THERMOPOROELASTICITY

PART IV

J. SANTOS

12/8/19

Time derivative in (16)

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$$(\rho \dddot{u}_m, v) + \left(\frac{\eta}{K}\ddot{u}^f, v^f\right) + \Lambda(\dot{u}_m, v)$$

$$- (\beta \dot{\theta}_m, \nabla\cdot v^s) - (\beta_f \dot{\theta}_m, \nabla\cdot v^f) \qquad (120)$$

$$+ (\tau \dddot{\theta}_m, w) + (C\ddot{\theta}_m, w) + (\gamma \nabla\dot{\theta}_m, \nabla w)$$

$$+ \left(\beta T_0 (\nabla\cdot \ddot{u}_m^s + \nabla\cdot\ddot{u}_m^f), w\right) = (\dot{f}, v) - (\dot{q}, w) + \langle \dot{g}, v\rangle + \langle \dot{x}, \dot{u}^f\cdot\nu\rangle + \langle \dot{h}, w\rangle$$

Take $v = \ddot{u}_m$, $w = \ddot{\theta}_m$

$$(\rho \dddot{u}_m, \ddot{u}_m) + \left(\frac{\eta}{K}\dddot{u}^f, \ddot{u}^f\right)$$

$$+ \Lambda(\dot{u}_m, \ddot{u}_m) - (\beta\dot{\theta}_m, \nabla\cdot\ddot{u}^s) \qquad (121)$$

$$- (\beta_f \dot{\theta}_m, \nabla\cdot\ddot{u}^f)$$

$$+ (\tau\dddot{\theta}_m, \ddot{\theta}_m) + (C\ddot{\theta}_m, \ddot{\theta}_m) + (\gamma\nabla\dot{\theta}_m, \nabla\ddot{\theta}_m)$$

$$+ \left(\beta T_0(\nabla\cdot\ddot{u}_m^s + \nabla\cdot\ddot{u}_m^f), \ddot{\theta}_m\right)$$

$$= (\dot{f}, \ddot{u}^s) + \langle \dot{g}, \ddot{u}^s\rangle + \langle \dot{x}, \ddot{u}^f\cdot\nu\rangle + \langle \dot{h}, \ddot{\theta}_m\rangle$$

$$- (\dot{q}, \ddot{\theta}_m)$$

or

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$$\frac{1}{2}\frac{d}{dt}\Big[(\rho \ddot{u}_m, \ddot{u}_m) + \Lambda(\dot{u}_m, \dot{u}_m) + (\tau \ddot{\theta}_m, \ddot{\theta}_m) + (\gamma \nabla\dot{\theta}_m, \nabla\dot{\theta}_m)\Big]$$

$$+ \left(\frac{\eta}{k}\dddot{u}^f_m, \dddot{u}^f_m\right) - (\beta \dot{\theta}_m, \nabla\cdot \dddot{u}^s_m) - (\beta_f \dot{\theta}_m, \nabla\cdot \dddot{u}^f_m)$$

$$+ (c\ddot{\theta}_m, \ddot{\theta}_m) + (\beta T_0 \nabla\cdot \dddot{u}^s_m, \ddot{\theta}_m) \qquad (122)$$

$$+ (\beta T_0 \nabla\cdot \dddot{u}^s_m, \ddot{\theta}_m) = (\ddot{f}^s, \dddot{u}^s_m) + \langle \ddot{g}, \dddot{u}^s_m\rangle$$

$$+ \langle \dot{\chi}, \dddot{u}^f_m \cdot \nu\rangle + \langle \dot{h}, \ddot{\theta}_m\rangle - (\dot{q}, \ddot{\theta}_m)$$

~~CHOOSE $v^s = \dddot{u}^s$, $v^f = 0$, $\omega = 0$ in~~

$$\cancel{(\mathcal{M}\,\dddot{u}, (\dddot{u}^s, 0)) + \mathcal{B}(\dot{u}, (\dddot{u}^s, 0))} \qquad (123)$$

OK

$$\cancel{-(\beta\dot{\theta}, e(\dddot{u}^s)) = (\dot{f}, \dddot{u}^s) + \langle \dot{g}, \dddot{u}^s\rangle}$$

~~Expanding (122):~~

$$(\rho \overset{\circ\circ\circ}{u}{}^{s}, \overset{\circ\circ\circ}{u}{}^{s}) + (\rho_f \overset{\circ\circ\circ}{u}{}^{f}, \overset{\circ\circ\circ}{u}{}^{s}) \qquad (102)$$

$$+ \left((\lambda_\mu + 2\mu)\, \varepsilon_{11}(\mathring{u}^s), \varepsilon_{11}(\overset{\circ\circ}{u}{}^s)\right)$$

$$+ \left((\lambda_\mu + 2\mu)\, \varepsilon_{33}(\mathring{u}^s), \varepsilon_{33}(\overset{\circ\circ}{u}{}^s)\right)$$

$$+ \left(\lambda_\mu\, \varepsilon_{33}(\overset{\circ}{\ddot{u}}{}^s), \varepsilon_{11}(\overset{\circ\circ}{\ddot{u}}{}^s)\right) + \left(\lambda_\mu\, \varepsilon_{11}(\overset{\circ\circ}{u}{}^s), \varepsilon_{33}(\overset{\circ\circ}{\ddot{u}}{}^s)\right)$$

$$+ \left(4\mu\, \varepsilon_{13}(\mathring{u}^s), \varepsilon_{13}(\overset{\circ\circ}{\ddot{u}}{}^s)\right) \qquad (124)$$

$$+ \left(B\, e(\mathring{u}^f), e(\overset{\circ\circ}{\ddot{u}}{}^s)\right)$$

$$- \left(\beta \mathring{\theta}, e(\overset{\circ\circ}{\ddot{u}}{}^s)\right) = \left(\mathring{f}^s, \overset{\circ\circ}{u}{}^s\right) + \langle \mathring{g}, \ddot{u}^s \rangle_\Gamma$$

Also, choose $v^s = 0$, $v^f = \overset{\circ\circ}{u}{}^f$ in (19):

$$(\rho_f \overset{\circ\circ\circ}{u}{}^s, \overset{\circ\circ}{u}{}^f) + (g \overset{\circ\circ\circ}{u}{}^f, \overset{\circ\circ}{u}{}^f) + \left(\tfrac{\eta}{k} \overset{\circ\circ}{u}{}^f, \overset{\circ\circ}{u}{}^f\right) + \left(B\, e(\overset{\circ\circ}{u}{}^s), e(\overset{\circ\circ}{u}{}^f)\right)$$

$$+ \left(M\, e(\mathring{u}^f), e(\overset{\circ\circ}{\ddot{u}}{}^f)\right) - \left(\beta_f \mathring{\theta}, e(\overset{\circ\circ}{u}{}^f)\right)$$

$$= \langle \mathring{\chi}, \overset{\circ\circ}{\ddot{u}}{}^f \cdot \nu \rangle_\Gamma + \left(\mathring{f}^f, \overset{\circ\circ}{u}{}^f\right) \qquad (125)$$

From ~~[illegible]~~ (125)

$$\left(\beta\dot{\theta}, e(\ddot{u}^f)\right) = \frac{\beta}{\beta_f}\left(\beta_f\dot{\theta}, e(\ddot{u}^f)\right)$$ 103

$$= \left(\frac{\beta}{\beta_f} e_f \dddot{u}^s, \ddot{u}^f\right) + \left(\frac{\beta}{\beta_f} g\, \dddot{u}^f, \ddot{u}^f\right) + \frac{\beta}{\beta_f}\left(\frac{\eta}{k}\ddot{u}^f, \ddot{u}^f\right)$$

$$+ \left(\frac{\beta}{\beta_f} B\, e(\dot{u}^s), e(\ddot{u}^f)\right)$$ (126)

$$+ \left(M\frac{\beta}{\beta_f} e(\dot{u}^f), e(\ddot{u}^f)\right) - \frac{\beta}{\beta_f}\langle \dot{X}, \ddot{u}^f\cdot\nu\rangle_\Gamma$$

$$- \frac{\beta}{\beta_f}\left(\dot{f}^f, \ddot{u}^f\right)$$

Using (124) and (126)

$$\left(\beta\dot{\theta}, e(\ddot{u}^s)\right) + \left(\beta\dot{\theta}, e(\ddot{u}^f)\right)$$

$$= \left((\lambda_u+2\mu)\,\varepsilon_{11}(\dot{u}^s), \varepsilon_{11}(\ddot{u}^s)\right)$$

$$+ \left((\lambda_u+2\mu)\,\varepsilon_{33}(\dot{u}^s), \varepsilon_{33}(\ddot{u}^s)\right)$$

$$+ \left(\lambda_u\,\varepsilon_{33}(\dot{u}^s), \varepsilon_{11}(\ddot{u}^s)\right) + \left(\lambda_u\,\varepsilon_{11}(\dot{u}^s), \varepsilon_{33}(\ddot{u}^s)\right)$$

$$+ \left(4\mu\,\varepsilon_{13}(\dot{u}^s), \varepsilon_{13}(\ddot{u}^s)\right) + \left(B\, e(\dot{u}^f), e(\ddot{u}^s)\right)$$

$$+ \left(\frac{\beta}{\beta_f} B\, e(\dot{u}^s), e(\ddot{u}^f)\right) + \left(\frac{\beta}{\beta_f} M\, e(\dot{u}^f), e(\ddot{u}^f)\right)$$

$$+ \left(\rho \overset{\circ\circ\circ}{u}{}^{s}, \overset{\circ\circ}{u}{}^{s}\right) + \left(\rho_f \overset{\circ\circ\circ}{u}{}^{f}, \overset{\circ\circ}{u}{}^{s}\right)$$

$$+ \left(\rho_f \frac{\beta}{\beta_f} \overset{\circ\circ\circ}{u}{}^{s}, \overset{\circ\circ}{u}{}^{f}\right) + \left(g \frac{\beta}{\beta_f} \overset{\circ\circ\circ}{u}{}^{f}, \overset{\circ\circ}{u}{}^{f}\right) + \left(\frac{\beta}{\beta_f}\frac{\chi}{k} \overset{\circ\circ}{u}{}^{f}, \overset{\circ\circ}{u}{}^{f}\right)$$

$$- \left(\overset{\circ}{f}{}^{s}, \overset{\circ\circ}{u}{}^{s}\right) - \left\langle \overset{\circ}{g}, \overset{\circ\circ}{u}{}^{s}\right\rangle_{\Gamma} - \left(\overset{\circ}{f}{}^{f}, \overset{\circ\circ}{u}{}^{f}\right)\frac{\beta}{\beta_f}$$

$$- \left\langle \overset{\circ}{\chi}, \overset{\circ\circ}{u}{}^{f} \cdot \nu \right\rangle_{\Gamma} \frac{\beta}{\beta_f}$$

set

$$a_\beta = \begin{pmatrix} \rho I & \rho_f I \\ \frac{\beta}{\beta_f}\rho_f I & \frac{\beta}{\beta_f} g I \end{pmatrix}$$

$$E_\beta = \begin{bmatrix} \lambda_\mu + 2\mu & \lambda_\mu & B & 0 \\ \lambda_\mu & \lambda_\mu + 2\mu & B & 0 \\ B\frac{\beta}{\beta_f} & B\frac{\beta}{\beta_f} & M\frac{\beta}{\beta_f} & 0 \\ 0 & 0 & 0 & 4\mu \end{bmatrix}$$

NOTE THAT a_β, E_β are positive definite since a, E are pos. def. and 1 row of a, E are multiplied by $\frac{\beta}{\beta_f}$.

$$B_\beta(u, v) = \left(E_\beta \tilde{\varepsilon}(u), \tilde{\varepsilon}(v)\right).$$

THEN

$$(\beta \dot{\theta}_m, \nabla\cdot \ddot{u}^s_m) + (\beta \dot{\theta}_m, \nabla\cdot \ddot{u}^f_m)$$

$$= (\mathcal{A}_\beta \dddot{u}_m, \ddot{u}_m) + \mathcal{B}_\beta(\dot{u}_m, \ddot{u}_m)$$

$$- (\dot{f}^s, \ddot{u}^s_m) - \langle \dot{g}, \ddot{u}^s_m \rangle - \langle \dot{\chi}, \ddot{u}^f_m\cdot\nu \rangle \frac{\beta}{\beta_f}$$

$$- \frac{\beta}{\beta_f}(\dot{f}^f, \ddot{u}^f_m) \qquad (127)$$

$$= \frac{1}{2}\frac{d}{dt}\left[(\mathcal{A}_\beta \ddot{u}_m, \ddot{u}_m) + \mathcal{B}_\beta(\dot{u}_m, \dot{u}_m)\right]$$

$$- (\dot{f}^s, \ddot{u}^s_m) - \langle \dot{g}, \ddot{u}^s_m \rangle - \langle \dot{\chi}, \ddot{u}^f_m\cdot\nu \rangle \frac{\beta}{\beta_f}$$

$$- \frac{\beta}{\beta_f}(\dot{f}^f, \ddot{u}^f_m)$$

Time derivative on (120), $w=0$ (106)

$$\left(\rho \ddddot{u}_m, v\right) + \left(\frac{\eta}{K} \dddot{u}^f, v^f\right) + \Lambda\left(\ddot{u}_m, v\right)$$

$$-\left(\beta \ddot{\theta}_m, \nabla\cdot v^s\right) - \left(\beta_f \ddot{\theta}_m, \nabla\cdot v^f\right)$$

$$= \left(\ddot{f}, v\right) + \left\langle \ddot{g}, v^s\right\rangle$$

$$+ \left\langle \ddot{\chi}, v^f\cdot\nu\right\rangle \qquad (128)$$

CHOOSE $v^s = \dddot{u}^s$, $v^f = 0$, in (128)

$v^s = 0$, $v^f = \dddot{u}^f$

and repeat the argument to get the analogue of (127)

$$\left(\beta \ddot{\theta}_m, \nabla\cdot \dddot{u}^s_m\right) + \left(\beta \ddot{\theta}_m, \nabla\cdot \dddot{u}^f_m\right)$$

$$= \frac{1}{2}\frac{d}{dt}\left[\left(a_\beta \dddot{u}_m, \dddot{u}_m\right) + B_\beta\left(\ddot{u}_m, \ddot{u}_m\right)\right]$$

$$-\left(\ddot{f}^s, \dddot{u}^s_m\right) - \left\langle \ddot{g}, \dddot{u}^s_m\right\rangle - \left\langle \ddot{\chi}, \dddot{u}^f\cdot\nu\right\rangle\frac{\beta}{\beta_f}$$

$$-\frac{\beta}{\beta_f}\left(\ddot{f}^f, \dddot{u}^f_m\right) \qquad (129)$$

CHECK:

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$$\left(\rho\, \overset{....}{u}^s_m, \overset{....}{u}^s_m\right) + \left(\rho_f\, \overset{....}{u}^f, \overset{...}{u}^s\right)$$

$$+ \left(\left(\lambda_u + 2\mu\right) \varepsilon_{11}\left(\overset{..}{u}^s\right), \varepsilon_{11}\left(\overset{...}{u}^s\right)\right)$$

$$+ \left(\left(\lambda_u + 2\mu\right) \varepsilon_{33}\left(\overset{..}{u}^s\right), \varepsilon_{33}\left(\overset{...}{u}^s\right)\right)$$

$$+ \left(\lambda_u\, \varepsilon_{33}\left(\overset{..}{u}^s\right), \varepsilon_{11}\left(\overset{...}{u}^s\right)\right) + \left(\lambda_u\, \varepsilon_{11}\left(\overset{..}{u}^s\right), \varepsilon_{33}\left(\overset{...}{u}^s\right)\right)$$

$$+ \left(4\mu\, \varepsilon_{13}\left(\overset{..}{u}^s\right), \varepsilon_{13}\left(\overset{...}{u}^s\right)\right) \qquad (124)$$

$$+ \left(B\, e\left(\overset{..}{u}^f\right), e\left(\overset{...}{u}^s\right)\right)$$

$$- \left(\beta\, \overset{..}{\theta}, e\left(\overset{...}{u}^s\right)\right) = \left(\overset{..}{f}^s, \overset{...}{u}^s_m\right) + \left\langle \overset{..}{g}, \overset{...}{u}^s_m \right\rangle_\Gamma$$

Also, choose $v^s = 0$, $v^f = \overset{...}{u}^f_m$ in (19):

$$\left(\rho_f\, \overset{....}{u}^s, \overset{...}{u}^f_m\right) + \left(g\, \overset{....}{u}^f_m, \overset{...}{u}^f_m\right) + \left(\frac{\eta}{k}\overset{...}{u}^f, \overset{...}{u}^f\right) + \left(B\, e\left(\overset{..}{u}^s\right), e\left(\overset{...}{u}^f_m\right)\right)$$

$$+ \left(M\, e\left(\overset{..}{u}^f_m\right), e\left(\overset{...}{u}^f_m\right)\right) - \left(\beta_f\, \overset{..}{\theta}_m, e\left(\overset{...}{u}^f_m\right)\right)$$

$$= \left\langle \overset{..}{\chi}, \overset{...}{u}^f_m \cdot \nu \right\rangle_\Gamma + \left(\overset{..}{f}^f, \overset{...}{u}^f\right) \qquad (125)$$

From ~~...~~: (125)

$$\left(\beta \overset{\circ\circ}{\theta}_m, e(\overset{\circ\circ\circ}{u}^f)\right) = \frac{\beta}{\beta_f}\left(\beta_f \overset{\circ\circ}{\theta}_m, e(\overset{\circ\circ\circ}{u}^f)\right)$$ (108)

$$= \left(\frac{\beta}{\beta_f} e_f \overset{\circ\circ\circ\circ}{u}^s, \overset{\circ\circ\circ}{u}^f\right) + \left(\frac{\beta}{\beta_f} g \overset{\circ\circ\circ\circ}{u}^f, \overset{\circ\circ\circ}{u}^f\right) + \frac{\beta}{\beta_f}\left(\frac{\eta}{K}\overset{\circ\circ\circ}{u}^f, \overset{\circ\circ\circ}{u}^f\right)$$

(126)

$$+ \left(\frac{\beta}{\beta_f} B\, e(\overset{\circ\circ}{u}^s), e(\overset{\circ\circ\circ}{u}^f)\right)$$

$$+ \left(M \frac{\beta}{\beta_f} e(\overset{\circ\circ}{u}^f), e(\overset{\circ\circ\circ}{u}^f)\right) - \frac{\beta}{\beta_f}\langle \overset{\circ\circ}{X}, \overset{\circ\circ\circ}{u}^f \cdot \nu\rangle_\Gamma$$

$$- \frac{\beta}{\beta_f}\left(\overset{\circ\circ}{f}, \overset{\circ\circ\circ}{u}^f\right)$$

Using (124) and (126)

$$\left(\beta \overset{\circ\circ}{\theta}, e(\overset{\circ\circ\circ}{u}^s)\right) + \left(\beta \overset{\circ\circ}{\theta}, e(\overset{\circ\circ\circ}{u}^f)\right)$$

$$= \left((\lambda_u + 2\mu)\, \varepsilon_{11}(\overset{\circ\circ}{u}^s), \varepsilon_{11}(\overset{\circ\circ\circ}{u}^s)\right)$$

$$+ \left((\lambda_u + 2\mu)\, \varepsilon_{33}(\overset{\circ\circ}{u}^s), \varepsilon_{33}(\overset{\circ\circ\circ}{u}^s)\right) + \left(\lambda_u\, \varepsilon_{11}(\overset{\circ\circ}{u}^s), \varepsilon_{33}(\overset{\circ\circ\circ}{u}^s)\right)$$

$$+ \left(\lambda_u\, \varepsilon_{33}(\overset{\circ\circ}{u}^s), \varepsilon_{11}(\overset{\circ\circ\circ}{u}^s)\right) + \left(B\, e(\overset{\circ\circ}{u}^f), e(\overset{\circ\circ\circ}{u}^s)\right)$$

$$+ \left(4\mu\, \varepsilon_{13}(\overset{\circ\circ}{u}^s), \varepsilon_{13}(\overset{\circ\circ\circ}{u}^s)\right)$$

$$+ \left(\frac{\beta}{\beta_f} B\, e(\overset{\circ\circ}{u}^s), e(\overset{\circ\circ\circ}{u}^f)\right) + \left(\frac{\beta}{\beta_f} M\, e(\overset{\circ\circ}{u}^f), e(\overset{\circ\circ\circ}{u}^f)\right)$$

$$+ \left(\rho \ddddot{u}^{s}, \dddot{u}^{s}\right) + \left(\rho_f \ddddot{u}^{f}, \dddot{u}^{s}\right)$$

$$+ \left(\rho_f \frac{\beta}{\beta_f} \ddddot{u}^{s}, \ddot{u}^{f}\right) + \left(g \frac{\beta}{\beta_f} \ddddot{u}^{f}, \dddot{u}^{f}\right) + \left(\frac{\beta}{\beta_f}\frac{\chi}{k} \dddot{u}^{f}, \ddddot{u}^{f}\right)$$

$$- \left(\ddot{f}^{s}, \dddot{u}^{s}\right) - \langle \ddot{g}, \dddot{u}^{s}\rangle_{\Gamma} - \left(\ddot{f}^{f}, \dddot{u}^{f}\right)\frac{\beta}{\beta_f}$$

$$- \langle \ddot{\chi}, \dddot{u}^{f} \cdot \nu \rangle_{\Gamma} \frac{\beta}{\beta_f}$$

Set

$$\mathcal{A}_\beta = \begin{pmatrix} \rho I & \rho_f I \\ \frac{\beta}{\beta_f}\rho_f I & \frac{\beta}{\beta_f} g I \end{pmatrix}$$

$$E_\beta = \begin{bmatrix} \lambda_\mu + 2\mu & \lambda_\mu & B & 0 \\ \lambda_\mu & \lambda_\mu + 2\mu & B & 0 \\ B\frac{\beta}{\beta_f} & B\frac{\beta}{\beta_f} & M\frac{\beta}{\beta_f} & 0 \\ 0 & 0 & 0 & 4\mu \end{bmatrix}$$

NOTE THAT $\mathcal{A}_\beta$, E_β are positive definite since $\mathcal{A}$, E are pos. def. and 1 row of $\mathcal{A}$, E are multiplied by $\frac{\beta}{\beta_f}$.

$$B_\beta(u, v) \equiv \left(E_\beta \tilde{\varepsilon}(u), \tilde{\varepsilon}(v)\right).$$

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$$(\beta \ddot{\theta}_m, \nabla\cdot \dddot{u}^s_m) + (\beta \ddot{\theta}_m, \nabla\cdot \dddot{u}^f_m)$$

$$= (A_\beta \ddddot{u}_m, \dddot{u}_m) + B_\beta(\ddot{u}_m, \dddot{u}_m)$$

$$- (\ddot{f}^s, \dddot{u}^s_m) - \langle \ddot{g}, \dddot{u}^s_m \rangle - \langle \ddot{\chi}, \dddot{u}^f_m\cdot\nu \rangle \frac{\beta}{\beta_f}$$

$$- \frac{\beta}{\beta_f}(\ddot{f}^f, \dddot{u}^f_m) \qquad (127)$$

$$= \frac{1}{2}\frac{d}{dt}\left[(A_\beta \dddot{u}_m, \dddot{u}_m) + B_\beta(\ddot{u}_m, \ddot{u}_m)\right]$$

$$- (\ddot{f}^s, \dddot{u}^s_m) - \langle \ddot{g}, \dddot{u}^s_m \rangle - \langle \ddot{\chi}, \dddot{u}^f_m\cdot\nu \rangle \frac{\beta}{\beta_f}$$

$$- \frac{\beta}{\beta_f}(\ddot{f}^f, \dddot{u}^f_m)$$

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Take time derivative in (120) and $w=0$ to get

$$\left(P\ddddot{u}_m, v\right) + \left(\frac{\eta}{k}\dddot{u}^f, v^f\right) + \Lambda\left(\ddot{u}_m, v\right) - \left(\beta\ddot{\theta}_m, \nabla\cdot v^s\right) - \left(\beta_f\ddot{\theta}_m, \nabla\cdot v^f\right) = \left(\ddot{f}, v\right) + \langle \ddot{g}, v^s\rangle + \langle \ddot{X}, v^f\cdot\nu\rangle \qquad (128)$$

CHOOSE $v^s = \dddot{u}^s$, $v^f = 0$ ~~to~~ in (128)

$$\left(P\ddddot{u}_m, (\dddot{u}^s, 0)\right) + \Lambda\left(\ddot{u}_m, (\dddot{u}^s_m, 0)\right) - \left(\beta\ddot{\theta}_m, \nabla\cdot\dddot{u}^s_m\right) = \left(\ddot{f}^s, \dddot{u}^s_m\right) + \langle \ddot{g}, \ddot{u}^s_m\rangle \qquad (129)$$

Expending (129)

Using (129) in (122) 112

$$\frac{1}{2}\frac{d}{dt}\Big[(\rho\,\ddot{u}_m,\ddot{u}_m)+\Lambda(\dot{u}_m,\dot{u}_m)$$

$$+(\tau\,\ddot{\theta}_m,\ddot{\theta}_m)+(\gamma\,\nabla\dot{\theta}_m,\nabla\dot{\theta}_m)$$

$$+(T_0\rho_\beta\,\dddot{u}_m,\dddot{u}_m)+T_0\Lambda_\beta(\ddot{u}_m,\ddot{u}_m)\Big]$$

$$+\left(\frac{\eta}{k}\ddot{u}^f_m,\ddot{u}^f_m\right)-(\beta\,\dot{\theta}_m,\nabla\cdot\ddot{u}^s_m)$$

$$-(\beta_f\,\dot{\theta}_m,\nabla\cdot\ddot{u}^f_m)+(c\,\ddot{\theta}_m,\ddot{\theta}_m)$$

$$-T_0(\ddot{f}^s,\dddot{u}^s_m)-T_0\langle\ddot{g},\dddot{u}^s_m\rangle$$

$$-\langle\ddot{\chi},\dddot{u}^f_m\cdot\nu\rangle\frac{\beta}{\beta_f}-\frac{\beta}{\beta_f}(\ddot{f}^f,\ddot{u}^f_m)$$

$$=(\dot{f}^s,\ddot{u}^s_m)+\langle\dot{g},\ddot{u}^s_m\rangle$$

$$+\langle\dot{\chi},\ddot{u}^f\cdot\nu\rangle+\langle\dot{h},\ddot{\theta}_m\rangle$$

$$-(\dot{q},\ddot{\theta}_m)$$