MA303 1rst. Mid Term Practice. Professor Juan E. Santos

Name: ______________________________

I.D. #: ______________________________

Instructions: Close books and notes. Show all your work in clear writing. No calculators. Use the back of the test for scrap paper. No credit in multiple choice questions. **Circle ONLY** the choice you believe is correct. If you circle or mark more than one option or you will get **zero credit**.

1. (10 pts) Determine the radious of convergence of the series

$$\sum_{n=0}^{\infty} \frac{n}{2^n}(x-3)^n$$

A) 1

B) 5

C) 3

D) 4

E) 2

2. (10 pts) The Taylor series of $f(x) = \dfrac{1}{1-x}$ about $x_0 = 2$ is

A) $-1 + (x-2) - (x-2)^2 + \ldots..$

B) $-1 + (x-2) - \dfrac{1}{2}(x-2)^2 + \ldots..$

C) $-1 + x - \dfrac{x^2}{2} + \ldots.$

D) $1 + \dfrac{1}{2}(x-2)^2 + \dfrac{1}{24}(x-2)^4 + \ldots\ldots..$

E) $-1 - (x-2)^2 + \dfrac{1}{12}(x-2)^4 + \ldots\ldots...$

3. (10 pts) The solution of the differential equation

$$y'' + xy' + 2y = 0$$

can be written in power series in the form

$$y(x) = \sum_{n=0}^{\infty} a_n x^n,$$

where the recurrence relation is

A) $(n+2)a_{n+2} - a_{n+1} - a_n = 0$, n=0,1,2,.....,

B) $a_{n+2} = -a_n/(n+1)$, n=0,1,2,.....,

C) $a_2 = 0$, $(n+2)(n+1)a_{n+2} = a_{n-1}$, n=1,2,.....,

D) $3(n+2)a_{n+2} - (n+1)a_n = 0$, n=0,1,2,.....,

E) $2(n+2)(n+1)a_{n+2} + (n+3)a_n = 0$, n=0,1,2,.....,

4. (10 pts) Let $y(x) = \sum_{n=0}^{\infty} a_n x^n$. The recurrence relation is

$$a_2 = -a_0/2, \quad a_{n+2} = \frac{n}{n+2} a_{n+1} - \frac{a_n}{(n+2)(n+1)}, n = 1, 2, ...$$

If $a_0 = 1, a_1 = 2$, the first four terms of the series solution is

A) $y(x) = 1 + 2x + 3x^2 + 4x^3 + ...$

B)$y(x) = 1 + 2x - \frac{1}{2}x^2 - 4x^3 + ...$

C) $y(x) = 1 + 2x - \frac{1}{2}x^2 - \frac{1}{2}x^3 + ...$

D) $y(x) = 1 + 2x - \frac{1}{2}x^2 - \frac{1}{4}x^3 + ...$

E) $y(x) = 1 + 2x - 3x^2 - 4x^3 + ...$

5. (10 pts) The general solution of

$$x^2\ y'' - 3\ x\ y' + 4y = 0, \quad x \neq 0$$

is

A) $y(x) = c_1 e^{2x} + c_2 x e^{2x}$

B) $y(x) = c_1 e^{4x} + c_2 x e^{-x}$

C) $y(x) = c_1 |x|^{-4} + c_2 |x|$

D) $y(x) = c_1 |x|^2 + c_2 |x|^2 \ln|x|$

E) $y(t) = c_1 |x| + c_2 |x| \ln|x|$

6. (10 pts) What is the value of $y(3)$ if $y(3)$ solves the initial value problem

$$x^2y'' + 4xy' + 2y = 0, \quad y(1) = 1, \ y'(1) = 2$$

A) -2

B) -1

C) 1

D) 5

E) 3

7. (10 pts) Classify the singular points of the given equation

$$x^2(2-x^2)y'' + \frac{2}{x}y' + 4y = 0$$

a) The regular singular points are :

b) The irregular singular points are:

8. (10 pts) What is the real part of the complex number 3^{4+5i}

9. (10 pts) Determine the radious of convergence of the Taylor series of $f(x) = \dfrac{1}{x(x-3)}$ about $x_0 = 1$

A) 1

B) 0

C) 6

D) ∞

E) 2

10. (10 pts). The inverse Laplace transform of

$$F(s) = \frac{3\ s+3}{s^2+2s+5}$$

is

A) $e^{-2t}\ \sin\ t$

B) $3\ e^{-t}\ \cos\ 2t$

C) $t+\ e^{2t}\ \cosh\ t$

D) $1-\cos\ 2t+\sin\ t$

E) $1+3e^t \cos\ 2t$

11. (10 pts). The solution of the initial value problem

$$y'' + y = g(t), \quad y(0) = 0, y'(0) = 1$$

with

$$g(t) = \begin{cases} t, & 0 \le t < 1, \\ 1, & t \ge 1, \end{cases}$$

is

A) $y(t) = t - u_2(t)\,[t - 2 - \cos(t - 2)]$

B) $y(t) = t - u_1(t)\,[t - 1 - \cos(t - 1)]$

C) $y(t) = t - u_2(t)\,[t - 2 - \cos(t - 2)]$

D) $y(t) = t - u_1(t)\,[t - 1 - \sin(t - 1)]$

E) $y(t) = u_1(t)\,[t - 1 - \cos(t - 1)]$

12. (10 pts). The inverse Laplace transform of

$$F(s) = \frac{2\ s + 2}{s^2 + 2s + 5}$$

is

A) e^{-t} sinh t

B) 2 e^{-t} cos $2t$

C) 2 e^{t} cosh t

D) $\frac{1}{2}$ e^{t} cos $2t$

E) $\frac{1}{2}$ e^{-t} sin $2t$

13. (10 pts). Use the Laplace transform to show that solution of the following initial value problem:

$$y^{(iv)} - y = \delta(t-2), \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0,\ y'''(0) = 0$$

is

A) $y(t) = \dfrac{1}{2}u_2(t)\,[\sinh(t-2) - \sin(t-2)]$

B) $y(t) = \dfrac{1}{4}u_2(t)\,[\sinh(t-2) + \cos(t-2)]$

C) $y(t) = \dfrac{1}{2}u_2(t)\,[\cosh(t-2) - \sin(t-2)]$

D) $y(t) = \dfrac{1}{4}u_2(t)\sinh(t-2)$

E) $y(t) = \dfrac{1}{2}u_2(t)\sin(t-2)$

14. (10 pts). The solution of the initial value problem

$$y'' + 4y = g(t), \quad y(0) = 0, y'(0) = 0$$

with

$$g(t) = \begin{cases} t, & 0 \leq t < \pi, \\ \pi, & t \geq \pi, \end{cases}$$

is

A) $y(t) = \frac{1}{8}t - \sin(t) - u_\pi(t)\left[\frac{1}{8}(t-\pi) - \sin(t-\pi)\right]$

B) $y(t) = u_\pi(t)\frac{1}{4}\left[2t - \pi - \sin(2t-\pi)\right]$

C) $y(t) = \frac{1}{4}t - \frac{1}{8}\sin(2t) - u_\pi(t)\left[\frac{1}{4}(t-\pi) - \frac{1}{8}\sin(2(t-\pi))\right]$

D) $y(t) = u_\pi(t)\left[\frac{1}{2}t - \sin(2t-\pi)\right]$

E) $y(t) = \frac{1}{3}t - \frac{1}{6}\cos(t) - u_\pi(t)\left[\frac{1}{3}(t-\pi) - \frac{1}{6}\cos(t-\pi)\right]$

15. (10 pts). The solution of the initial value problem

$$y'' - y = 2\delta(t-1), \quad y(0) = 1, y'(0) = 0$$

is

A) $y(t) = \frac{1}{8}\cosh(t) - 2u_1(t)\sinh(t-1)$

B) $y(t) = \frac{1}{2}\sinh(t) + u_1(t)\cosh(t-1)$

C) $y(t) = \cosh(2t) + u_1(t)\cosh(t-1)$

D) $y(t) = 2\sinh(t) + u_1(t)\sinh(t-1)$

E) $y(t) = \cosh(t) + 2\ u_1(t)\sinh(t-1)$

Answers

1. E
2. A
3. B
4. C
5. D
6. C
7. a) $\pm\sqrt{2}$
   7. b) 0

8) $3^4\cos(5\ln(3))$

9) A

10) B

11) D

12) B

13) A

14) C

15) E