

Wave Propagation in Fractured Poroelastic Media

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Fractured media. I

- **Fractures** are common in the earth's crust due to different factors, for instance, tectonic stresses and natural or artificial hydraulic fracturing caused by a pressurized fluid.
- **Seismic wave propagation** through **fractures and cracks** is an important subject in exploration and production geophysics, earthquake seismology and mining.
- In geophysical prospecting, reservoir development and CO₂ storage in geological formations, knowledge of **fracture orientation, densities and sizes** is essential since these factors control hydrocarbon production.

Fractured media. II

- A planar fracture embedded in a Biot background is a particular case of the thin layer problem, when one of the layers is very thin, highly permeable and compliant.
- A Biot medium containing a dense set of aligned fractures behaves as an effective transversely isotropic and viscoelastic (TIV) medium at the macroscale when the predominant wavelength is much larger than the average distance between fractures.
- One important mechanism in Biot media at seismic frequencies is wave-induced fluid flow generated by fast compressional waves at mesoscopic-scale heterogeneities, generating slow diffusion-type Biot waves.

Fractured media. III

- In the context of **Numerical Rock Physics**, we present and analyze a set of time-harmonic finite element (FE) experiments that take into account the effects of the presence of aligned fractures and interlayer fluid flow occurring at the mesoscale.
- These numerical experiments allow to determine the complex and frequency dependent stiffnesses of the **effective TIV medium** at the macroscale. They are defined as **BVP** on representative samples of the fractured material, with boundary conditions associated with compressibility and shear tests, which are solved using the FE method.

- For Biot's media, White et al. (1975) were the first to introduce the mesoscopic-loss mechanism in the framework of Biot's theory.
- For fine layered poroelastic materials, the theories of Gelinsky and Shapiro (GPY, 62, 1997) and Krzikalla and Müller (GPY, 76, 2011) allow to obtain the **five** complex and frequency-dependent stiffnesses of the **equivalent TIV medium**.
- To test the model and provide a more general modeling tool, we present a **FE upscaling procedure** to obtain the complex stiffnesses of the **equivalent TIV medium**.
- The samples contain mesoscopic-scale heterogeneities due to patchy brine-CO₂ saturation and fractal porosity (fractal frame properties).

Let us consider isotropic fluid-saturated poroelastic layers.

$\mathbf{u}^s(\mathbf{x}), \mathbf{u}^f(\mathbf{x})$: time Fourier transform of the displacement vector of the solid and fluid relative to the solid frame, respectively.

$$\mathbf{u} = (\mathbf{u}^s, \mathbf{u}^f)$$

$\sigma_{kl}(\mathbf{u}), \mathbf{p}_f(\mathbf{u})$: Fourier transform of the total stress and the fluid pressure, respectively.

$\mathbf{e}(\mathbf{u}_s)$ the strain tensor of the solid phase.

On each plane layer n in a sequence of N layers, the **frequency-domain stress-strain relations** are

$$\begin{aligned}\sigma_{kl}(\mathbf{u}) &= 2\mu e_{kl}(\mathbf{u}^s) + \delta_{kl} \left(\lambda_G \nabla \cdot \mathbf{u}^s + \alpha M \nabla \cdot \mathbf{u}^f \right), \\ \mathbf{p}_f(\mathbf{u}) &= -\alpha M \nabla \cdot \mathbf{u}^s - M \nabla \cdot \mathbf{u}^f.\end{aligned}$$

Biot's equations in the diffusive range:

$$\nabla \cdot \sigma(\mathbf{u}) = 0,$$

$$i\omega \frac{\eta}{\kappa} \mathbf{u}^f(\mathbf{x}, \omega) + \nabla p_f(\mathbf{u}) = 0,$$

$\omega = 2\pi f$: angular frequency

η : fluid viscosity

κ : frame permeability

TIV media and fine layering. III

$\tau_{ij}(\tilde{\mathbf{u}}_s)$, $\epsilon_{ij}(\tilde{\mathbf{u}}_s)$: stress and strain tensor components of the equivalent TIV medium,

$\tilde{\mathbf{u}}_s$: solid displacement vector at the macroscale.

The stress-strain relations (assuming a closed system ($\nabla \cdot \mathbf{u}_f = 0$))

$$\tau_{11}(\tilde{\mathbf{u}}_s) = p_{11} \epsilon_{11}(\tilde{\mathbf{u}}_s) + p_{12} \epsilon_{22}(\tilde{\mathbf{u}}_s) + p_{13} \epsilon_{33}(\tilde{\mathbf{u}}_s),$$

$$\tau_{22}(\tilde{\mathbf{u}}_s) = p_{12} \epsilon_{11}(\tilde{\mathbf{u}}_s) + p_{11} \epsilon_{22}(\tilde{\mathbf{u}}_s) + p_{13} \epsilon_{33}(\tilde{\mathbf{u}}_s),$$

$$\tau_{33}(\tilde{\mathbf{u}}_s) = p_{13} \epsilon_{11}(\tilde{\mathbf{u}}_s) + p_{13} \epsilon_{22}(\tilde{\mathbf{u}}_s) + p_{33} \epsilon_{33}(\tilde{\mathbf{u}}_s),$$

$$\tau_{23}(\tilde{\mathbf{u}}_s) = 2 p_{55} \epsilon_{23}(\tilde{\mathbf{u}}_s), \quad \tau_{13}(\tilde{\mathbf{u}}_s) = 2 p_{55} \epsilon_{13}(\tilde{\mathbf{u}}_s),$$

$$\tau_{12}(\tilde{\mathbf{u}}_s) = 2 p_{66} \epsilon_{12}(\tilde{\mathbf{u}}_s).$$

This approach provides the complex velocities of the fast modes and takes into account **interlayer flow effects..**

The harmonic experiments to determine the stiffness coefficients. I

To determine the complex stiffness we solve Biot's equation in the 2D case on a reference square $\Omega = (0, L)^2$ with boundary Γ in the (x_1, x_3) -plane. Set $\Gamma = \Gamma^L \cup \Gamma^B \cup \Gamma^R \cup \Gamma^T$, where

$$\Gamma^L = \{(x_1, x_3) \in \Gamma : x_1 = 0\}, \quad \Gamma^R = \{(x_1, x_3) \in \Gamma : x_1 = L\},$$

$$\Gamma^B = \{(x_1, x_3) \in \Gamma : x_3 = 0\}, \quad \Gamma^T = \{(x_1, x_3) \in \Gamma : x_3 = L\}.$$

The harmonic experiments to determine the stiffness coefficients. II

The sample is subjected to **harmonic compressibility and shear tests** described by the following sets of **boundary conditions**.

$p_{33}(\omega)$:

$$\sigma(\mathbf{u})\boldsymbol{\nu} \cdot \boldsymbol{\nu} = -\Delta P, \quad (x_1, x_3) \in \Gamma^T,$$

$$\sigma(\mathbf{u})\boldsymbol{\nu} \cdot \boldsymbol{\chi} = 0, \quad (x_1, x_3) \in \Gamma^T \cup \Gamma^L \cup \Gamma^R,$$

$$\mathbf{u}^s \cdot \boldsymbol{\nu} = 0, \quad (x_1, x_3) \in \Gamma^L \cup \Gamma^R,$$

$$\mathbf{u}^s = 0, \quad (x_1, x_3) \in \Gamma^B, \quad \mathbf{u}^f \cdot \boldsymbol{\nu} = 0, \quad (x_1, x_3) \in \Gamma.$$

$\boldsymbol{\nu}$: the unit outer normal on Γ

$\boldsymbol{\chi}$: a unit tangent on Γ so that $\{\boldsymbol{\nu}, \boldsymbol{\chi}\}$ is an orthonormal system on Γ .

Denote by V the original volume of the sample and by $\Delta V(\omega)$ its (complex) oscillatory volume change.

The harmonic experiments to determine the stiffness coefficients. III

In the quasistatic case

$$\frac{\Delta V(\omega)}{V} = -\frac{\Delta P}{p_{33}(\omega)},$$

Then after computing the average $u_3^{s,T}(\omega)$ of the vertical displacements on Γ^T , we approximate

$$\Delta V(\omega) \approx Lu_3^{s,T}(\omega)$$

which enable us to compute $p_{33}(\omega)$

To determine $p_{11}(\omega)$ we solve an identical boundary value problem than for p_{33} but for a 90° rotated sample.

The harmonic experiments to determine the stiffness coefficients. IV

$p_{55}(\omega)$: the boundary conditions are

$$-\sigma(u)\nu = g, \quad (x_1, x_3) \in \Gamma^T \cup \Gamma^L \cup \Gamma^R,$$

$$u^s = 0, \quad (x_1, x_3) \in \Gamma^B,$$

$$u^f \cdot \nu = 0, \quad (x_1, x_3) \in \Gamma,$$

where

$$g = \begin{cases} (0, \Delta G), & (x_1, x_3) \in \Gamma^L, \\ (0, -\Delta G), & (x_1, x_3) \in \Gamma^R, \\ (-\Delta G, 0), & (x_1, x_3) \in \Gamma^T. \end{cases}$$

The harmonic experiments to determine the stiffness coefficients. V

The change in shape suffered by the sample is

$$\tan[\theta(\omega)] = \frac{\Delta G}{p_{55}(\omega)}. \quad (1)$$

$\theta(\omega)$: the angle between the original positions of the lateral boundaries and the location after applying the shear stresses.

Since

$\tan[\theta(\omega)] \approx u_1^{s,T}(\omega)/L$, where $u_1^{s,T}(\omega)$ is the average horizontal displacement at Γ^T , $p_{55}(\omega)$ can be determined from

to determine $p_{66}(\omega)$ (shear waves traveling in the (x_1, x_2) -plane, we rotate the layered sample 90° and apply the shear test as indicated for $p_{55}(\omega)$.

The harmonic experiments to determine the stiffness coefficients. VI

$p_{13}(\omega)$: the boundary conditions are

$$\sigma(u)\nu \cdot \nu = -\Delta P, \quad (x_1, x_3) \in \Gamma^R \cup \Gamma^T,$$

$$\sigma(u)\nu \cdot \chi = 0, \quad (x_1, x_3) \in \Gamma,$$

$$u^s \cdot \nu = 0, \quad (x_1, x_3) \in \Gamma^L \cup \Gamma^B, \quad u^f \cdot \nu = 0, \quad (x_1, x_3) \in \Gamma.$$

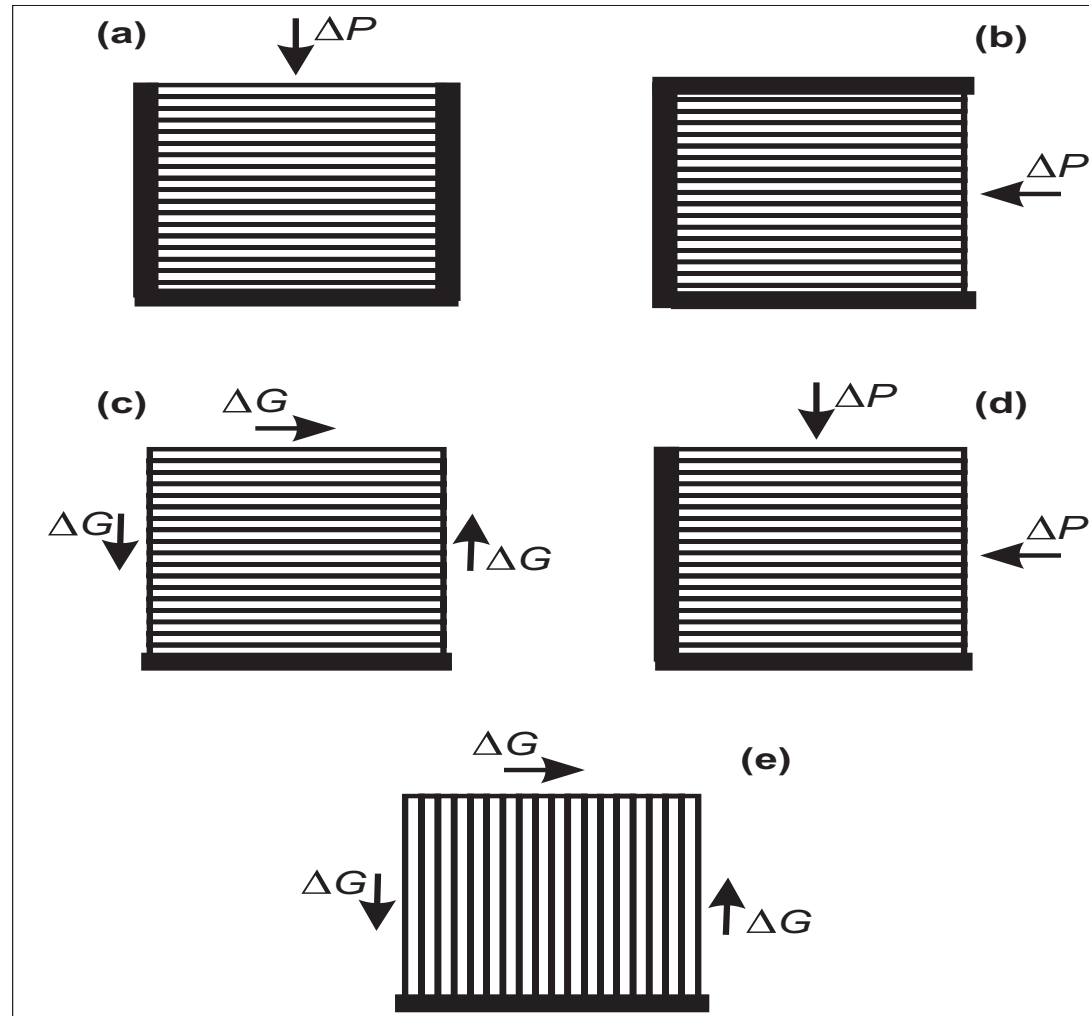
In this experiment $\epsilon_{22} = \nabla \cdot u^f = 0$, so that

$$\tau_{11} = p_{11}\epsilon_{11} + p_{13}\epsilon_{33}, \quad \tau_{33} = p_{13}\epsilon_{11} + p_{33}\epsilon_{33}, \quad (2)$$

$\epsilon_{11}, \epsilon_{33}$: the strain components at the right lateral side and top side of the sample, respectively. Then,

$$p_{13}(\omega) = (p_{11}\epsilon_{11} - p_{33}\epsilon_{33}) / (\epsilon_{11} - \epsilon_{33}).$$

Schematic representation of the oscillatory compressibility and shear tests in Ω



a): p_{33} , b): p_{11} , c): p_{55} , d): p_{13} e): p_{66}

Examples. I

A set of numerical examples consider the following cases for a square poroelastic sample of 160 cm side length and 10 periods of 1 cm fracture, 15 cm background:

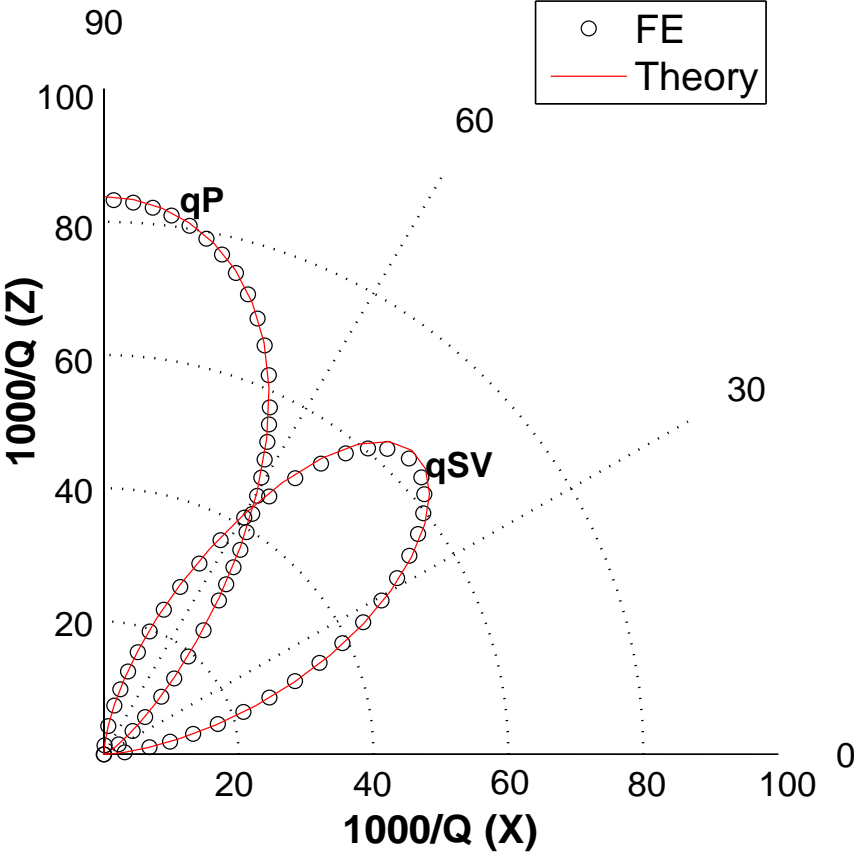
- Case 1: A brine-saturated sample with fractures.
- Case 2: A brine-CO₂ patchy saturated sample without fractures.
- Case 3: A brine-CO₂ patchy saturated sample with fractures.
- Case 4: A brine saturated sample with a fractal frame and fractures.

The discrete boundary value problems to determine the complex stiffnesses $p_{IJ}(\omega)$ were solved for 30 frequencies using a public domain sparse matrix solver package.

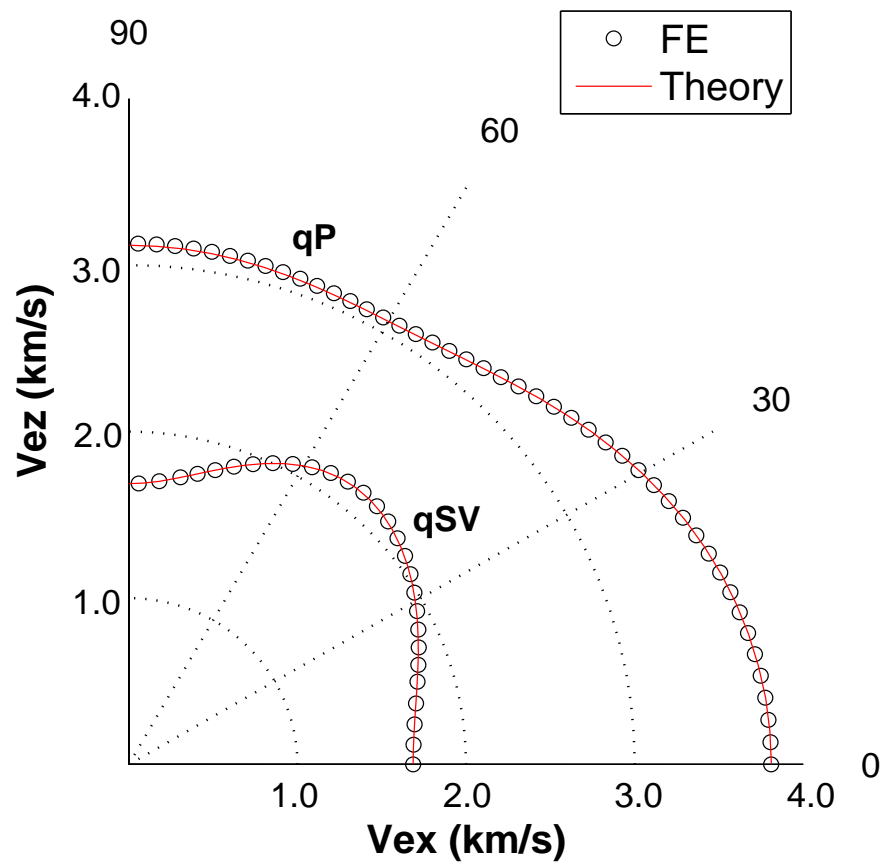
Using relations not included for brevity, the $p_{IJ}(\omega)$'s determine in turn the energy velocities and dissipation coefficients shown in the next figures.

Examples. II

- **Both background and fractures:**
grain density is $\rho_s = 2650 \text{ kg/m}^3$,
bulk modulus is $K_s = 37 \text{ GPa}$,
shear modulus is $\mu_s = 44 \text{ GPa}$.
- **for the background:**
Porosity is $\phi = 0.25$, Permeability is $\kappa = 0.247 \text{ Darcy}$.
dry bulk modulus is $K_m = 1.17 \text{ GPa}$,
shear modulus is $\mu = 1.45 \text{ GPa}$.
- **for the fractures:**
Porosity is $\phi = 0.5$, Permeability is $\kappa = 4.44 \text{ Darcy}$,
dry bulk modulus is $K_m = 0.58 \text{ GPa}$,
shear modulus is $\mu = 0.68 \text{ GPa}$.

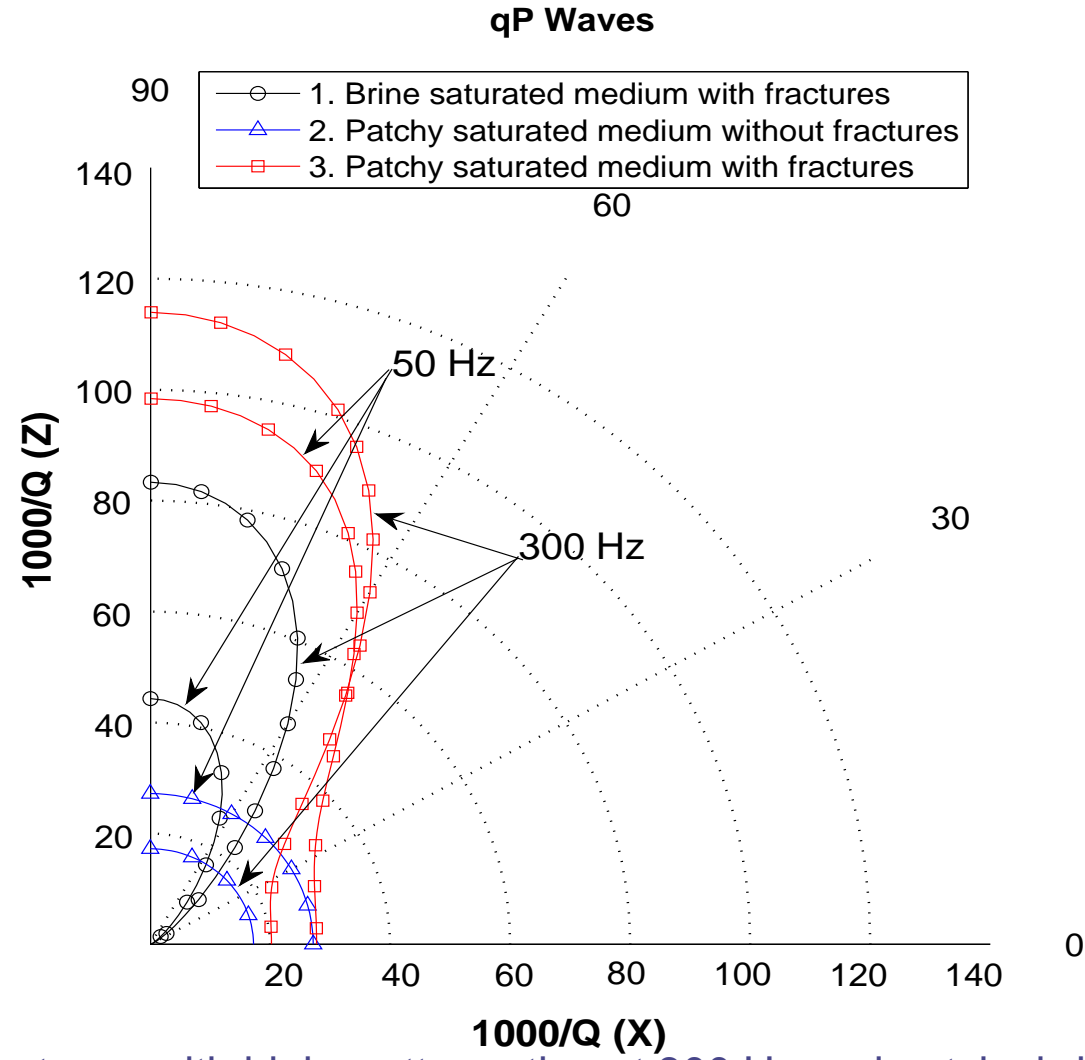


A very good match between the theoretical and numerical results is observed.

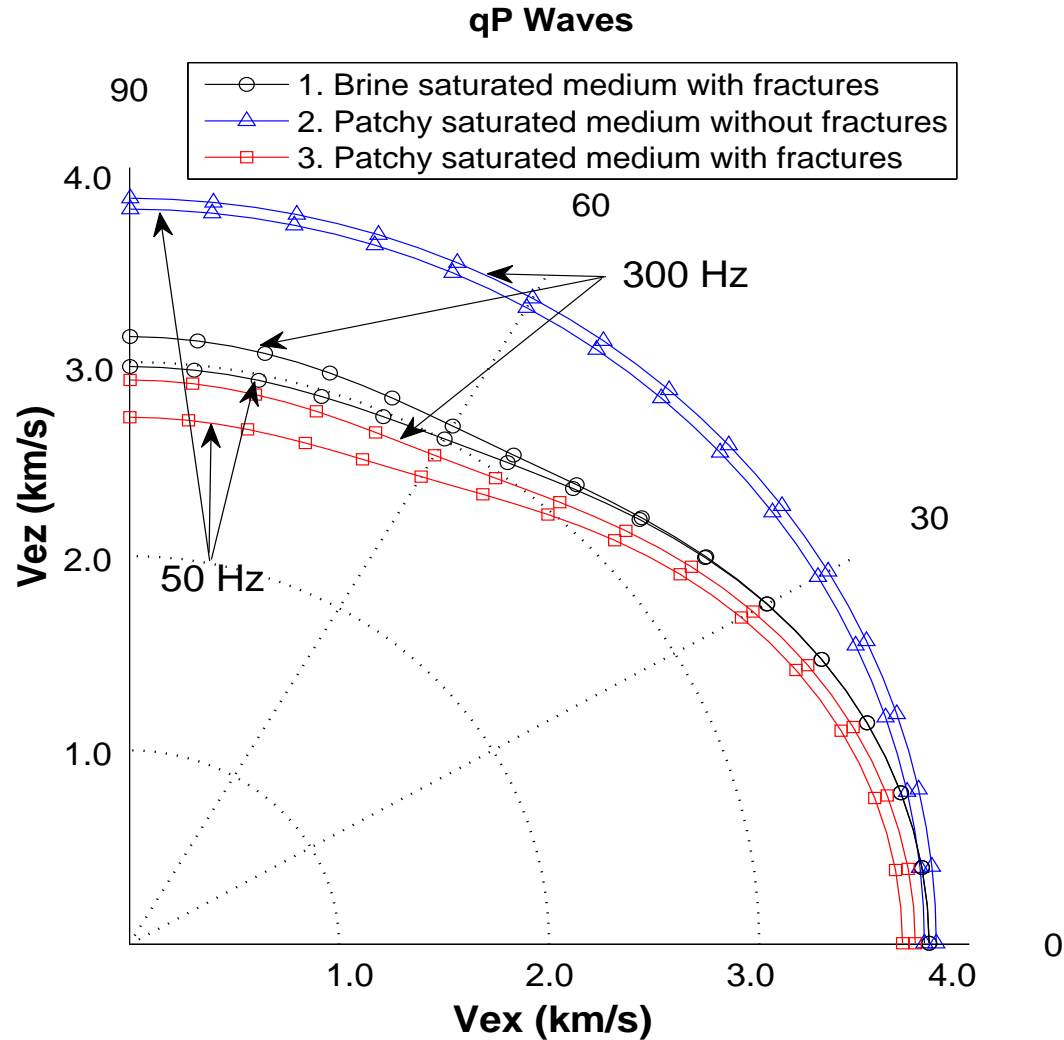


A very good match between the theoretical and numerical results is observed.

Dissipation factors of qP waves at 50 Hz and 300 Hz. Cases 1, 2 and 3

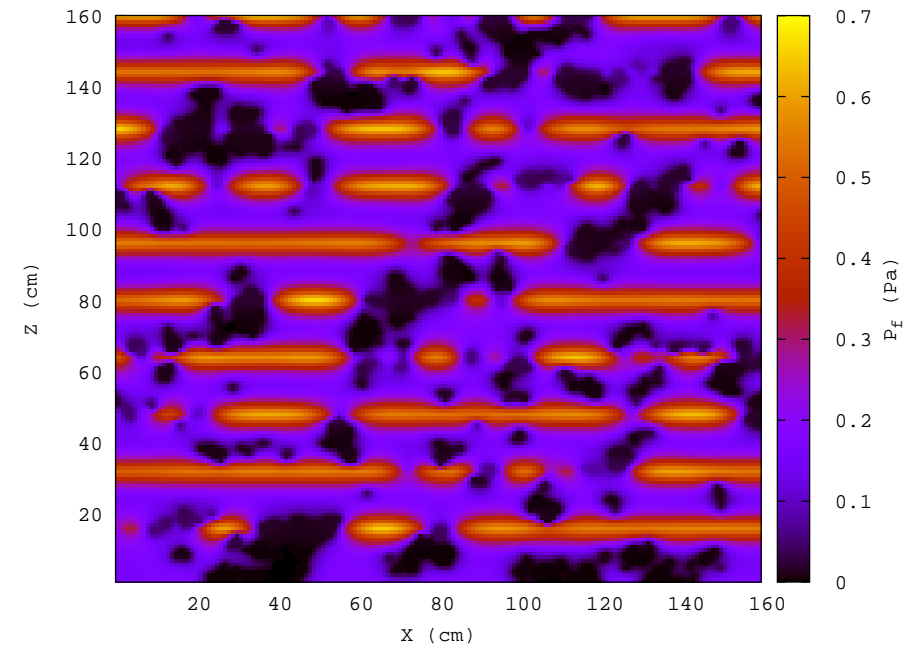
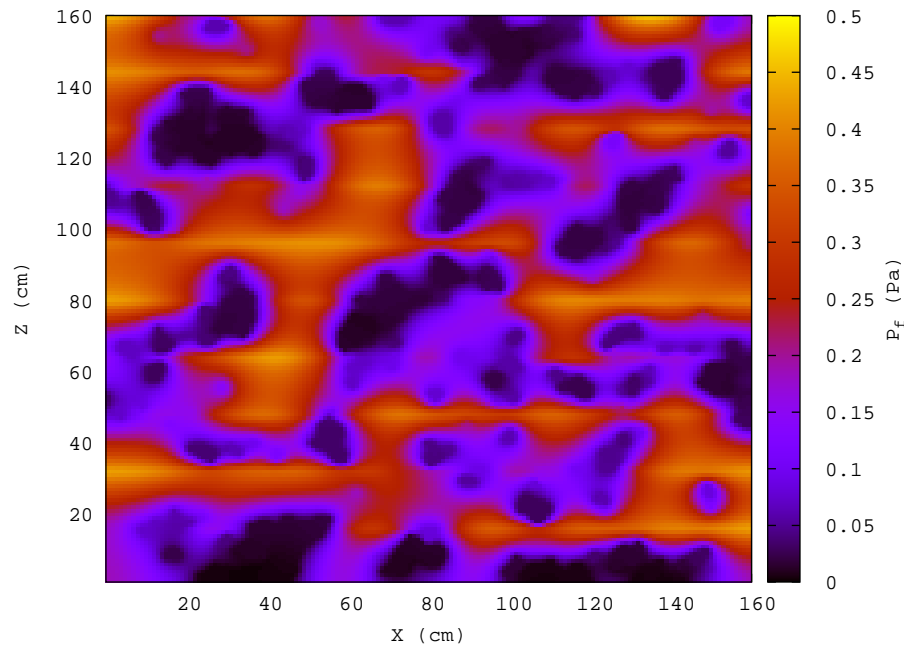


Note strong Q anisotropy, with higher attenuation at 300 Hz and patchy brine- CO_2 saturation. Energy losses are much higher for angles between 60 and 90 degrees (waves incident normal to the fracture layering)



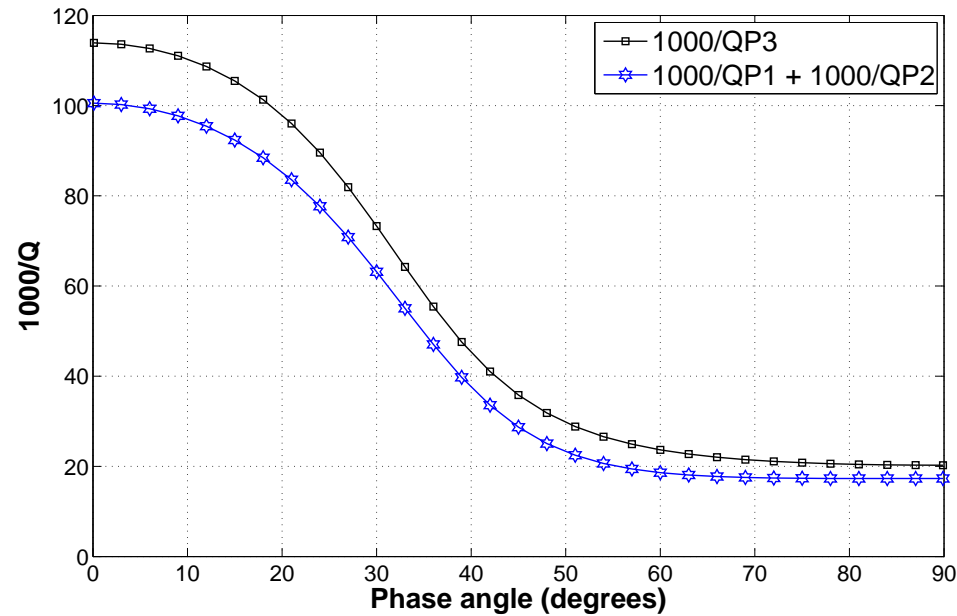
Velocity anisotropy caused by the fractures in cases 1 and 3 is enhanced for the case of patchy saturation, with lower velocities when patches are present. Velocity behaves isotropically in case 2

Fluid pressure distribution at 50 Hz and 300 Hz for case 3 and compressions normal to the fracture layering.



Note that pressure gradients take their highest values at the fractures (mesoscopic losses), and at 300 Hz remain always higher than at 50 Hz. This explains the higher losses for qP waves at 300 Hz as compared with the 50 Hz experiment observed before.

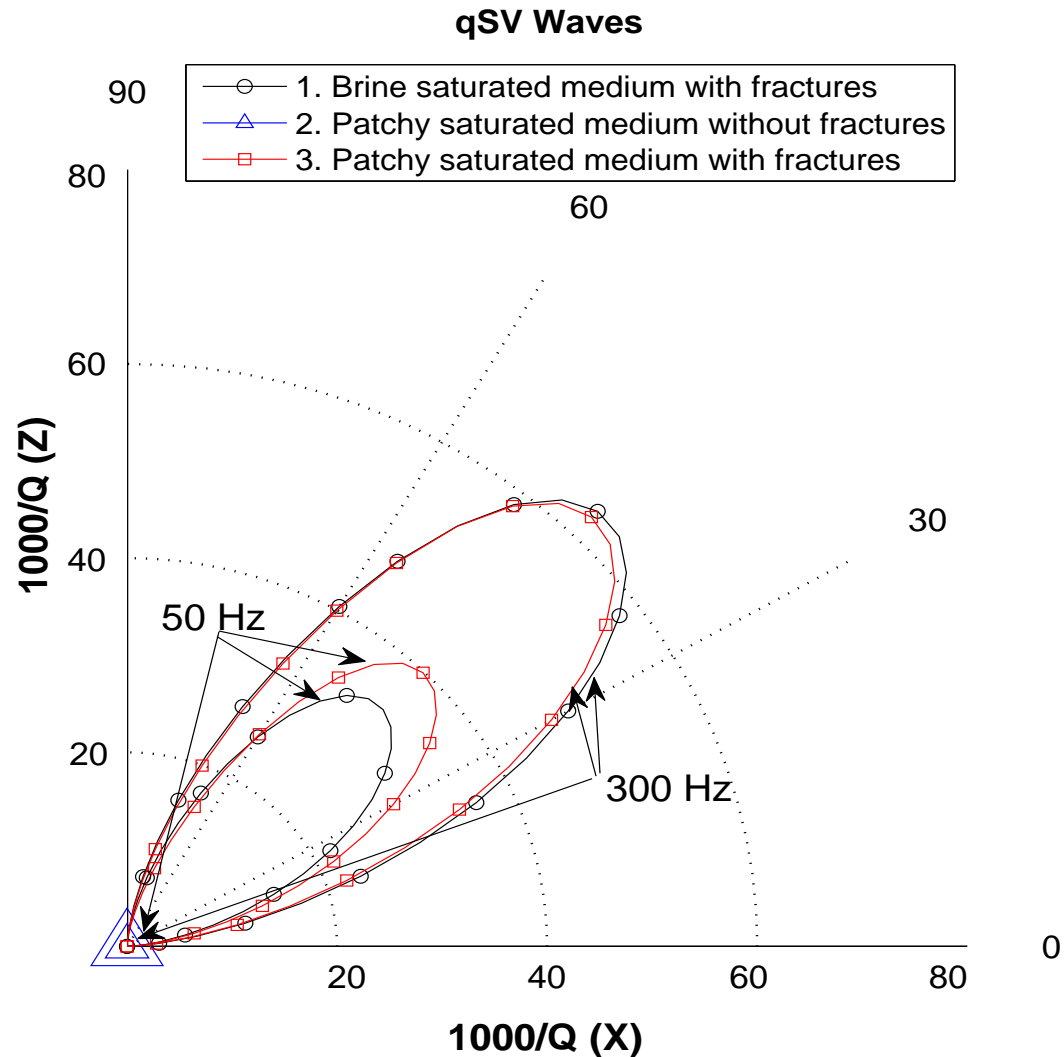
Relation among different attenuation mechanisms for cases 1, 2 and 3 at 300



Q_{P1} , Q_{P2} and Q_{P3} : the qP-quality factors associated with cases 1, 2 and 3. Approximate validity of the commonly used approximation

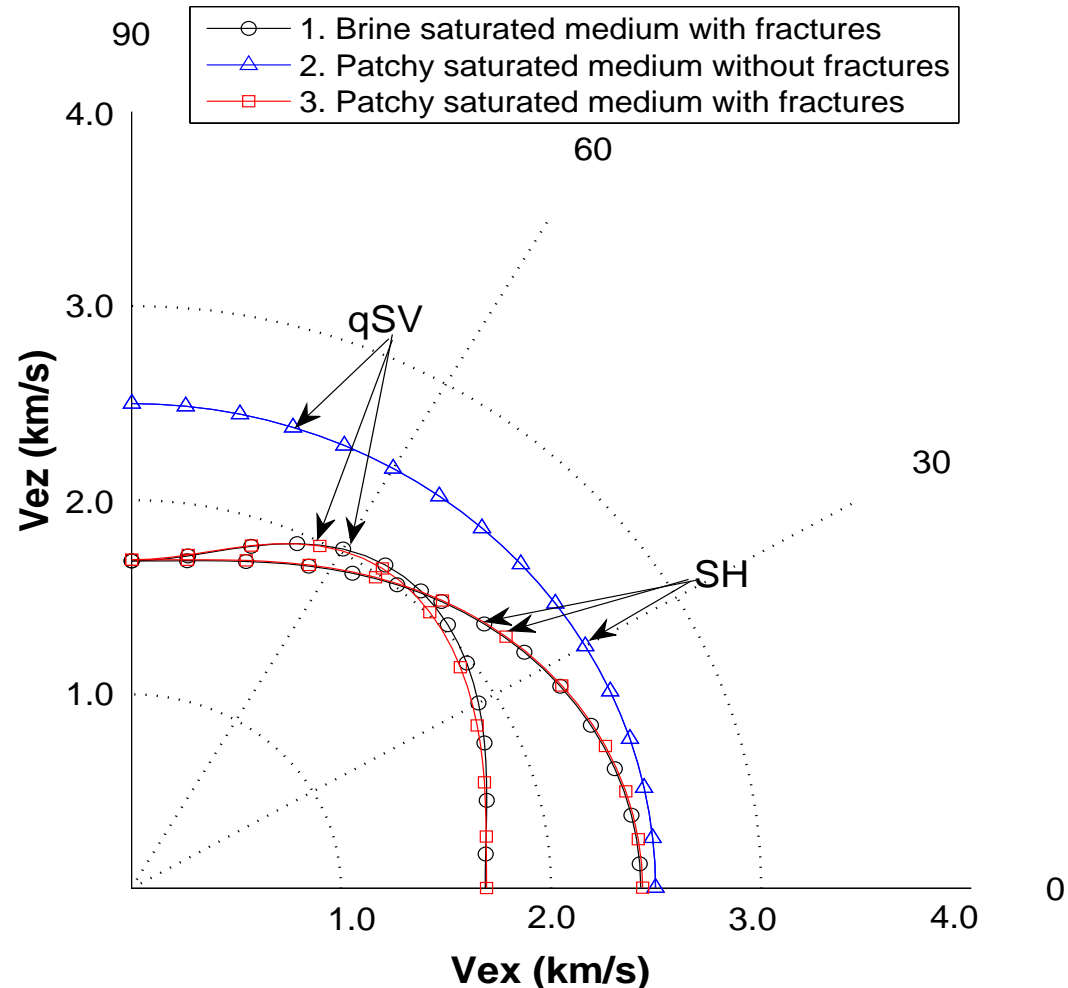
$$Q_{P3}^{-1} = Q_{P1}^{-1} + Q_{P2}^{-1}$$

for the dissipation factors for qP waves.



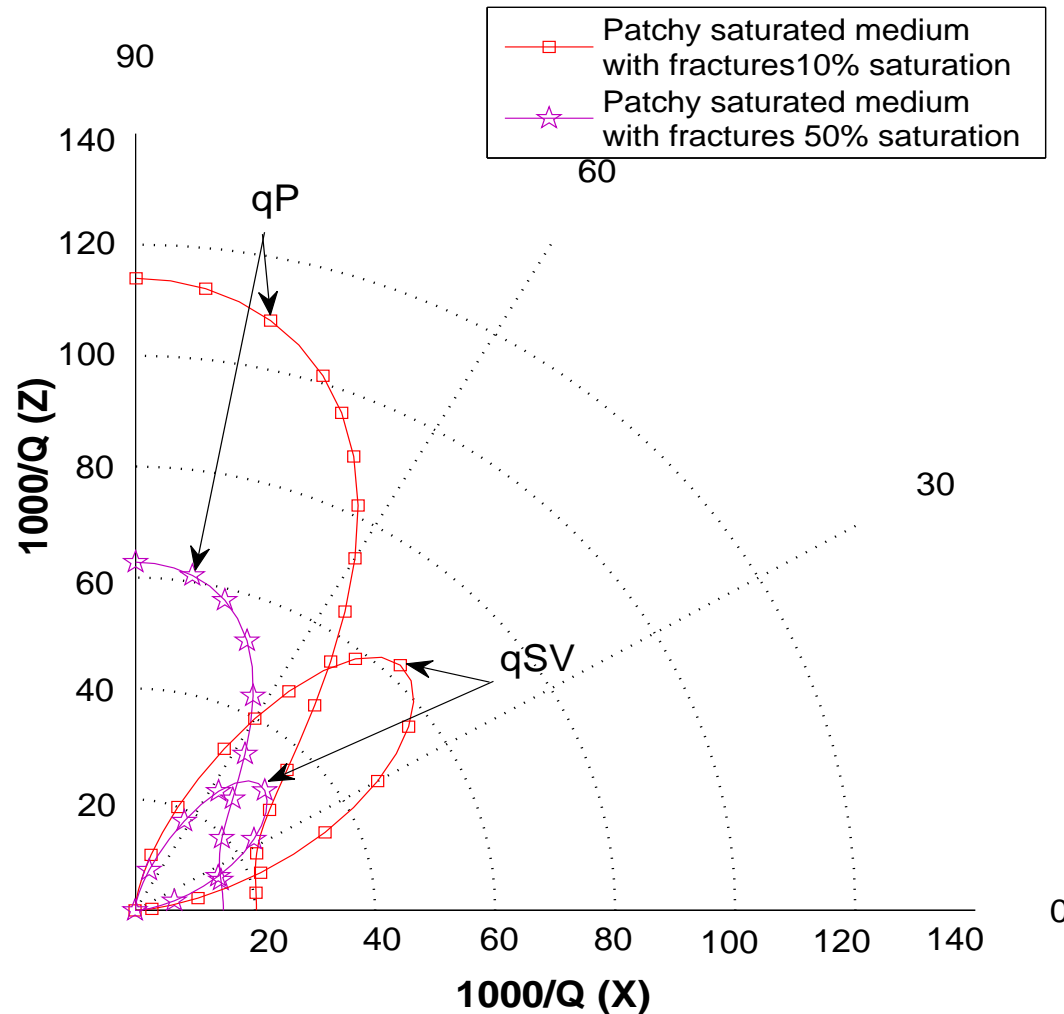
Case 2 is lossless, while for a fractured sample brine or patchy saturated (cases 1 and 3), Q anisotropy is strong for angles between 30 and 60 degrees, with about a 50 % increase in attenuation at 300 Hz with respect to 50 Hz

Energy velocity of qSV and SH waves at 50 Hz. Cases 1, 2 and 3



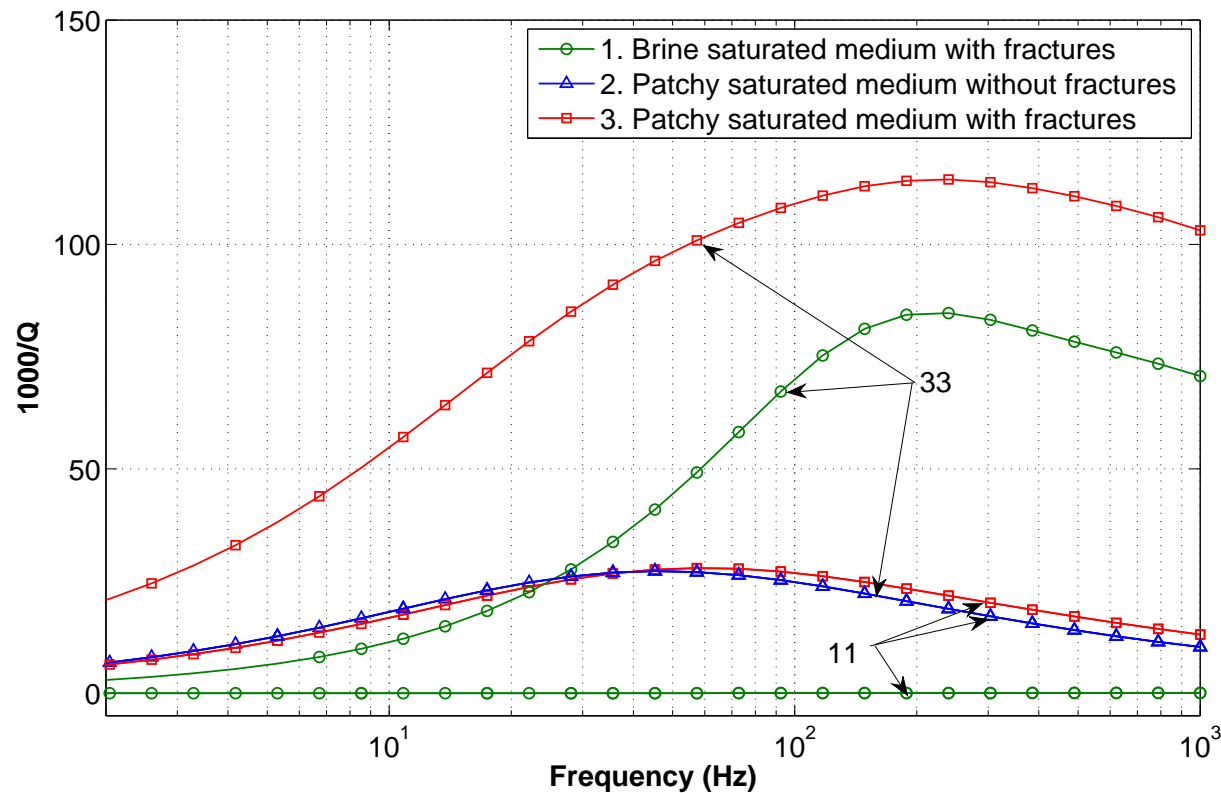
Case 2 shows isotropic velocity for both waves. Velocity anisotropy is observed to be induced by fractures (cases 1 and 3). Patchy saturation does not affect the anisotropic behavior of the velocities. At 300 Hz the behavior is almost identical for qSV and SH waves.

Dissipation factors of qP and qSV waves for case 3 at 300 Hz as function of CO₂ saturation.

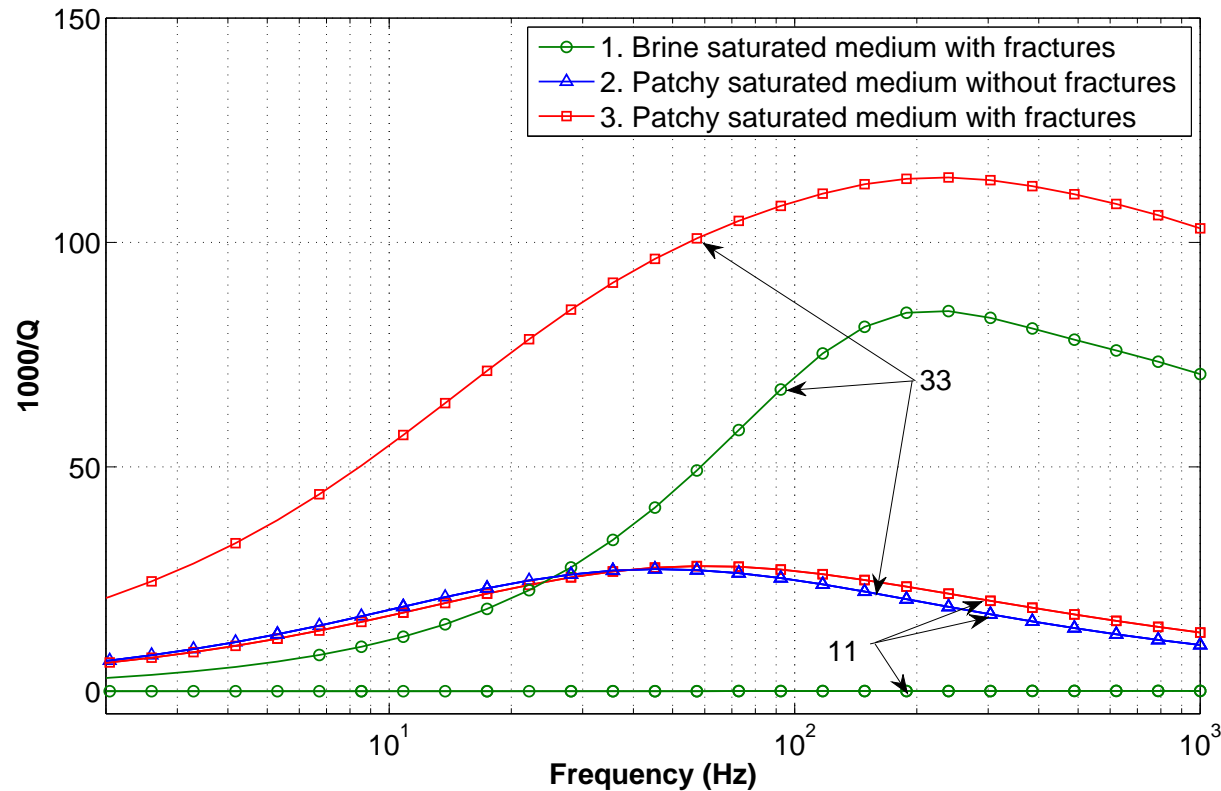


For qP waves, an increase of CO₂ saturation from 10% to 50% induces a strong decrease in attenuation for angles close to the normal orientation of the fractures. For qSV waves the same decrease in attenuation is observed, but for angles between 30 and 60 degrees.

- **qP energy velocities decrease for increasing CO₂ saturation, with the greater decreases for angles closer to the normal layering of the fractures.**
- **For qSV and SH waves, energy velocities show almost no change between the two CO₂ saturations.**

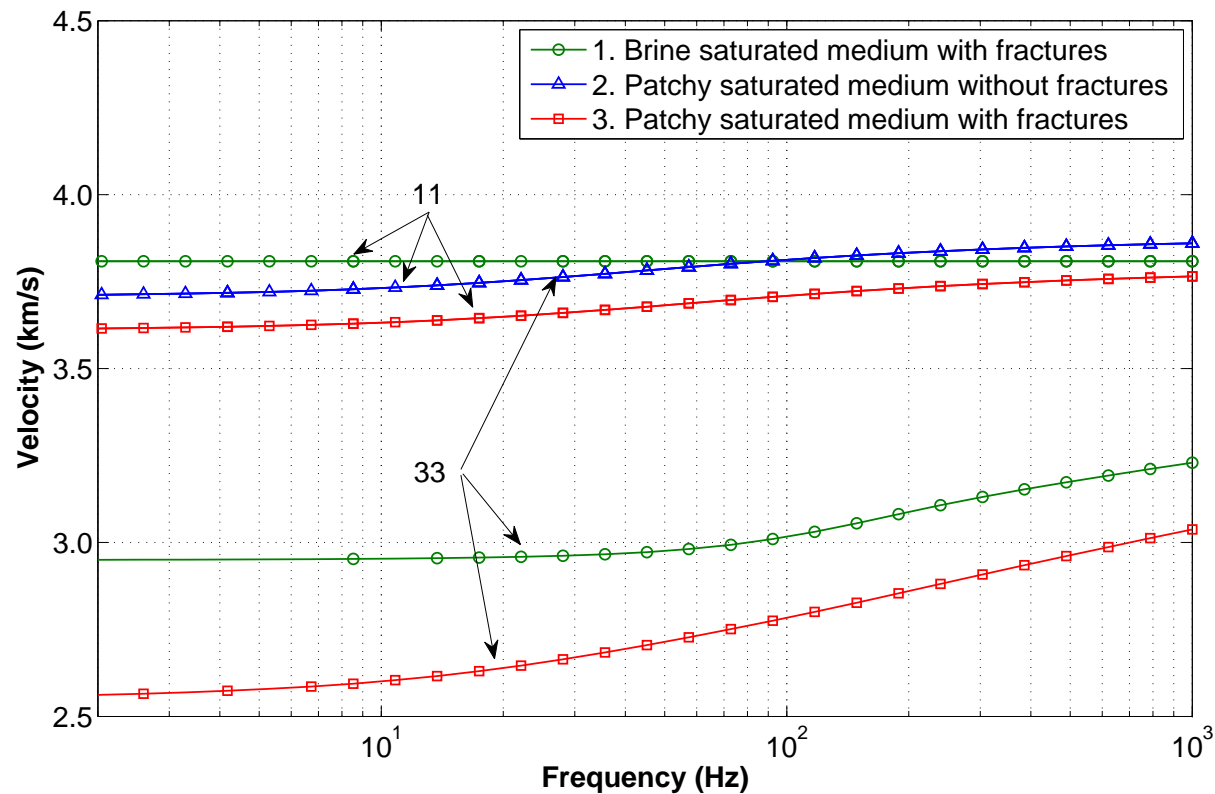


'11' waves for case 1 (brine-saturated homogeneous background with fractures) are lossless, while the cases of patchy saturation with and without fractures suffer similar attenuation.



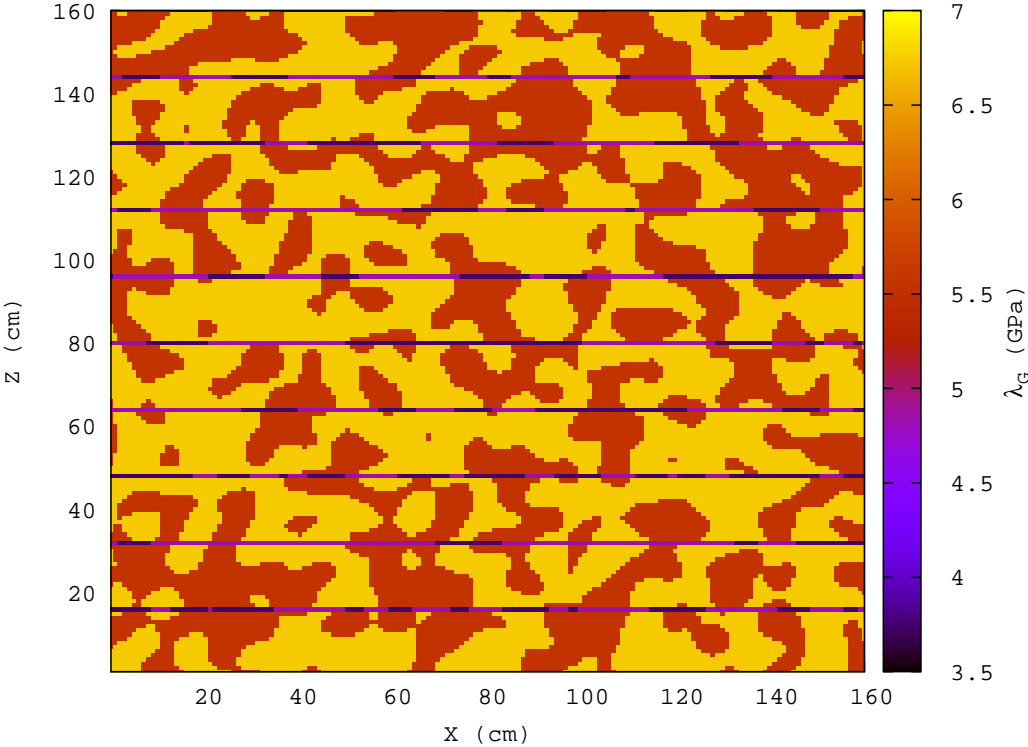
'11' waves for case 1 (brine-saturated homogeneous background with fractures) are lossless, while the cases of patchy saturation with and without fractures suffer similar attenuation. '33' show much higher attenuation than those for '11' waves for the three cases.

Velocities for cases 1, 2 and 3 for waves parallel ('11' waves) and normal ('33' waves) to the fracture layering

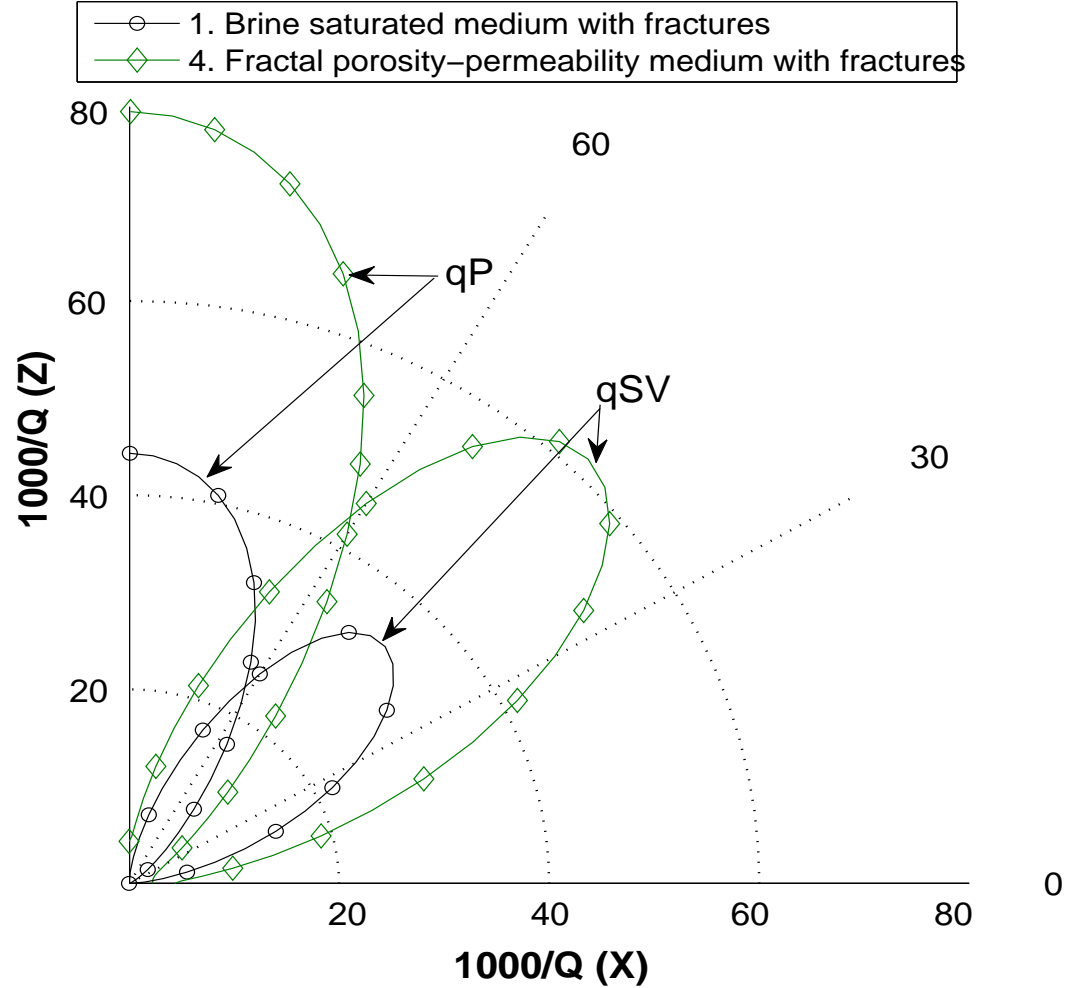


In case 1, '11' velocities are essentially independent of frequency. In the case of patchy saturation with fractures (case 3), velocities are always smaller than in case 1. For '33' waves the presence of fractures induces a noticeable reduction of velocities normal to the fracture plane, either for brine or patchy saturation.

Lamé coefficient λ_G of the brine saturated fractal sample used in case 4



Dissipation factors of qP and qSV waves at 50 Hz for cases 1 and 4



Energy velocity of qP and qSV waves at 50 Hz for cases 1 and 4

