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APPLICATION OF FRACTIONAL CALCULUS FOR MODELLING OF TWO-PHASE GAS/LIQUID FLOW SYSTEM

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Abstract. *In recent years the use of fractional calculus in control system identification is becoming popular and it has found new applications. The paper presents application of fractional calculus for modelling of two-phase gas/liquid flows in a test rig. The installation consists of three horizontal and vertical measuring segments with different diameters, which allow to investigate flows in a wide range of parameters. Flow components supply is measured/controlled by NI PXI system and a set of flow meters/controllers. The paper presents model of the two-phase flow in the above described installation, which leads to precise and accurate flow mathematical model. The main goal of the flow model is to describe steady flow parameters, especially the flow fractions, or type of the flow. The model describes flows more accurately, that classical second order system model.*

Keywords: tomography, electrical capacitance tomography, fractional calculus

ZASTOSOWANIE RACHUNKU RÓŻNICZKOWEGO NIECAŁKOWITEGO RZĘDU DO MODELOWANIA PRZEPŁYWÓW DWUFAZOWYCH GAZ/CIECZ

Streszczenie*. W ostatnich latach wykorzystanie rachunku różniczkowego niecałkowitego rzędu staje się coraz bardziej popularne i znajduje nowe obszary zastosowań. W pracy przedstawiono zastosowanie powyższego rachunku różniczkowego do modelowania przepływów dwufazowych gaz / ciecz. Instalacja badawcza składa się z trzech poziomych i pionowych odcinków pomiarowych o różnych średnicach, które umożliwiają badanie przepływu w szerokim zakresie parametrów. Przepływ komponentów mieszaniny jest mierzony / sterowany przez system NI PXI oraz zestaw przepływomierzy i sterowników. W artykule przedstawiono modelowanie przepływu dwufazowego w wyżej opisanej instalacji, które prowadzi do określenia precyzyjnego modelu matematycznego przepływu. Opracowany model opisuje przepływy dwufazowe dokładniej w porównaniu z klasycznym model opisanym równaniami różniczkowymi drugiego rzędu.*

Słowa kluczowe: tomografia, elektryczna tomografia pojemnościowa, rachunek różniczkowy niecałkowitego rzędu

Introduction

Two-phase gas/liquid flows are important and commonly found in the chemical, food, pharmaceutical and other industries. Examples include installations for hydraulic and pneumatic transport of gas and liquid, apparatus for separating gas from liquid, heat-exchange units, bio-reactors, cooling installations. In nuclear power stations cooling systems, the understanding of two-phase flows as well as monitoring and identification of the flow regime is essential [11]. Another emerging environment of two-phase flows is high computing power electronics, where power dissipation and heat is becoming a bottleneck. Two-phase cooling systems for electronics are exposed for strong flow instabilities, density-wave flow oscillations, which require efficient control systems [12]. Design of hardware configuration and an appropriate mode of operation is a complex task, because the induced fluid motion causes instability of the jet, which may in turn result in pressure and flow rate drop. Therefore, determining properties of the flow, share of the gas phase, parameters of gas bubbles like shape, dimensions, allows making an appropriate tuning control to the process. Application of Electrical Capacitance Tomography (ECT) system acts as a non-invasive method of obtaining information, which allows using it as a tool for imaging of different industrial processes. In the case of gas/liquid flow it is possible to reach higher spatial resolution and shorter time to collect the measurements (more than 100 measurements per second) than conventional measuring methods. An ECT measuring system does not require placing sensors inside the process. This allows to get information from previously inaccessible locations such as a cross-section of a pipeline, interior of a tank or reactor. By using correlation techniques and twin-plane ECT sensors, the velocity and character of the flow can be determined and the flow can be calculated [10].

A semi-industrial test rig for two-phase flows, was built to demonstrate the ECT technique [1]. The process fluid supply system is equipped with Kobold liquid mass flow meters. Installation of these meters allows measurement of different liquid flows, regardless their viscosities and electrical properties, like polypropylene glycol. For measurement of gas flow, two precise mass flow meters/controllers manufactured by Brooks are used. Elements of flow rig control system are shown in Figure 1.

For over 40 years fractional calculus has been a subject of growing interest [5], with successful applications in many scientific and technical fields. Mathematical modelling of real-time physical processes by linear or non-linear, time variant or time invariant fractional-order differential equations can be mentioned. The interest ranges from physics and chemistry to technical like mechanical and electrical to economical and biological processes, analysis and control strategies synthesis. Fractional calculus is becoming popular in many areas, especially in system identification, image processing, automation and control systems [9]. Review of available literature show a lack of publications concerning applications of fractional calculus in process tomography. This article presents preliminary results of research into the use of fractional calculus on two-phase flow modelling.

a) b)

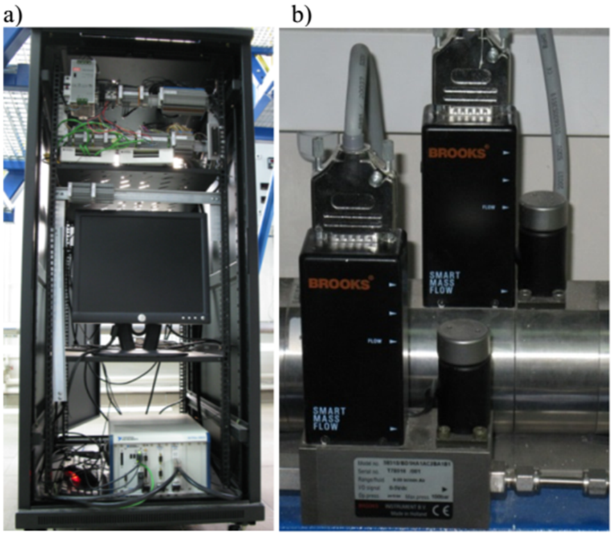

Fig. 1. a) overview of control system, b) Brooks flow meters/controllers

1. Flow Measurement System

Experiment of two-phase flow measurement were performed using the installation as mentioned before [1]. The flow rig is fully controlled by a NI PXI system, which in connection with other data acquisition systems (including ECT) and data processing units allows the measurement of flow parameters like pressures, components supply, ECT measurements with concurrent recordings of the flow image in the transparent segment of the installation located next to an ECT sensor and storing data for off-line post-processing.

Data collected are then processed using a software package "TomoKIS Studio" [3, 4]. The program allows 3D reconstruction of the flow pattern and calculation of the liquid fraction of the flow. Depending on the computer hardware used, it is possible to perform on-line 3D reconstruction with the performance of 10–12 frames per second (FPS) using nVidia Tesla graphics cards and applying CUDA technology implemented in the TomoKIS Studio. Part of the experiments were performed using a computer with nVidia Tesla cards, part using a standard PC and data post processing at approximately 2 FPS. As an example, a TomoKIS Studio screenshot in Figure 2 shows flow pattern reconstruction and liquid fraction computation.

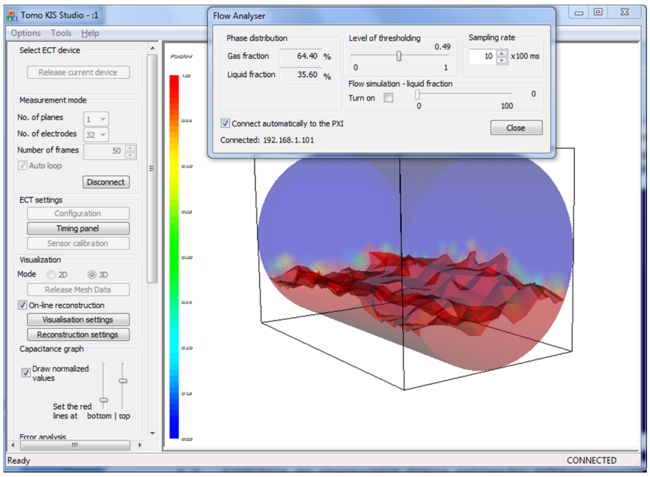

Fig. 2. TomoKIS Studio screenshot of flow image reconstruction

From the reconstructed tomographic images, different flow parameters like bubbles shape and dimensions, share of the various phases in the flow are calculated. The calculated values are used in a control system running LabVIEW by a specially developed dll plug-in and PXI control system. The aim of this research is to identify the flow parameters to build an appropriate model of the flow control system.

Following successful research on fractional calculus in mobile robotics and image processing, authors decided to apply this approach for flow identification and measurements using tomography techniques.

2. Flow Model Based on Fractional Calculus

2.1. Grünwald-Letnikov Form of the Fractional-Order Backward Difference

A definition of the Grünwald – Letnikov fractional-order difference (FOBD) is given below.

Definition: The Grünwald – Letnikov backward difference/sum of a fractional-order $\nu \in \mathbb{R}_+$ is defined as a sum

$$ {}_{0}^{GL}\Delta_k^{(\nu)} f_k = \sum_{i=0}^{k} a_i^{(\nu)} f_{k-i}, \tag{1}$$

where coefficients $a_i^{(\nu)}$ are defined as follows

$$a_i^{(\nu)} = \begin{cases} 1 & \text{for} \quad i=0 \\ (-1)^i \frac{\nu(\nu-1)\dots(\nu-i+1)}{i!} & \text{for} \quad i=1,2,\dots \end{cases} \tag{2}$$

and $\nu \in \mathbb{R}_+$ is an order. Applying to both sides the one-sided $\mathcal{Z}$-Transform one immediately gets

$$\mathcal{Z}\left\{{}_{0}^{GL}\Delta_k^{(\nu)} f(k)\right\} = \mathcal{Z}\left\{\sum_{i=0}^{k} a_i^{(\nu)} f_{k-i}\right\} = (1-z^{-1})^{\nu}. \tag{3}$$

A fractional-order linear time-invariant differential equation is

$$\sum_{i=0}^{p} \bar{A}_i \, {}_{0}^{GL}\Delta_k^{(\nu_i)} y(k) = \sum_{i=0}^{q} \bar{B}_i \, {}_{0}^{GL}\Delta_k^{(\mu_i)} u(k), \tag{4}$$

where:

$\bar{A}_p = 1, \quad \bar{A}_i, \bar{B}_i \in \mathbb{R}$

$0 = \nu_0 > \nu_1 < \dots < \nu_p$

$0 = \mu_0 > \mu_1 < \dots < \mu_q$

$\mu_q < \nu_p$

An application of the one-sided Z-Transform to both sides of equation (4) yields

$$\sum_{i=0}^{p} \bar{A}_i (1-z^{-1})^{\nu_i} Y(z) = \sum_{i=0}^{q} \bar{B}_i (1-z^{-1})^{\mu_i} U(z), \tag{5}$$

with $Z\{y(k)\} = Y(z)$, $Z\{u(k)\} = U(z)$. From equation (5) one gets discrete transfer function [2,6,8]

$$G(z) = \frac{Y(z)}{U(z)} = \frac{\sum_{i=0}^{q} \bar{B}_i (1-z^{-1})^{\mu_i}}{\sum_{i=0}^{p} \bar{A}_i (1-z^{-1})^{\nu_i}}. \tag{6}$$

Now on orders $\mu_i, \nu_j \in \mathbb{R}_+$ one imposes a condition that they are the rational numbers. Hence the orders can be expressed as fractions

$$\nu_i = \frac{e_i}{d_i} \quad \text{for} \quad i = 1,2,\cdots,p, \tag{7}$$
$$\mu_i = \frac{g_i}{f_i} \quad \text{for} \quad i = 1,2,\cdots,q,$$

where $e_i, d_i, g_i, f_i \in \mathbb{Z}_+$. Next one assumes that $d_m \in \mathbb{Z}_+$ is the least common denominator of fractions (7). Then

$$\nu_i = \frac{n_i}{d_m} \quad \text{for} \quad i = 1,2,\cdots,p, \tag{8}$$

$$\mu_i = \frac{m_i}{d_m} \quad \text{for} \quad i = 1,2,\cdots,q. \tag{9}$$

Denoting

$$\nu = \frac{1}{d_m} \tag{10}$$

such that

$$0 < \nu = \frac{1}{d_m} < 1 \tag{11}$$

one express orders

$$\nu_i = \nu n_i \quad \text{for} \quad i = 1,2,\cdots,p \tag{12}$$

$$\mu_i = \nu m_i \quad \text{for} \quad i = 1,2,\cdots,q \tag{13}$$

By inequalities in formula (4)

$$0 = n_0 < n_1 < \dots < n_p \tag{14}$$

$$0 = m_0 < m_1 < \dots < m_q \tag{15}$$

$$m_q < n_p \tag{16}$$

In the light of equations (12) and (13) formula (5) takes the form

$$\sum_{i=0}^{p} \bar{A}_i (1-z^{-1})^{\nu n_i} Y(z) = \\ = \sum_{i=0}^{q} \bar{B}_i (1-z^{-1})^{\nu m_i} U(z) \tag{17}$$

3. Modelling of the Flow System

Following the presented theoretical assumptions the following models of the system were assumed.

3.1. Classical first-order model

A classical first-order difference equation is

$$\frac{{}_{0}^{GL}\Delta_k^{(1)} y(kh)}{h} + a_0 y(kh) = b_0 u(kh) \tag{18}$$

$${}_{0}^{GL}\Delta_k^{(1)} y(kh) + h a_0 y(kh) = h b_0 u(kh) \tag{19}$$

$${}_{0}^{GL}\Delta_k^{(1)} y(kh) + A_0 y(kh) = B_0 u(kh) \tag{20}$$

with a classical first-order model discrete transfer function:

$$G_1(z) = \frac{B_0}{(1-z^{-1})^1 + A_0} = \frac{B_0}{(1-z^{-1}) + A_0} = \frac{B_0}{-z^{-1} + A_0 + 1} = \\ = \frac{B_0 z}{(A_0+1)z - 1} = \frac{\frac{B_0}{A_0+1} z}{z - \frac{1}{A_0+1}} \tag{21}$$

Coefficients a_0, b_0 or $a_0 = b_0$ have to be identified when changing the level of $u(kh)$. h – denotes simulation step.

Fractional-order model

$$\frac{{}_{0}^{GL}\Delta_k^{(\nu)} y(kh)}{h^\nu} + a_0 y(kh) = b_0 u(kh) \tag{22}$$

$${}_{0}^{GL}\Delta_k^{(\nu)} y(kh) + h^\nu a_0 y(kh) = h^\nu b_0 u(kh) \tag{23}$$

$${}_{0}^{GL}\Delta_k^{(\nu)} y(kh) + A_0 y(kh) = B_0 u(kh) \text{ with} \\ A_0 = h^\nu a_0, B_0 = h^\nu b_0 \tag{24}$$

Coefficients and fractional order a_0, b_0, ν or only a_0, ν when $a_0 = b_0$ have to be identified and one can change the level of $u(kh)$.

A fractional-order model discrete transfer function is

$$G_\nu(z) = \frac{B_0}{(1-z^{-1})^\nu + A_0} \tag{25}$$

The fractional-model solution is

$$y(kh) = \\ -\frac{1}{1+A_0} \begin{bmatrix} a_1^{(\nu)} & a_2^{(\nu)} & \cdots & a_{k-1}^{(\nu)} \end{bmatrix} \begin{bmatrix} y(kh-h) \\ y(kh-2h) \\ \vdots \\ y(0h) \end{bmatrix} + \\ \frac{B_0}{1+A_0} u(kh) \tag{26}$$

where

$$a_i^{(\nu)} = \begin{cases} 1 & \text{for} \quad i = 0 \\ a_{i-1}^{(\nu)} \left(1 - \frac{\nu+1}{i}\right) & \text{for} \quad i = 1,2,\cdots \end{cases} \tag{27}$$

Integer first-order model solution. One should only put $\nu = 1$ and use the above equations

A fractional-order oscillation element is

$$G_{2\mu}(z) = \frac{D_0}{(1-z^{-1})^{2\mu} + C_1(1-z^{-1})^{\mu} + C_0} \tag{28}$$

for $0 < \mu \le 1$

The linear model and non-linear model (FO-inertia & oscillation) are shown in Figure 3.

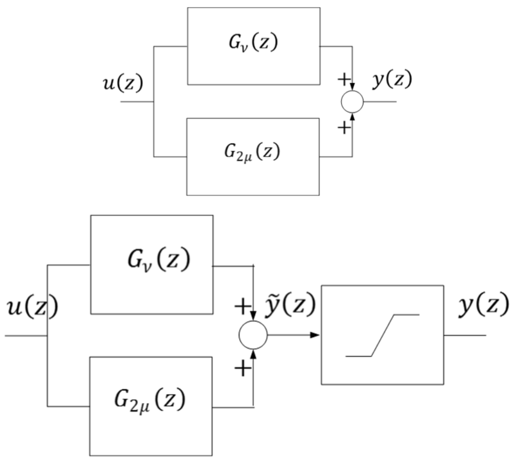

Fig. 3. The linear model and non-linear model (FO-inertia & oscillation)

Elements to evaluation: $A_0, B_0, \nu, C_0, D_0, \mu$, S_l - saturation level, s_l – range of linearity.

4. Flow Model Identification

The above theoretical solution was implemented in MatLab to determine the parameters of the fractional derivative model, which fits the best to the measured flow phenomena. Figure 4 shows results of liquid fraction calculations based on ECT measurements of an air/propylene glycol two-phase flow. The horizontal axis presents measurement frame number. The average acquisition speed was 11 FPS. During experiments the supply parameters of flow components were set to obtain flow, which was recognised as regularly repeating long bubbles of air in the liquid. An example of such a case is shown in Figure 5.

To identify model parameters, part of the flow pattern was selected, presented in Figure 6. For this set of measurement data, calculations performed in MatLab showed that the differential equation with fractional order of 0,82 – shown with small rectangles on Figure 7 RMS matches best to the originally measured data.

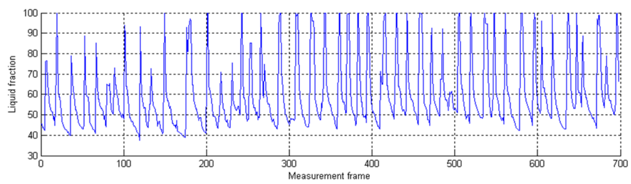

Fig. 4. Liquid fraction calculation from ECT measurements

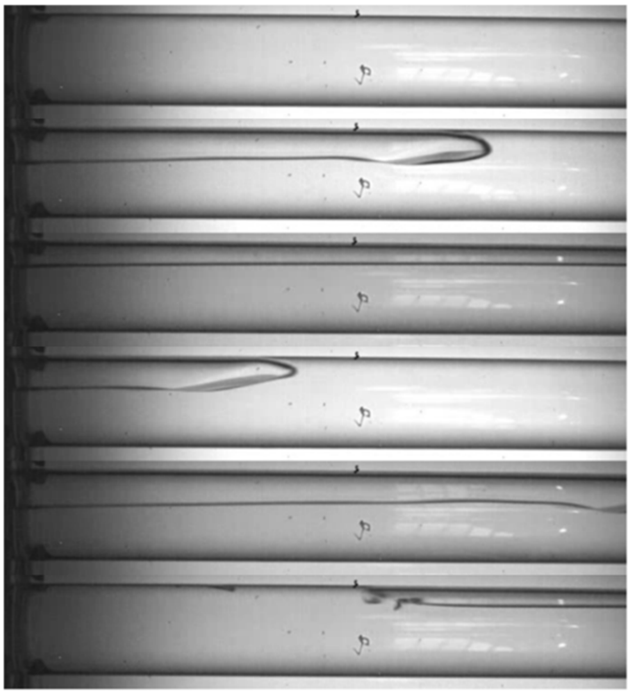

Fig. 5. Sequence of images of the measured flow

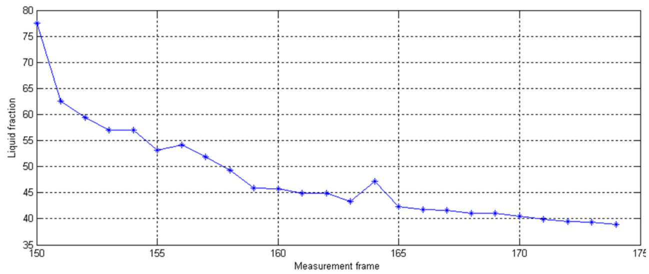

Fig. 6. Part of selected diagram from Figure 4

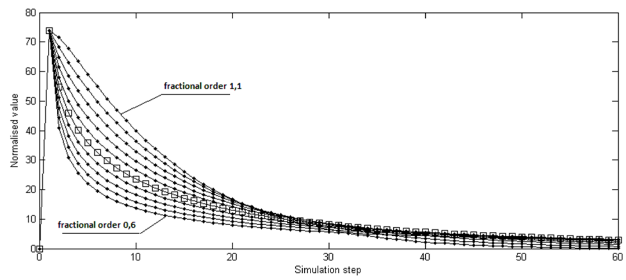

Fig. 7. Matlab simulation of the model for different fractional order values

5. Conclusions

More complex models based on fractional calculus allowed accurate system identification and allowed developing efficient control and data processing. The above presented case shows that application of fractional calculus allowed accurate system identification and preparing efficient control. The above presented example is part of research on flow identification and control, which covers among others flow regime identification using AI tools based on different measurement data acquired by different ECT systems and imaging cameras. The results confirmed the applicability of fractional calculus in two-phase flow modelling.

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