A simple approximation for seismic attenuation and dispersion in a fluid-saturated porous rock with aligned fractures

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Summary

Physical properties of many reservoir rocks can be modelled using the concept of poroelasticity. Many reservoir rocks, in addition to the network of pores, contain larger fractures or cracks. Galvin and Gurevich (2006) solved the single scattering problem for a crack in a poroelastic medium and then estimated the effective properties for a distribution of cracks. However this problem requires the solution of a Fredholm integral equation of the 2nd kind which in general has no analytical solution for intermediate frequencies. We propose a simple analytical approximation of this solution using the branching function approach. Quantitative comparison shows good agreement between the two solutions. Our analytical solution exhibits a relaxation peak at a frequency where the fluid diffusion length is of the order of the crack diameter. The diffusion length is proportional to $\omega^{-1/2}$ (where ω is frequency) and is usually much smaller than the wavelength of the normal compressional or shear wave. This shows that the presence of cracks in a fluid-saturated porous medium can cause significant attenuation and dispersion at very low frequencies, well before the onset of elastic (Rayleigh) scattering.

Introduction

Physical properties of many reservoir rocks can be modelled using the concept of poroelasticity. A poroelastic material consists of an elastic frame permeated by an interconnected pore space filled with a Newtonian fluid (Biot 1962). Many reservoir rocks, in addition to the network of pores, contain larger fractures or cracks. A common method of detecting such cracks is based on the use of elastic wave scattering. Attenuation and dispersion of the passing wave due to a distribution of cracks can be estimated using multiple-scattering theory. This approach requires an understanding of how an elastic wave interacts with a single crack. For fluid-saturated reservoir rocks this interaction differs from the corresponding elastic scattering, as it involves flow of the pore fluid between the crack and the host medium, induced by the passing wave. This effect is particularly significant for thin cracks, as their high compliance (compared to that of the relatively stiff pores) causes the fluid to flow in and out of the crack during rarefaction and compression wave cycles.

Galvin and Gurevich (2006) solved the single scattering problem for a crack in a poroelastic medium and then estimated the effective properties for a distribution of cracks. However this problem requires the solution of a Fredholm integral equation of the 2nd kind which in general has no analytical solution and must be solved numerically for every frequency. There are asymptotic analytical solutions, however, in the limits of high and low frequency. A model involving numerical solution of an integral equation is cumbersome for practical purposes and especially for any inversion procedure. Therefore some analytical approximation of this solution is desirable.

In this paper we derive such an approximate solution using a branching function approach. This approach consists in connecting low- and high frequency asymptotic solutions by a function that ensures that the result is physically consistent for all frequencies. The branching function approach has been utilized in many different applications. Johnson et al (1987) employed this approach to build a model for dynamic permeability and tortuosity in fluid saturated porous rock subjected to a mechanical wave. Models for attenuation and dispersion due to patchy saturation (Johnson, 2001) and double porosity and dual permeability (Pride and Berryman, 2003,a,b) have also utilized a branching function. Pride et al (1987) have investigated the effect of replacing different branching functions in Johnson et al. (1987) model for dynamic permeability, whilst Toms et al (2006) compared Johnson's (2001) branching function model for patchy saturation against an exact analytical solution, and showed that the approach is reasonably accurate at intermediate frequencies. In this paper we apply the branching function approach to the low and high frequency asymptotes of dispersion and attenuation in porous rocks with aligned cracks to get an approximate solution for all intermediate frequencies.

Galvin and Gurevich (2006) presented a theoretical study of the problem of the interaction of a plane longitudinal elastic wave in a poroelastic medium with an open circular oblate spheroidal crack of radius a and small thickness $2b << a$ placed perpendicular to the direction of wave propagation. They restricted the analysis to so-called mesoscopic cracks whose radius is small compared to the wavelength $2\pi / k_1$ of the normal compressional wave, but large compared to the individual pore size. Crack thickness $2b$ was assumed smaller than the fluid diffusion length (wavelength of Biot's slow wave). In elastic media (say, in a dry porous material) such cracks would have very little effect on wave propagation. However in porous media there may be a significant effect due to fluid diffusion in and out of the crack (as the fluid diffusion length is much smaller than the wavelength).

Single Scattering

The poroelastic single scattering problem can be solved in the same fashion as the corresponding problem for elasticity (Robertson, 1967). Consider an incident plane normal (fast) compressional wave propagating in a porous medium with porosity ϕ along the z-axis of the cylindrical co-ordinate system with the axial solid displacement $u_z^{in} = u_0 \exp(ik_1 z)$, where k_1 is the wavenumber (time dependency $\exp(-i\omega t)$ is assumed). We obtain the secondary (scattered) field $\mathbf{u}(\mathbf{r})$ resulting from interaction of the incident wave with the crack occupying the circle $r \le a$ in the plane $z = 0$. The total field is therefore $\mathbf{u}^T(\mathbf{r}) = u_z^{in}\mathbf{e}_z + \mathbf{u}(\mathbf{r})$, where $\mathbf{e}_z$ is a unit vector in the z-direction. We assume that the crack is in hydraulic communication with the surrounding pore space. Both the scattered and total fields must each satisfy Biot's (1962) equations of poroelasticity in the semi-infinite poroelastic medium $z \ge 0$. The distribution of displacements and stresses in the neighborhood of the crack is the same as that produced in a semi-infinite porous medium $z \ge 0$ when its free surface $z = 0$ is subject to the following boundary conditions:

$$\sigma_{rz} = 0, \qquad 0 \le r < \infty \tag{1}$$

$$u_z = 0, \qquad a < r < \infty \tag{2}$$

$$w_z = 0, \qquad a < r < \infty \tag{3}$$

$$u_z + w_z = 0, \qquad 0 \le r < a \tag{4}$$

$$\sigma_{zz} + p = -ik_1\left(H - \alpha M\right)u_0, \quad 0 \le r < a \tag{5}$$

where σ_{ij} is a component of the total stress tensor, p is the fluid pressure, w_z is the z-component of the so-called relative fluid displacement, M and H are poroelastic material constants related to the bulk moduli of the fluid K_f, solid K_g, and dry skeleton K by the Gassmann equations $M = \left[(\alpha - \phi)/K_g + \phi/K_f\right]^{-1}$ and $H = K + 4\mu/3 + \alpha^2 M$, and $\alpha = 1 - K/K_g$. The general solution of the equations of motion in cylindrical coordinates can be obtained by representing the four axial and radial components u_z, u_r, w_z, and w_r of the solid and relative fluid displacements in the form of an inverse Hankel transform with respect to the radial coordinate r. Then the boundary conditions yield a pair of dual integral equations for the unknown wave amplitudes (in the frequency-wavenumber domain). As shown by Noble (1963), such integral equations are equivalent to a single Fredholm equation of the second kind in the unknown amplitude function:

$$B\left(x\right) + \frac{1}{\pi}\int_0^\infty R(x,y)F\left(y\right)B\left(y\right)dy = -p_0 S(x), \tag{6}$$

where

$$R(x,y) = \frac{\sin a(x-y)}{x-y} - \frac{\sin a(x+y)}{x+y}, \tag{7}$$

$$S(x) = \frac{2}{\pi}\frac{\sin ax - ax\cos ax}{x^2}, \tag{8}$$

$p_0 = ik_1\left(H - \alpha M\right)u_0$, $g = \mu\big/\left(K + 4\mu/3\right)$, μ is the shear modulus of the solid skeleton, $q_2 = \sqrt{y^2 - k_2{}^2}$, k_2 is the wavenumber of Biot's slow wave and

$$F\left(y\right) = M\frac{\left(2\alpha gy^2 - k_2^2\right)^2 - 2yq_2\alpha g\left[k_2^2\left(\alpha g - 2\right) + 2\alpha y^2 g\right]}{2Hg\left(g-1\right)yq_2k_2^2}. \tag{9}$$

For frequencies much smaller than Biot's characteristic frequency $\omega_B = \phi\eta\big/\kappa\rho_f$, Biot's slow wave has a diffusion-like character and its wavenumber (inverse of the fluid diffusion length) is proportional to the square root of frequency.

A Random Distribution of Aligned Cracks

The theory presented above can be used to estimate attenuation and dispersion of an elastic wave propagating in a medium with a random distribution of aligned cracks. This can be done by using a Foldy-type approximation of multiple scattering (Waterman and Truell 1961) which expresses the effective wavenumber k^* for the medium with cracks in terms of the number of scatterers per unit volume n_0 and the far-field forward scattering amplitude $f(0)$ for a single scatterer. For a small concentration of aligned cracks the scattering theorem takes the form

$$k^* \approx k_1\left[1 + \frac{2\pi n_0 f\left(0\right)}{k_1^2}\right]. \tag{10}$$

Using equation (10) one can compute effective wave velocity $c(\omega) = \omega / \mathrm{Re}\, k^*$ and attenuation (reciprocal quality factor) $Q^{-1} = 2\,\mathrm{Re}\, k^* / \mathrm{Im}\, k^*$ as functions of frequency due to fluid flow between the cracks and surrounding pores. The relationship between $f(0)$ and the solution $B(y)$ of the integral equation (6) is

$$f(0) = -\frac{ik_1}{u_0}\frac{(H-\alpha M)}{2\mu H(1-g)}\lim_{y\to 0}\frac{B(y)}{y}. \qquad (11)$$

For low frequencies such that $|k_2 a| << 1$ equation (6) can be solved analytically by using an asymptotic expression for the transfer function $T(y)$ in the limit $k_2 << y$ and expanding $R(x,y)$ in powers of x. This yields an expression for effective velocity in the static limit:

$$c_0 = c_1\left[1 - \frac{2\varepsilon(H-\alpha M)^2}{3\mu H(1-g)}\right]. \qquad (12)$$

In equation (12) $c_1 = \omega / k_1 = (H/\rho)^{1/2}$ is the velocity of the fast compressional wave in the porous host (crack-free porous medium) and $\varepsilon = n_0 a^3 = (3/4\pi)(a/b)\phi_c$ is the crack density parameter (Hudson 1980) where $\phi_c = (4/3)\pi a^2 b n_0$ is the additional porosity present due to the cracks. For dry open cracks $K_f = M = 0$, $H = K + 4\mu/3$ and equation (12) simplifies to

$$c_0 = c_1\left[1 - \frac{2\varepsilon}{3g(1-g)}\right], \qquad (13)$$

which coincides with the well-known expression for the velocity of compressional waves propagating perpendicular to a system of dry open cracks in an elastic medium in the limit of low crack density (Hudson 1980). Furthermore, equation (12) coincides with the expression for the compressional wave velocity obtained from Gassmann's exact static result for the undrained elastic moduli of an anisotropic fluid-saturated porous medium with low crack density (Gurevich 2003). This Gassmann consistency is an important feature of the model presented here. Low-frequency attenuation Q^{-1} is given by

$$Q^{-1}_{low} = \frac{2M(H-\alpha M)^2\left(2-4\alpha g+3\alpha^2 g^2\right)|k_2 a|^2\varepsilon}{15\mu H^2 g(1-g)^2}, \qquad (14)$$

and is proportional to $|k_2 a|^2$, that is, to the first power of frequency.

In the limit of high frequencies such that crack radius is large compared to the fluid diffusion length, but smaller than the incident wavelength, $|k_1 a| << 1 << |k_2 a|$ (while crack thickness is still smaller then the diffusion length, $|k_2 b| << 1$ and the frequency is still smaller than Biot's characteristic frequency ω_B), a similar analysis yields

$$f_{high}(0) = \frac{i(k_1 a)^2 (H-\alpha M)^2 g}{3\mu M k_2}. \qquad (15)$$

By substituting this expression into the dispersion equation (10) one can see that its relative contribution to the real part of the effective wavenumber vanishes in the high frequency limit, implying that the velocity in that limit tends to the velocity in the porous crack-free medium. This result is logical as at sufficiently high frequencies the fluid has no time to move between pores and cracks, and therefore the cracks behave as if they were isolated (Thomsen 1995). Note however that this result is a consequence of assuming that there is an incompressible fluid occupying the cracks and a small aspect ratio b/a; the more precise validity condition is $K_f/\mu >> b/a$. In particular, the dry case is excluded, except in the static limit (13). Attenuation at high frequencies reads

$$Q^{-1}_{high} = \frac{2\sqrt{2}\pi\varepsilon(H-\alpha M)^2 g}{3\mu M|k_2 a|} \qquad (16)$$

and thus scales with $\omega^{-1/2}$.

Branching Function Approximation

In the previous section we have derived asymptotic solutions for attenuation and dispersion in the low- and high-frequency limits. For intermediate frequencies an analytical solution does not exist and attenuation and velocity can only be obtained by solving integral equation (6) numerically for every frequency. Figure 1 shows this numerical solution (asterisks) and the analytical low and high frequency asymptotes.

In order to get a simple analytical approximation of this complicated solution, we connect the exact low- and high-frequency asymptotic solutions using a branching function, which ensures causality of the solution. Following Johnson (2001) we model the dynamic saturated P wave modulus $\tilde{H}$ as

$$\tilde{H}(\omega) = H - (H - H_0)b(\omega), \qquad (17)$$

where

$$H_0 = c_0^2 \rho = H_1 \left[1 - \frac{2\varepsilon (H - \alpha M)^2}{3\mu H (1-g)} \right]^2 \quad (18)$$

and H are the saturated P wave moduli at low and high frequency limits, b is the branching function:

$$b(\omega) = 1 \Big/ \left(1 - \zeta + \zeta \sqrt{1 - \frac{i\omega\tau}{\zeta^2}} \right). \quad (19)$$

In equation (19) ζ and τ are the shape and frequency scaling parameters, given by

$$\tau = \left(\frac{H - H_0}{HG} \right)^2, \quad \zeta = \frac{(H - H_0)^3}{2 H_0 H^2 T G^2}. \quad (20)$$

Our saturated P-wave modulus $\tilde{H}$ converges at low frequencies as

$$\tilde{H}_{\omega \to 0} = H_0 (1 - i\omega T) \quad (21)$$

and at high frequencies as

$$\tilde{H}_{\omega \to \infty}(\omega) = H \left(1 - \frac{G}{\sqrt{-i\omega}} \right). \quad (22)$$

By comparing our exact expressions (18) and (19) with (21) and (22) we have

$$T = \frac{2M (H - \alpha M)^2 (2 - 4\alpha g + 3\alpha^2 g^2) \eta H a^2 \varepsilon}{15 \mu H^2 g (1-g)^2 \kappa M \left(K + \frac{4}{3}\mu \right)} \quad (23)$$

and

$$G = \frac{4\pi\varepsilon (H - \alpha M)^2 g \sqrt{\kappa M \left(K + \frac{4}{3}\mu \right)}}{3\mu M \sqrt{\eta H}} \quad (24)$$

Equations (17) – (20), (23) and (24) give our branching function solution for attenuation and dispersion at all frequencies. Figure 1 shows the comparison of this analytical approximation (solid lines) with the original results based on numerical solution of the integral equation (asterisks). One can see that the branching function gives a good approximation of the numerical solution.

Conclusions

We have presented an approximate analytical solution for attenuation and dispersion of elastic wave in a porous fluid-saturated medium with aligned fractures of finite size. This solution exhibits a relaxation peak at a frequency $f = \omega / 2\pi \simeq 2\kappa M (K + 4\mu/3) / H\eta a^2$, the frequency where the fluid diffusion length $1/|k_2|$ is of the order of the crack diameter a. Note that the diffusion length is proportional to $\omega^{-1/2}$ and is usually much smaller than the wavelength of the normal compressional or shear wave. This shows that the presence of cracks in a fluid-saturated porous medium can cause significant attenuation and dispersion at very low frequencies, well before the onset of elastic (Rayleigh) scattering.

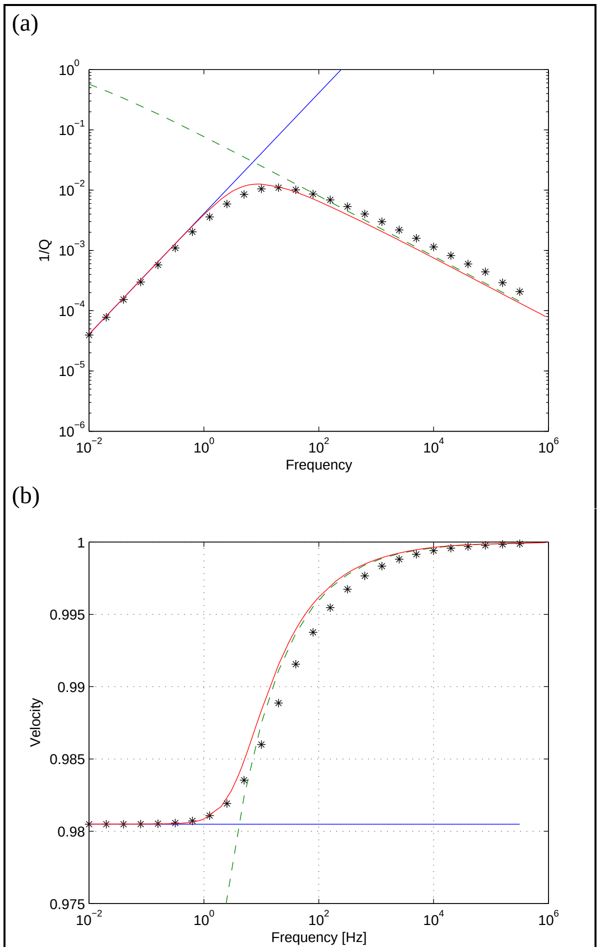

Figure 1: P-wave attenuation (a) and velocity (b) vs frequency: asterisks - numerical solution, and red line – analytical approximation

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EDITED REFERENCES

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REFERENCES

Biot, M. A., 1962, Mechanics of deformation and acoustic propagation in porous media: Journal of Applied Physics, 33, 1482–1498.

Galvin, R. J., and B. Gurevich, 2006, Interaction of an elastic wave with a circular crack in a fluid-saturated porous medium: Applied Physics Letters, 88, 061918.

Gurevich, B., 2003, Elastic properties of saturated porous rocks with aligned fractures: Journal of Applied Geophysics, 54, 203–218.

Hudson, J. A., 1980, Overall properties of a cracked solid: Mathematical Proceedings of the Cambridge Philosophical Society, 88, 371–384.

Johnson, D. L., 2001, Theory of frequency dependent acoustics of patchy-saturated porous media: Journal of the Acoustical Society of America, 110, 682–694.

Johnson, D. L, J. Koplik, and R. Dashen, 1987, Theory of dynamic permeability and tortuosity in fluid-saturated porous media: Journal of Fluid Mechanics, 176, 379–402.

Noble, B., 1963, The solution of Bessel function dual integral equations by a multiplying-factor method: Proceedings of the Cambridge Philosophical Society, 59, 351–362.

Pride, S. R., and J. G. Berryman, 2003, Linear dynamics of double-porosity and dual permeability materials. II. Fluid transport equations: Physical Review E, 68, 036,604.

Robertson, I. A., 1967, Diffraction of a plane longitudinal wave by a penny-shaped crack: Proceedings of the Cambridge Philosophical Society, 63, 229–238.

Thomsen, L., 1995, Elastic anisotropy due to aligned cracks in porous rock: Geophysical Prospecting, 43, 805–829.

Toms, J., T. Müller, B. Gurevich, and D. L. Johnson, 2006, Attenuation and dispersion in partially saturated porous rock: Random vs Periodic Models: 68th Conference and Exhibition, EAGE.

Waterman, P. C., and R. Truell, 1961, Multiple scattering of waves: Journal of Mathematical Physics, 2, 512–537.

reference deleted due to duplication