BOUNDS FOR $\ddot{\theta}_m(0)$, $\|\ddot{u}_m(0)\|_V$

Regularity:

$$\|\ddot{u}^s\|_2 + \|\ddot{u}^f\|_{H^1(\mathrm{div},\Omega)} + \|\ddot{\theta}\|_2$$

$$\le C\left[\|\ddot{f}\|_0 + \|\ddot{g}\|_{1/2,\Gamma} + \|\ddot{\lambda}\|_{1/2,\Gamma} + \|\ddot{h}\|_{1/2,\Gamma}\right] \qquad (A)$$

At $t=0$

$$\|\ddot{u}^s(0)\|_2 + \|\ddot{u}^f(0)\|_{H^1(\mathrm{div},\Omega)} + \|\ddot{\theta}(0)\|_2$$

$$\le C\left[\|\ddot{f}(0)\|_0 + \|\ddot{g}(0)\|_{1/2,\Gamma} + \|\ddot{\lambda}(0)\|_{1/2,\Gamma} + \|\ddot{h}(0)\|_{1/2,\Gamma}\right] \qquad (B)$$

$$= C\, H(0)$$

Then,

$$\|\ddot{u}^s(0)\|_1 + \|\ddot{u}^f(0)\|_{H(\mathrm{div},\Omega)} \leq C\,H(0)$$

or

$$\|\ddot{u}(0)\|_V \leq C\,H(0)$$

$$\|\ddot{u}_m(0)\|_V = \|\ddot{u}_m(0) - \ddot{u}(0)\|_V + \|\ddot{u}(0)\|_V$$

Assume that in (31)

$$\ddot{u}_m(0) \xrightarrow[m\to\infty]{} \dot{u}^1 \text{ in } H^1(\Omega)$$

Then

$$\|\ddot{u}_m(0)\|_V \leq \varepsilon + C\,H(0), \quad m \geq m_0(\varepsilon).$$

Also,

$$\cancel{\|\ddot{\theta}_m(0)\|_2 \leq \|\ddot{\theta}_m(0) - \dot{\theta}^1 + \|\dot{\theta}^1\|_2}$$

$$\cancel{\leq \|\ddot{\theta}_m(0) - \dot{\theta}^1\|_2 + \|\dot{\theta}^1\|_2}$$

$$\cancel{\leq \varepsilon + \|\dot{\theta}^1\|_2}$$

$$\|\ddot{\theta}_m(0)\|_2 \leq \|\ddot{\theta}_m(0) - \ddot{\theta}(0)\|_2 + \|\ddot{\theta}(0)\|_2$$

$$\leq \varepsilon + \|\ddot{\theta}(0)\|_2$$

$$\leq \varepsilon + C\,H(0), \qquad m \geq m_0(\varepsilon)$$

assuming

$$\ddot{\theta}_m(0) \longrightarrow \ddot{\theta}(0) \quad \text{in} \quad H^2(\Omega).$$

BOUND FOR $\|\ddot{u}_m(0)\|_0$

$$\left(\rho\, \ddot{u}_m(0), \ddot{u}_m(0)\right) + \left(\frac{\eta}{\kappa}\dot{u}^f_{m}(0), \ddot{u}^f_m(0)\right)$$

$$- \left(\mathcal{L}(u_m^{(0)}, \theta_m^{(0)}), \ddot{u}_m(0)\right) = \left(f(0), \ddot{u}_m(0)\right)$$

Then

$$\|\ddot{u}_m(0)\|_0^2 \le C\Big[\|\dot{u}_m(0)\|_0\|\ddot{u}_m(0)\|_0$$

$$+\left(\|u_m(0)\|_2 + \|\theta_m(0)\|_1\right)\|\ddot{u}_m(0)\|_0$$

$$+\|f(0)\|_0\|\ddot{u}_m(0)\|_0\Big]$$

Then,

$$\|\ddot{u}_m(0)\|_0 \le C\Big(\|\dot{u}_m(0)\| + \|u_m(0)\|_2$$

$$+\|\theta_m(0)\|_1 + \|f(0)\|_0\Big)$$