

September 26, 2020

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We are sending the Revised Version of Book Chapter “An SEIR epidemic model of fractional order to analyze the evolution of the COVID-19 epidemic in Argentina” (Section-II-Ch-03) to be consider for publication in the book volume “Mathematical Modelling and Analysis of Infectious Disease Problems (COVID-19)”.

All suggestions given by the Reviewers have been included in this Revised Version.

The following files are being sent:

The files

santos-book-chapter-Section-II-Ch-03.tex,

santos-book-chapter-Section-II-Ch-03.pdf

have the revised version of the Chapter with the corrected parts indicated in boldface

The files

clean-santos-book-chapter-Section-II-Ch-03.tex,

clean-santos-book-chapter-Section-II-Ch-03.pdf

have the SAME revised version of the Chapter but the corrections are NOT indicated in boldface

The Covering letter including the response to the Reviewers

Sincerely,

Juan E. Santos

## Response to Review-Report-Section-II-Ch-03-1

### General Remarks and Observations

*We know that that in viscoelasticity the fractional order introduces dissipation and proper creep and relaxation behavior. What is the physical process in the case of the epidemic equations? A sentence in the Introduction will be enough.*

We added a sentence at the end of the 3rd paragraph.

*Indicate the end of the epidemic in the table.*

The end of the epidemic is defined as the day in which the number of infected individuals is smaller than one. This date makes sense only for the classical SEIR model, for the fractional order SEIR model the date has no sense due to the extremely extended period of the epidemic predicted by the simulations.

*For instance, when you report an incubation period of 3 in Table 1. This is actually the incubation period or the incubation period power  $\nu$ ? In all the tables you should give the real incubation period, e.g.  $(\epsilon^{-1})^\nu = 3^\nu$ . This 3 is correct. Do the same for all the other parameters. Then, in ALL the tables there is no need to indicate the power  $\nu$ , only in the equations.*

The Table has been fixed accordingly.

*Conclusions section. Write one, even if short.*

A new Section with conclusiones is included

### Minor points:

#### Abstract

*6th line: there are two “the”*

Fixed.

*$N_0$  is not defined and in validation has a different symbol.*

Fixed.

#### Introduction

*(page 2), First paragraph, 5th line: “still increasing at July 15th”: I assume that your are updating the data. This should change after the update.*

This paragraph has been fixed by using the updated results.

*page 2, I see that you assume 100 infectious initially. What happens if you change this number?*

We added a paragraph at the end of subsection 4.2:

*Last paragraph, last line: I think that “dead individuals” is “dead individuals per day”*

Fixed.

**Section 2. The Caputo derivative and initial value problems** (page 3)  
*first line: Replace  $D_c^\nu(u(t))$  by  $D_c^\nu(u(t))$  (remove parenthesis)*

Fixed.

*Eq. (2): Replace  $D_c^\nu(f(t))|_{t_{n+1}}$  by  $D_c^\nu(f(t))|_{t_{n+1}}$ ; these expressions have an additional parenthesis*

Fixed.

**Section 3. The classical and fractional-order SEIR models**

*(page 4) first line of Eq.(6):  $\lambda^\nu$  should be  $\mu^\nu N$  ;*

*Eq. (7):  $\mu^\mu$  should be  $\mu^\nu$ .*

Fixed.

**Subsection 4.1. Validation of the GMMP algorithm**

*(page 5) first paragraph, 2nd line: fractional orders  $\nu = 1, 0.9$  and  $0.8$  is  $\nu = 0.9$  and  $0.8$ ?*

Yes, fixed.

**Section 5. Analysis of the COVID-19 epidemic in the RMBA**

*(page 9): As mentioned above, update the analysis using data at least up to July 31st. Therefore update all the dates.*

All dates were updated, the analysis uses data until September 22th, 2020 (Section 5 is now Subsection 4.2). Besides, we added a new Case 4, considering 30 % more casualties to date, taking into account that the reported number of deceased people could have been underestimated due to delays in the upload of official data.

*Moreover, the authors should include an analysis of variations in the results associated with changes in the initial number of infected individuals;*

As indicated in the **General Remarks and Observations** a paragraph is included at the end of Subsection 4.2

*(page 10) 3rd paragraph, line 3: Complete the sentence “shows a decay of the in the simulated curves”*

Done

*(page 14) last sentence, fix “and the peak infected individuals and number of casualties increase”*

Done

**6 Appendix**

*(page 16) All lines of Eq.(13):  $b_{j,n+1}$  by  $b_{j,n+1}$*

Done.

(page 17) All lines of Eq.(14):  $f_j^\nu(S_{n+1}^p, E_{n+1}^p, I_{n+1}^p, R_{n+1}^p))$ ; there are additional parenthesis.

Done.

(page 17): Update reference [2]

Done

## Response to Review-Report-Section-II-Ch-03-2.

*A section with the conclusions is needed,*

We added this section.

*The effect of the density of population needs some discussion*

We added relevant sentences in the 1st paragraph of the Introduction.

*In page 4 is stated that "the epidemic has a relatively short period" I presume that they mean that the epidemic has a relative short time duration but it should be made clear to what is relatively short.*

We changed this paragraph. See end of 1st paragraph in Section 3.

*Top of page 6. It is necessary to quantify the agreement.*

See new sentences at the beginning of the 3rd paragraph of Section 4.1.

*In page 6 how realistic is to assume  $R < 1$ . Quote cases discussed.*

See new sentences at the end of the 3rd paragraph of Section 4.1.

*In page 6 and following ones the insert in the figures does seem consistent. Something seems missing..*

We added a missing sentence about Figure 5. See 4th paragraph of Section 4.1.

*In page 17 Please specify what means " we refer to."*

We clarified a sentence at the end of the appendix.

*Concerning Section 6. The paper adds to theoretical cases which we may not rule out as possible which are infinite. I would make this section shorter. This would add to the significance of the first part of the paper.*

Section 6 is an Appendix where the ABM method is outlined in order that the paper is self-contained. There are no theoretical cases analyzed in Section 6, so we keep this Appendix as it is essential.

*The reference [5] for the Caputo Fractional Derivative is fine as is the reference [12] but I would add Kochubei A.N., General fractional calculus evolution equations and renewal processes, Integral Equations of fractional order, theory, 2011, 71, 585-600, as well the reference to the 1967 Caputo original paper*

*Caputo, M. (1967). Linear models of dissipation whose  $Q$  is almost frequency independent, Part II, Geophys. J. R. Astr. Soc. 13, 529-539. [Reprinted in Fract. Calc. Appl. Anal. 11, 4-14 (2008)]*

These two references were included.

Finally we want to remark that all data was updated, so the analysis is done until September 22th, 2020 (Section 5 is now Subsection 4.2). Besides, we added a new Case 4, considering 30 % more casualties to date, taking into account that the reported number of deceased people could have been underestimated due to delays in the upload of official data.