Do Core Sample Measurements Record Group or Phase Velocity?

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ABSTRACT

Laboratory core-sample measurements do not record elastic constants directly; they record traveltimes from which the elastic parameters must be deduced. Do the recorded traveltimes represent group velocities, phase velocities, or something else? For propagation down symmetry directions group and phase velocity are the same and there is no ambiguity. However, propagation in *non*-symmetry directions is key to determining a complete set of elastic constants. In nonsymmetry directions there is no guarantee the energy radiated from the source will travel straight up the axis of the core to the receiver; the leading planar portion of the wavefront may crab sideways and partially or completely miss the receiver-transducer target. Our model results show that unless the miss is complete, picking first breaks gives a reasonably good phase-velocity arrival time.

INTRODUCTION

Elastic parameters of rock samples are typically measured in the laboratory by cutting cylinders out of samples of the rock, affixing a transducer to either end, and measuring the traveltime of ultrasonic waves across the sample. Layered rocks such as shale (transversely isotropic) are usually cut at angles to the layering of 0°, 90°, and 45° (shown diagrammatically in Figure 1). Elastic constants are then determined from the set of recorded travel times.

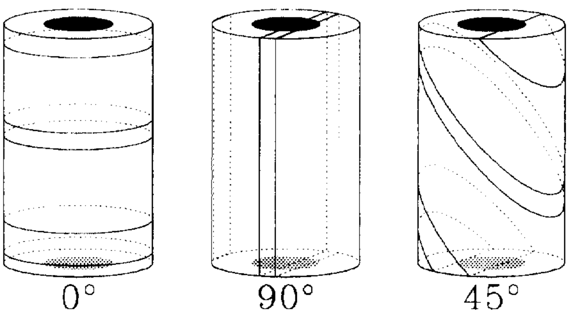

Figure 1: Shale cores cut at 0°, 90°, and 45°. The disks at the top and bottom of each core show the relative size of the P-wave transducers.

Before we can determine accurate elastic constants from the recorded traveltimes, we need to know what velocities the traveltimes are measuring. Theoretically we know that if we could somehow do the experiment using ideal *point* sources and receivers, we really would be measuring *group* velocities along the direction from the point source to the point receiver. Similarly, if we could somehow do the experiment using infinite parallel *planar* sources and receivers, we really would be measuring *phase* velocities along the direction normal to the source and receiver planes. (See Figure 2.)

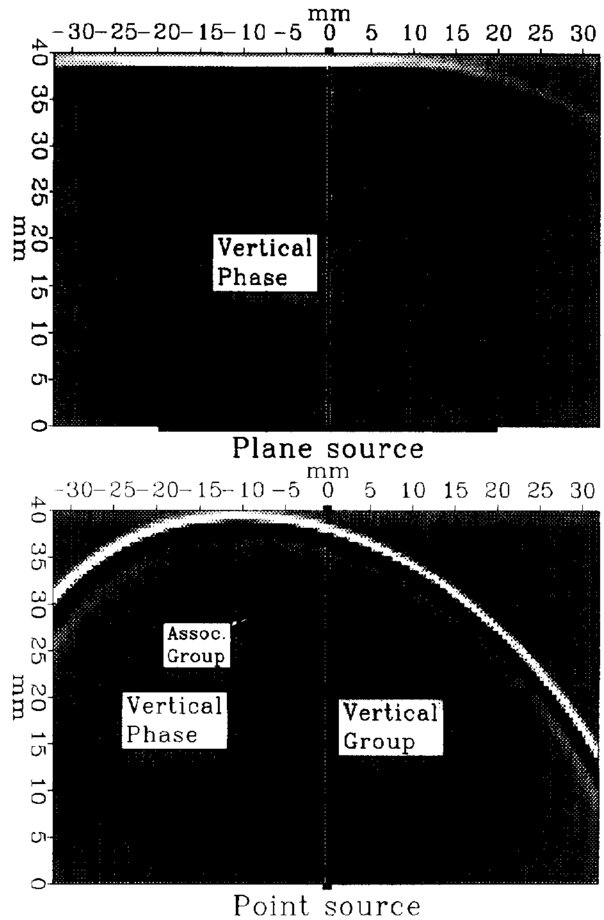

Figure 2: Snapshots demonstrating the two "ideal" core-sample experiments. The positions and sizes of the source and receiver transducers are indicated respectively by thick horizontal lines at the bottom and top of the model. Top: an infinite-source experiment for measuring vertical phase velocity. Bottom: a point-source to point-receiver experiment for measuring vertical group velocity. (As you can see the two numerical experiments shown are really approximations, the "infinite" transducer is only 40mm wide and the "point" transducers are 2mm wide.) The labeled vertical and near-vertical distances mark the progress of key points on the radiated wavefront during its travels from the source up to the top of the model. Three distinct velocities are indicated: the vertical group (energy) velocity, the vertical phase (plane-wave) velocity, and finally the (nonvertical) group velocity *associated with* the vertical phase velocity. (Note the "associated group velocity" is faster than the vertical phase velocity, which in turn is faster than the vertical group velocity. If the medium were isotropic, all three of these would be equivalent.)

While neither of these idealized experiments is possible in practice, we usually expect to be close enough to one extreme or the other to know what we are measuring, and we hope the error should be small. But is this assumption correct? In particular, should we expect the traveltimes measured in a typical laboratory experiment to give us accurate vertical phase velocities, accurate vertical group velocities, or something in between?

To find out, we examine the results of a computer finite-difference model patterned after a laboratory experiment done by Vernik and Nur (1992). The anisotropy of their Bakken Shale sample was quite severe, about the worst we might expect to encounter in a geological sample. Furthermore, the P-wave transducers in their experiment were 12 millimeters wide but 40 millimeters apart, so the distance the waves traveled was fully three times greater than the source and receiver size. Despite these vicissitudes, approaching what we might consider a "worst-case" scenario for geological samples, Vernik and Nur proceeded on the assumption that their measured traveltimes represented phase velocities. Were they correct to do so?

LABORATORY AND NUMERICAL MODELS

Figure 1 shows the laboratory experimental configuration. The cylinders were cut from a sample of Bakken shale (Vernik and Nur, 1992); Figure 3 shows the shape of qP, qSV, and SH wavefronts emanating from a point source in this shale. The aspect ratios of the cores in Figure 1 are correct (40mm tall and 26mm wide); the disks at the top and bottom of each core show the true relative widths of the P-wave transducers (12mm). The SV and SH transducers were almost twice as wide, 20mm, nearly as wide as the core itself.

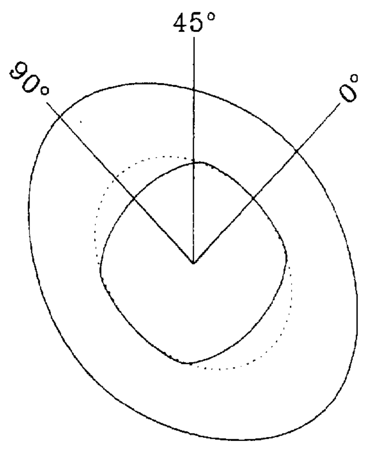

Figure 3: Impulse-response curves showing the shapes of wavefronts propagating in the medium used in our numerical models: qP (outer curve), qSV (inner solid curve), and SH (dotted). The 90°, 45°, and 0° labels show the direction of vertical for the corresponding shale-core orientations.

We wish to model Vernik and Nur's laboratory experiment numerically. Since we are only interested in seeing how anisotropy may have affected the direct wave from the source to the receiver transducer, a very simplified numerical model is more than adequate. We will not clutter the model by attempting to include the rather complex boundary conditions entailed by tilted-axis anisotropy interacting with a truncated cylindrical surface; we will also keep the model two-dimensional. (The elastic constants we used for the numerical simulation are $C_{11} = 20.16$, $C_{33} = 11.97$, $C_{55} = 4.00$, $C_{66} = 6.86$, and $C_{13} = 5.51$. The density has been normalized out so these are all in units of $(\mathrm{mm}/\mu\mathrm{s})^2$.)

THE TWO IDEAL EXPERIMENTS

Before examining our simplified numerical simulation of Vernik and Nur's experiment, we will first show two complementary "ideal" simulated experiments, one designed for measuring the vertical group velocity and the other for measuring the vertical phase velocity.

The lower plot in Figure 2 shows how vertical group velocity could be properly measured by using extremely small source and receiver transducers. The anisotropic wavefront radiates out from the point source at the bottom; the point receiver at the top detects the part of the wavefront with vertically traveling energy as it passes by. The distance between the two transducers divided by the measured traveltime gives the vertical group velocity.

The upper plot in Figure 2 shows how true phase velocity could be properly measured. The source must be wide enough to launch a reasonable facsimile of a plane wave. Since the source is not infinite, the "plane wave" is truncated; the receiver on the top must be positioned where it can sample the flat central part of the wavefront, away from the diffracting truncated edge. Note that while the source transducer runs from -20 to $+20$ mm, the flat part of the wavefront in the figure runs from -30 to $+10$. While the wavefront has traveled vertically 40mm from the bottom of the model to the top, it has also slipped sideways 10mm. The receiver in the upper plot in Figure 2, while OK, is perilously close to the edge; a position 10 or 15 millimeters further to the left would have been better. Note that if the source transducer had been infinite, there would be no such complications: the position of the receiver would be irrelevant.

The upper plot in Figure 2 was constructed by summing multiple copies of the lower plot in Figure 2 shifted from -20 to $+20$ mm. The flat part of the wavefront in the upper plot is the sum of all the shifted copies of the highest point on the wavefront in the lower plot. From this basic relationship, we can see that we could have measured the vertical phase velocity directly from the lower plot by simply shifting the receiver over to where the wavefront first encountered the upper surface (around the -11mm position) and using the *vertical* distance from source to receiver to calculate the velocity (40mm in the figure) instead of the true Euclidean distance ($\sqrt{40^2+11^2}$mm here).

P-WAVE MODEL RESULTS

So much for "ideal" experiments; what did the P waves do in Vernik and Nur's core samples? Figure 4 shows a snapshot of the situation for two different core-layer orientations in their experiment.

In the upper plot the layers run vertically; there is no tendency for the wave energy to "slip sideways" and the flat part of the wavefront impacts the receiver transducer well centered. We can see from the figure that things are working correctly in this case. Mathematically this is because the group and phase velocities happen to be the same for this propagation direction. The same good behavior also occurs when the layers in the core run horizontally, because this also sends the waves along a symmetry direction where the group and phase velocities are identical (for all wavetypes).

The situation is not so happy in the lower plot. Here the layers are at a 45° angle, running from the lower right to the upper left. The P wave travels faster along the layers than across them; as a result the "flat" part of the wavefront (containing the main focus of energy) tends to follow the layers and slips sideways to the left. (See Green (1973) for striking photographs of side-slipping wavefronts in a block of quartz.) In our particular example the accumulated sideways slip from bottom to top happens to be about the same as the transducer widths, so the "flat" part of the wavefront only just grazes the edge of the receiver. What traveltime will be measured in this borderline case?

The answer is contained in Figure 5. The left part of this figure shows the result of a tiny "seismic survey" over the shale-core model. The source transducer is held fixed. The horizontal axis shows how the trace recorded at the receiver would vary with offset if the receiver were moved around (instead of glued in place). The first break on the earliest arrival occurs at $10.29\mu s$ for an offset of -12mm. (The "correct" phase-velocity arrival time as defined by running the model with infinite-length transducers is very slightly later, $10.30\mu s$.) The first break at zero offset (and thus what corresponds to the signal recorded in the actual experiment) occurs at $10.34\mu s$. If this time were used to calculate the vertical phase velocity it would cause an error of only .5%, which is smaller than the typical errors Vernik and Nur encountered in picking first breaks in their experiment, about 1% for P waves and 2% for S waves. (The only significant effect of the borderline miss in our simulated experiment is a 48% drop in trace amplitude.)

The right part of Figure 5 shows the results of re-running the numerical experiment for a range of transducer sizes. Zero-offset traces corresponding to what would be measured by a laboratory experiment are shown. With a point source and point receiver, the first break measures the group-velocity arrival time, $10.63\mu s$. As the transducer size is increased towards 12mm the first break moves rapidly earlier to within .5% of the phase-velocity arrival time. As the transducer size is increased yet further the first-break time closes in on the phase-velocity arrival time more slowly, finally reaching it

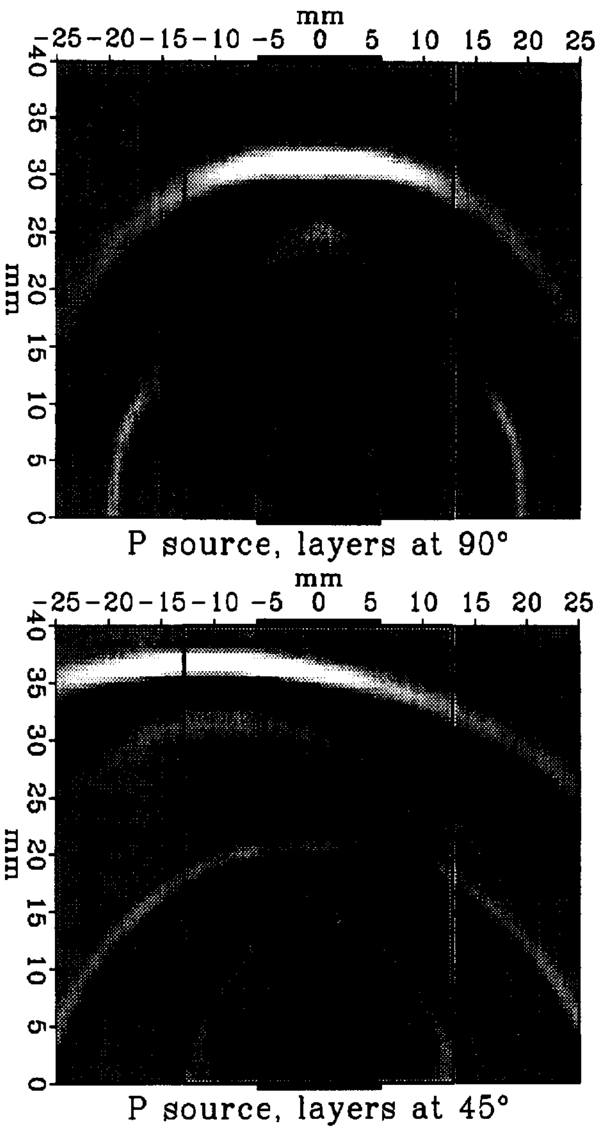

Figure 4: Snapshots showing the behavior of qP waves in our 90° (top) and 45° (bottom) core-sample simulations. (The top snapshot shows the situation at $7.5\mu s$, the bottom at $10.\mu s$.) The vertical bars show the relative width of Vernik and Nur's cores, while the thick solid lines at the bottom and top show the size and positions of the P-wave source and receiver transducers. Note in the 45° case how the leading part of the wavefront is aiming to miss its intended target, hitting the top of the core somewhat to the left of the receiver instead.

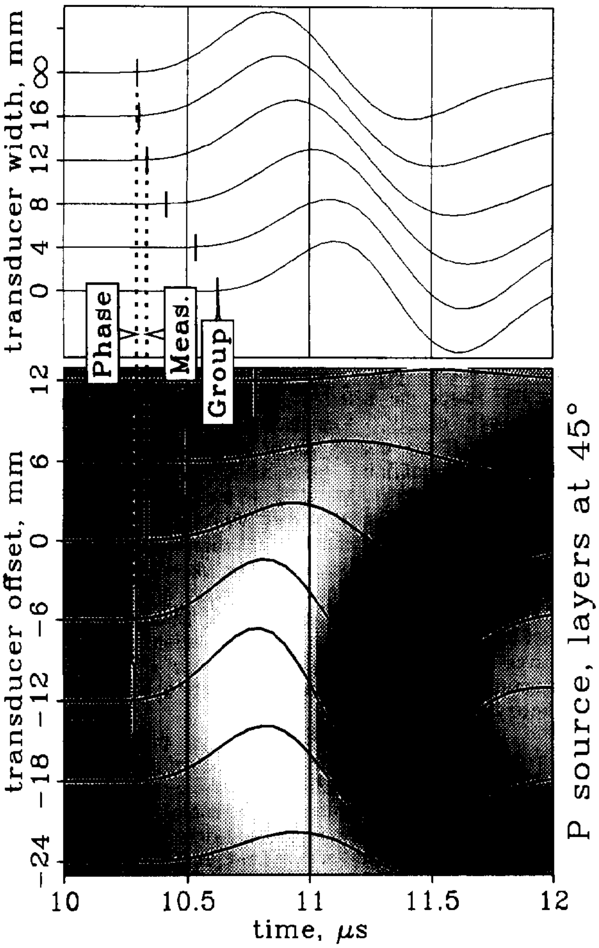

Figure 5: (Turn your head to the left when looking at this plot.) A double plot showing how the signal recorded in the 45° P-wave case (lower plot in Figure 4) would change as a function of two normally fixed parameters, source-receiver transducer offset (left subplot) and source and receiver transducer width (right subplot). In the "real life" case the transducer offset was fixed at 0mm and the transducer width was fixed at 12mm. In the left subplot the transducer width is held at 12mm, but the transducer offset is allowed to vary from −24mm to +12mm (with individual wiggle traces shown every 6mm). In the right subplot the transducer offset is held at 0mm, but the transducer width is allowed to vary by 4mm steps from 0mm to 16mm (and then is abruptly jumped to ∞). First-break times on various traces are marked by short horizontal lines. Three significant first-break times are labeled: The 0mm-width 0mm-offset trace defines the group-velocity arrival time. The 12mm-width 0mm-offset trace occurs in both subplots and shows the measured arrival time for the "real life" case. The ∞-width trace (offset is irrelevant) defines the phase-velocity arrival time. Except for a tiny mismatch (caused by a wavelet phase shift) the ∞-width first-break arrival time in the right subplot is the same as the −12mm-offset first-break arrival time in the left subplot.

when the transducer width is 20mm. All wider transducers measure the phase-velocity arrival time.

S WAVES

Our model results for shear waves were similar, although there were a few surprises. Unfortunately there is not sufficient space here to present our S-wave results; you'll have to come to the presentation or wait for the complete version to appear in GEOPHYSICS.

CONCLUSIONS

Our numerical modeling shows that Vernik and Nur were indeed correct in their assumption; they did measure phase velocity. The only error due to the severe anisotropy that might have affected their P-wave results was a .5% delay in the 45° measurement. Given that they found typical random errors on their measurements of about 1% to 2%, an additional .5% of error is fairly insignificant. The numerical model furthermore shows that for the experiment to have measured P *group* velocities to a similar level of accuracy, the transducers would have had to have been at most *2mm* wide!

We conclude that laboratory experiments of this kind should almost always measure phase velocity. In the most extreme cases they may measure something hard to interpret in the never-never land between group and phase.

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