An SEIR epidemic model of fractionl order to analyze the evolution of the COVID-19 epidemic in Argentina

by Santos, et al

This paper presents a case study of the evolution of the COVID-19 epidemic in Argentina, specifically in Buenos Aires, where the process is mainly active. The analysis is performed by applying an SEIR diffusion model of fractional order in time, which allows for the incorporation of hereditary properties in the system. It is very important that the authors have used two different algorithms to solve the fractional equations. This cross-check verifies the correctness of the results. In general, I see that most of the papers dealing with epidemics use only one algorithm and some of the results in these papers seem suspicious. For instance, the fact that the key isolation parameter β has to be modified by initializing each time the individuals is not evident and has been shown by Santos et al, while this procedure is not mentioned in other works. Another very important point is the calibration with the dead individuals instead of the infectious, because this number can be highly variable and wrong. Instead the deaths are more constrained. I have not seen calibrations with the number of casualties in other papers. This is the first time and can make the results very reliable.

Therefore, in my opinion the material is suitable to be included in the book "Mathematical Modelling and Analysis of Infectious Disease Problems (COVID-19)", since it presents a highly relevant case history, taking into account also that the pandemic is in the process of full evolution in South America and never more than now models have to be tested to predict possible outbreaks. In any case, the paper can still be improved.

Thus, this material can be published after the following revisions. First of all update the data till present. This is important because the update can modify the conclusions. The abstract lacks values of the results, as for instance the R_0 and the fatality rate IFR, even if these values are hypothetical, readers would like to have and idea of the most possible figures. I see that there is a table with 3 cases. We know that that in viscoelasticity the fractional order introduces dissipation and proper creep and relaxation behavior. What is the physical process in the case of the epidemic equations? A sentence in the Introduction will be enough. I see that you assume 100 infectious initially. What happens if you change this number? Indicate the end of the epidemic in the table. This date is important. I have a problem with the values of the parameters: you report values elevated to the power of the fractional order. This is not physical For instance, when you report an incubation period of 3 in Table 1. This is actually the incubation period or the incubation period power ν?. In all the tables you should give the real incubation period, e.g. $(\epsilon^{-1})^\nu = 3^\nu$. This 3 is correct. Do the same for all the other parameters. Then, in ALL the tables there is no need to indicate the power ν, only in the equations. I do not see a Conclusions section. Write one, even if short.

Minor points:

- N_0 is not defined and in validation has a different symbol.
- **Abstract**, 6th line: there are two "the"
- **Introduction** (page 2), First paragraph, 5th line: "still increasing at July 15th": I assume that your are updating the data. This should change after the update. Last paragraph, last line: I think that "dead individuals" is "dead individuals per day"
- **Section 2 The Caputo derivative and initial value problems** (page 3) first line: "$D_c^\nu(u(t)$"; Eq. (1): "$D_c^\nu(f(t)$ "; Eq. (2): Replace "$D_c^\nu(f(t)|_{t_{n+1}}$ "; these expressions have an additional parenthesis
- **Section 3 The classical and fractional-order SEIR models** (page 4) first line of Eq.(6): "Λ^ν" should be "$\mu^\nu N$"; Eq.(7): "μ^μ" should be "μ^ν".
- **Subsection 4.1 Validation of the GMMP algorithm** (page 5) first

paragraph, 2nd line: "fractional orders $\nu = 1, 0.9$ and 0.8 is $\nu = 0.9$ and 0.8?

- **Section 5 Analysis of the COVID-19 epidemic in the RMBA** (page 9): As mentioned above, update the analysis using data at least up to July 31st. Therefore update all the dates. Moreover, the authors should include an analysis of variations in the results associated with changes in the initial number of infected individuals; (page 10) 3rd paragraph, line 3: Complete the sentence "shows a decay of the in the simulated curves"; (page 14) last sentence, fix "and the peak infected individuals and number of casualties increase"

- **6 Appendix** (page 16) All lines of Eq.(13): "$b_{j,n+1)}$" by "$b_{j,n+1}$"; (page 17) All lines of Eq.(14): "$f_j^\nu(S_{n+1}^p, E_{n+1}^p, I_{n+1}^p, R_{n+1}^p))$"; there are additional parenthesis.

  (page 17): Update reference [2]