

BOUNDS FOR $\ddot{\theta}_m(0)$, $\|\ddot{u}_m(0)\|_V$ (1)

Regularity:

$$\|\ddot{u}^s\|_2 + \|\ddot{u}^f\|_{H^1(\text{div}, \Omega)} + \|\ddot{\theta}\|_2$$

$$\leq C [\|\ddot{f}\|_0 + \|\ddot{g}\|_{1/2, \Gamma} + \|\ddot{\chi}\|_{1/2, \Gamma} + \|\ddot{h}\|_{1/2, \Gamma}] \quad (A)$$

At $t=0$

$$\|\ddot{u}^s(0)\|_2 + \|\ddot{u}^f(0)\|_{H^1(\text{div}, \Omega)} + \|\ddot{\theta}(0)\|_2$$

$$\leq C [\|\ddot{f}(0)\|_0 + \|\ddot{g}(0)\|_{1/2, \Gamma} + \|\ddot{\chi}(0)\|_{1/2, \Gamma} + \|\ddot{h}(0)\|_{1/2, \Gamma}] \quad (B)$$

$$= C H(0)$$

Then,

(2)

$$\|\ddot{u}^s(0)\|_L + \|\ddot{u}^f(0)\|_{H(\text{div}, \Omega)} \leq C H(0)$$

or

$$\|\ddot{u}(0)\|_V \leq C H(0)$$

$$\|\ddot{u}_m(0)\|_V = \|\ddot{u}_m(0) - \ddot{u}(0)\|_V + \|\ddot{u}(0)\|_V$$

Assume that in (31)

$$\ddot{u}_m(0) \xrightarrow{m \rightarrow \infty} \ddot{u}' \text{ in } H'(\Omega)$$

Then

$$\|\ddot{u}_m(0)\|_V \leq \varepsilon + C H(0), \quad m \geq m_0(\varepsilon).$$

Also,

$$\begin{aligned} \|\ddot{\theta}_m(0)\|_2 &\leq \|\ddot{\theta}_m(0) - \ddot{\theta}'\|_2 + \|\ddot{\theta}'\|_2 \\ &\leq \|\ddot{\theta}_m(0) - \ddot{\theta}'\|_2 + \|\ddot{\theta}'\|_2 \\ &\leq \varepsilon + \|\ddot{\theta}'\|_2 \end{aligned}$$

$$\|\ddot{\Theta}_m(0)\|_2 \leq \|\ddot{\Theta}_m(0) - \ddot{\Theta}(0)\|_2 + \|\ddot{\Theta}(0)\|_2 \quad (3)$$

$$\leq \varepsilon + \|\ddot{\Theta}(0)\|_2$$

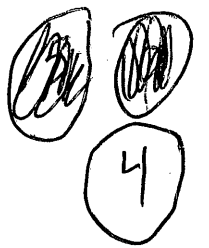
$$\leq \varepsilon + C H(0), \quad m \geq m_0(\varepsilon)$$

assuming

$$\ddot{\Theta}_m(0) \rightarrow \ddot{\Theta}(0) \quad \text{in } H^2(\Omega).$$

BOUND FOR $\|\ddot{u}_m(0)\|_0$

$$\left(\rho \ddot{u}_m(0), \ddot{u}_m(0) \right) + \left(\frac{\gamma}{\kappa} \dot{u}_m^f(0), \ddot{u}_m^f(0) \right)$$



$$- \left(\alpha (u_m^{(0)}, \theta_m^{(0)}), \ddot{u}_m(0) \right) = \left(f(0), \ddot{u}_m(0) \right)$$

Then

$$\begin{aligned} \|\ddot{u}_m(0)\|_0^2 \leq & C \left[\|\dot{u}_m(0)\|_0 \|\ddot{u}_m(0)\|_0 \right. \\ & + \left(\|u_m(0)\|_2 + \|\theta_m(0)\|_1 \right) \|\ddot{u}_m(0)\|_0 \\ & \left. + \|f(0)\|_0 \|\ddot{u}_m(0)\|_0 \right] \end{aligned}$$

Then,

$$\begin{aligned} \|\ddot{u}_m(0)\|_0 \leq & C \left(\|\dot{u}_m(0)\|_0 + \|u_m(0)\|_2 \right. \\ & \left. + \|\theta_m(0)\|_1 + \|f(0)\|_0 \right) \end{aligned}$$