathrm{Im}(V_{sc}(\omega)^2)}{\mathrm{Re}(V_{sc}(\omega)^2)}. \tag{28}$$

3 A variational formulation

In order to state a variational formulation for (5) and either (14)-(19) or (23)-(25) we need to introduce some notation. For $X \subset \mathbb{R}^d$ with boundary ∂X, let $(\cdot,\cdot)_X$ and $\langle \cdot,\cdot \rangle_{\partial X}$ denote the complex $L^2(X)$ and $L^2(\partial X)$ inner products for scalar, vector, or matrix valued functions. Also, for $s \in \mathbb{R}$, $\|\cdot\|_{s,X}$ and $|\cdot|_{s,X}$ will denote the usual norm and seminorm for the Sobolev space $H^s(X)$. In addition, if $X = \Omega$ or $X = \Gamma$, the subscript X may be omitted such that $(\cdot,\cdot) = (\cdot,\cdot)_\Omega$ or $\langle \cdot,\cdot \rangle = \langle \cdot,\cdot \rangle_\Gamma$.

Also, let us introduce the spaces

$$\mathcal{W}_{11}(\Omega) = \{v \in [L^2(\Omega)]^2 : v|_{R^{(l)}} \in [H^1(R^{(l)})]^2, l = 1,2 \ v{\cdot}\nu = 0 \ \text{ on } \Gamma^B \cup \Gamma^L \cup \Gamma^R\},$$

$$H_0(\mathrm{div};\Omega) = \{v \in [L^2(\Omega)]^2 : \ v|_{R^{(l)}} \in H(\mathrm{div}, R^{(l)}), v \cdot \nu = 0 \ \text{ on } \Gamma\},$$

Multiply equation (5) by $v = \left(v^s, v^f\right) \in \mathcal{W}_{11}(\Omega) \times H_0(\mathrm{div};\Omega)$, use integration by parts and apply the boundary conditions (14), (15) and (16) (13) to see that the solution of (5) and (14)-(19) and (13) satisfies *the weak form*:

Set

$$
\begin{aligned}
&\Lambda(u,v) = -\omega^2 \left(\mathcal{P}u, v\right) + i\omega \left(\mathcal{B}u, v\right) \\
&+ \sum_{l=1}^{2} \sum_{s,t} \left(\tau_{st}(u), \varepsilon_{st}(v^s)\right)_{R^{(l)}} - \Big(p_f(u), \nabla \cdot v^f\Big)_{R^{(l)}} \\
&\Big\langle \mathbf{F}\left([u_s]\cdot\nu_{1,2}, [u_s]\cdot\chi_{1,2}, [u_f]\cdot\nu_{1,2}\right)^T, \left([v_s]\cdot\nu_{1,2}, [v_s]\cdot\chi_{1,2}, [v_f]\cdot\nu_{1,2}\right)\Big\rangle \\
&= \langle \Delta P, v^s \cdot \nu \rangle_{\Gamma^T}, \qquad \forall v = \Big(v^s, v^f\Big) \in \mathcal{W}_{11}(\Omega) \times H_0(\mathrm{div};\Omega)
\end{aligned} \tag{29}
$$

Note that in (29), we can write

$$
\sum_{l=1}^{2} \sum_{s,t} \left(\tau_{st}(u), \varepsilon_{st}(v^s)\right)_{R^{(l)}} - \Big(p_f(u), \nabla \cdot v^f\Big)_{R^{(l)}} = \sum_{l=1}^{2} \left(\mathbf{E}\ \widetilde{\epsilon}(u), \widetilde{\epsilon}(v)\right)_{R^{(l)}} \tag{30}
$$

In (30), the complex matrix $\mathbf{E}(\omega) = \mathbf{E}(\omega)_R + i\mathbf{E}(\omega)_I$ and the column vector $\widetilde{\varepsilon}((u))$ are defined by

$$
\mathbf{E} = \begin{pmatrix} \lambda_c + 2\mu & \lambda_c & \alpha K_{av} & 0 \\ \lambda_c & \lambda_c + 2\mu & \alpha K_{av} & 0 \\ \alpha K_{av} & \alpha K_{av} & K_{av} & 0 \\ 0 & 0 & 0 & 4\mu \end{pmatrix}, \ \widetilde{\varepsilon}(u) = \begin{pmatrix} \varepsilon_{11}(u^s) \\ \varepsilon_{22}(u^s) \\ \nabla \cdot u^f \\ \varepsilon_{12}(u^s) \end{pmatrix}.
$$

It will be assumed that the real part $\mathbf{E}_R(\omega)$ is positive definite since in the elastic limit it is associated with the strain energy density. Furthermore, the imaginary parts $\mathbf{E}_I(\omega)$ are assumed to be positive definite because of the restriction imposed on our system by the first and second laws of thermodynamics.

Jose: aqui hay que asumir que $\mathbf{E}_I(\omega)$ es definida positiva, o sea poro-viscoelasticidad, para poder demostrar que la solucion de los p_{IJ} es unica. Despues en los exprimentos, se hara como en el paper en GJI.

Lo que importa es que creas el argumento que lleva a las boundary conditions, y si tenes ganas, escribi la demostracion de que ese termino Θ es positivo en el caso general. Lo interesante es que la weak form queda muy parecida a la del caso viscoelastico fracturado.

demas, hay que generalizar las B. C. para que la matriz F tenga parte imaginaria (positiva), asi tenemos atenuacion en las fracturas, como en el sfrac.pdf. para el η_T es facil, es lo mismo del sfrac.pdf.

todavia no escribi la matematica de ese paper nuesto en GJI, quedo alli en la cola. La de este contiene a esa, asi que ahora a escribir los teoremas antes que me agarre la licuadora de los K.....

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Figure 1: P-wave phase velocity obtained from the compressibility test (dots) and using White's theory (line) for frequencies lying between 0 and 100 Hz.

Figure 2: P-wave inverse quality factor obtained from the compressibility test (dots) and using White's theory (line) for frequencies lying between 0 and 100 Hz.

Figure 3: Gas-water distribution for a given realization. Black zones correspond to pure gas saturation and the white ones to pure water saturation. The overall gas saturation is 0.1.

Figure 4: Normalized fluid-pressure amplitude for the fluid distribution shown in Figure 3. The excitation frequency is 190 Hz.

Figure 5: Averaged variance of the compressional phase velocity as function of the total number of realizations.

Figure 6: Effective compressional phase velocity as function of frequency (solid lines). Dotted lines indicate the corresponding standard deviations.

Figure 7: Effective compressional inverse quality factor as function of frequency. Dotted lines indicate the corresponding standard deviations.

Figure 8: Distribution of shale and sandstone. Black zones correspond to pure shale, and the white ones to pure sandstone.

Figure 5: Equivalent P-wave phase velocity as function of frequency.

Figure 10: Equivalent compressional inverse quality factor as function of frequency.

Figure 11: Equivalent shear inverse quality factor as function of frequency.