

Static and dynamic behavior of a porous solid saturated by a two-phase fluid

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A method is presented to determine the elastic constants for an isotropic, porous, elastic solid saturated by a two-phase fluid. Assuming that the shear modulus of the empty matrix is known, it is shown that the six additional coefficients in the stress-strain relations can be uniquely determined by performing two ideal experiments referred to as "generalized jacketed and partially jacketed compressibility tests," in analogy with the single-phase theory of Biot. Under reasonable assumptions on the behavior of the material, the experiments yield expressions for the coefficients in terms of the material properties of the individual phases and the capillary pressure function relating the pressures in the two fluid phases. Finally, numerical results showing properties of the phase velocities and attenuations for the four different types of body waves are presented and analyzed.

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INTRODUCTION

A linear theory describing deformation and elastic wave propagation in an elastic porous solid saturated by a two-phase fluid was presented in Ref. 1. Here we first derive a method for determining the elastic constants in the stress-strain relations for such a medium. The procedure yields values for the coefficients in terms of the properties of the solid and individual fluid phases and the capillary pressure function relating the two fluid pressures. Then, the phase velocities and attenuations for the different types of body waves propagating in this kind of media are computed and analyzed. The theory of deformation and elastic wave propagation in an elastic porous solid saturated by a single-phase fluid was presented by Biot in several classical papers²⁻⁵; later, Burridge and Keller⁶ rederived the model using homogenization. Existence, uniqueness, and regularity of the solution of Biot's equations of motion were analyzed in Ref. 7. Some numerical procedures for obtaining approximate solutions were given in Ref. 8.

The organization of the paper is as follows. In Sec. I we present a brief review of the stress-strain relations for the single-phase case and give a new method to determine the elastic constants in these relations. In Sec. II we begin by stating the stress-strain relations for an isotropic elastic porous solid saturated by a two-phase fluid as derived in Ref. 1. We then extend the method described in Sec. I for determining the single-phase elastic constants to the two-phase case. By performing ideal experiments referred to as "generalized jacketed and partially jacketed compressibility tests," we first determine the bulk modulus of the two-phase system and then obtain a set of equations that yield the desired expressions for the elastic coefficients. Finally, in Sec. III a

parameter study of the different types of waves propagating in a two-phase fluid saturated porous media is given. First, the elastic coefficients for two different formations saturated by mixtures of oil and water and gas and water are computed. Then, the phase velocities and attenuations for the four different types of body waves propagating in this type of media are computed for one of the formations, as functions of both frequency and saturation of the nonwetting phase.

I. REVIEW OF THE STRESS-STRAIN RELATIONS FOR THE SINGLE-PHASE ISOTROPIC CASE

Let us consider a porous elastic solid saturated by a viscous compressible fluid. Let $u^s = (u_1^s, u_2^s, u_3^s)$ and $\tilde{u}^f = (\tilde{u}_1^f, \tilde{u}_2^f, \tilde{u}_3^f)$ be the averaged displacement vectors of the solid and fluid parts of the medium. Here, \tilde{u}^f is defined such that for any face S of a cube Q of bulk material, $\int_S \phi \tilde{u}^f \cdot \nu \, d\sigma$ represents the amount of fluid displaced through S , where ν is the unit outer normal to S , ϕ is the effective porosity, and $d\sigma$ the surface measure on S . Set

$$u^f = \phi(\tilde{u}^f - u^s)$$

$$\xi = -\nabla \cdot u^f,$$

$$e = \nabla \cdot u^s$$

$$\epsilon_{ij}(u^s) = \frac{1}{2} \left(\frac{\partial u_i^s}{\partial x_j} + \frac{\partial u_j^s}{\partial x_i} \right), \quad 1 \leq i, j \leq 3.$$

Next, let $\tau_{ij} = \bar{\tau}_{ij} + \Delta_{ij}$ and $p_f = \bar{p}_f + \Delta p_f$ be the total stress tensor and the fluid pressure, respectively, with $\Delta \tau_{ij}$ and Δp_f being increments in τ_{ij} and p_f with respect to the reference stress $\bar{\tau}_{ij}$ and the reference fluid pressure \bar{p}_f associated with the initial equilibrium state. For the isotropic case, the stress-strain relations were given by Biot⁴:

$$\Delta\tau_{ij} = 2N\epsilon_{ij} + \delta_{ij}(\lambda_c e - B\xi) \quad (1a)$$

$$\Delta p_f = -Be + M\xi. \quad (1b)$$

The inverse relations for (1) that give us the strains ϵ_{ij} and ξ as linear functions of the stresses $\Delta\tau_{ij}$ and Δp_f can be written in the form

$$\epsilon_{ij} = (1/2N)\Delta\tau_{ij} + \delta_{ij}(D\Delta\tau - F\Delta p_f), \quad (2a)$$

$$\xi = -F\Delta\tau + H\Delta p_f, \quad (2b)$$

where

$$\Delta\tau = \Delta\tau_{11} + \Delta\tau_{22} + \Delta\tau_{33}.$$

Let us analyze the physical significance of the strain variables e and ξ . Let \bar{V}_b , \bar{V}_s , and \bar{V}_f denote the bulk, solid, and fluid volumes of a homogeneous part Ω of bulk material in the initial equilibrium state. Since u^s is the average solid displacement vector over the whole bulk material, e represents the change $\Delta V_b = V_b - \bar{V}_b$ in bulk volume per unit volume of bulk material; i.e.,

$$e = \Delta V_b / \bar{V}_b. \quad (3)$$

Next, let ΔV_f^c be part of the total change $\Delta V_f = V_f - \bar{V}_f$ in fluid volume due to changes in fluid pressure. Then, if K_f denotes the bulk modulus of the fluid,

$$\frac{\Delta V_f^c}{V_f} = -\frac{\Delta p_f}{K_f}. \quad (4)$$

Now we observe that the change in fluid content is the difference between ΔV_f and ΔV_f^c . Since for uniform porosity ξ represents the change in fluid content per unit volume of bulk material, it follows that

$$\xi = (\Delta V_f - \Delta V_f^c) / \bar{V}_b = \bar{\phi}(\Delta V_f - \Delta V_f^c) / \bar{V}_f, \quad (5)$$

where

$$\bar{\phi} = \bar{V}_f / \bar{V}_b.$$

For the analysis that follows it is convenient to decompose any tensional change $\Delta\tau_{ij}$ into the form

$$\Delta\tau_{ij} = -\Delta p_f \delta_{ij} + \Delta\hat{\tau}_{ij}, \quad (6)$$

where $\hat{\tau}_{ij}$ is the so-called residual or effective stress of the material. Following the ideas in Refs. 5 and 8, the elastic coefficients in the right-hand side of (1) can be determined as follows. First, since the fluid does not support any shear, N is identical to the shear modulus of the empty matrix. To determine the remaining coefficients in (1), it is sufficient to consider tensional changes $\Delta\tau_{ij}$ such that

$$\begin{aligned} \Delta\tau_{11} = \Delta\tau_{22} = \Delta\tau_{33} &= \frac{1}{3}\Delta\tau = -\Delta p, \quad \Delta p > 0, \\ \Delta\tau_{ij} &= 0, \quad i \neq j. \end{aligned}$$

Set

$$\frac{1}{3}\Delta\hat{\tau} \equiv \Delta\hat{\tau}_{11} = \Delta\hat{\tau}_{22} = \Delta\hat{\tau}_{33} = -\Delta\hat{p}. \quad (7)$$

Then the decomposition (6) becomes

$$-\frac{1}{3}\Delta\tau = \Delta p = \Delta p_f + \Delta\hat{p}, \quad (8)$$

and (1)-(2) reduce to

$$\frac{1}{3}\Delta\tau = -\Delta p = Ge - B\xi, \quad (9a)$$

$$\Delta p_f = -Be + M\xi, \quad (9b)$$

and

$$e = (3D + 1/2N)\Delta\tau - 3F\Delta p_f, \quad (10a)$$

$$\xi = -F\Delta\tau + H\Delta p_f, \quad (10b)$$

where

$$G = \lambda_c + \frac{2}{3}N. \quad (11)$$

Let us consider the special case in which no fluid is allowed to flow in or out of the bulk material (closed system), and let K_c , the bulk modulus of the closed system, be defined by the equation

$$e = -\Delta p / K_c, \quad (12)$$

corresponding to a compressibility test in which a sample of bulk material is enclosed in an impermeable jacket and then subjected to an additional external pressure Δp . Since for a closed system $\xi = 0$, it follows from (9a) and (12) that

$$G = K_c. \quad (13)$$

Now, we shall obtain expressions for the elastic coefficients $G = K_c, B$, and M in (9) using an argument that can be generalized to treat the two-phase problem. Let K_s and K_m denote the bulk modulus of the solid grains and the empty matrix, respectively. We shall eventually determine K_c using (10). So, first we shall derive expressions for the coefficients $3D + 1/2N$ and F using the jacketed compressibility test as described in Ref. 5. This test corresponds to a tensional state such that

$$\Delta p_f = 0, \quad e = -\Delta p / K_m = -\Delta\hat{p} / K_m; \quad (14)$$

i.e., the internal fluid pressure is held constant and the external applied pressure $\Delta\hat{\tau}_{11} = \Delta\hat{\tau}_{22} = \Delta\hat{\tau}_{33} = -\Delta\hat{p}$ is supported by the solid matrix. Note that, ignoring second order terms, (4)-(5) imply that for this experiment

$$\xi = \frac{\Delta V_f}{\bar{V}_b} = \frac{\Delta(\phi V_b)}{\bar{V}_b} = \bar{\phi} \frac{\Delta V_b}{\bar{V}_b} + \Delta\hat{\phi}.$$

According to a formula on page 13 of Ref. 9,

$$\Delta\phi = [1/K_s - (1 - \bar{\phi})/K_m] \Delta\hat{p}. \quad (15)$$

Hence,

$$\xi = (1/K_s - 1/K_m) \Delta\hat{p}. \quad (16)$$

Now, using (14) and (16) in (10), we see that

$$3D + 1/2N = 1/3K_m, \quad (17a)$$

$$F = \frac{1}{3}(1/K_s - 1/K_m). \quad (17b)$$

Next, we shall derive an expression for the bulk modulus K_c of the closed system. Note that (4), (5), and the fact that for the closed system $\xi = 0$ give us the relation

$$\Delta V_f / \bar{V}_f = -\Delta p_f / K_f.$$

On the other hand, (15) implies that

$$\begin{aligned} \frac{\Delta V_f}{\bar{V}_f} &= \frac{\Delta V_b}{\bar{V}_b} + \frac{\Delta\phi}{\bar{\phi}} \\ &= -\frac{\Delta p}{K_c} + \frac{1}{\bar{\phi}} \left(\frac{1}{K_s} - \frac{1 - \bar{\phi}}{K_m} \right) \Delta\hat{p}. \end{aligned}$$

Thus, using (8) we obtain the following relation between Δp_f and Δp :

$$\Delta p_f = \frac{1/K_s - 1/K_m + \bar{\phi}(1/K_m - 1/K_c)}{1/K_s - 1/K_m + \bar{\phi}(1/K_m - 1/K_f)} \Delta p. \quad (18)$$

Now, using the equation above we see that for the closed system (10a) yields

$$\frac{1}{K_c} = \frac{1}{K_m} + \left(\frac{1}{K_s} - \frac{1}{K_m} \right) \times \frac{1/K_s - 1/K_m + \bar{\phi}(1/K_m - 1/K_c)}{1/K_s - 1/K_m + \bar{\phi}(1/K_m - 1/K_f)}$$

Thus,

$$K_c = K_s [(K_m + Q)/(K_s + Q)], \quad (19)$$

where

$$Q = K_f(K_s - K_m)/\bar{\phi}(K_s - K_f), \quad (20)$$

which coincides with the expression for K_c in Ref. 9.

The remaining elastic constants B and M in (9) can be determined as functions of K_c using the jacketed compressibility test described before. For that test (9) yields the relations

$$1 = \frac{K_c}{K_m} + B \left(\frac{1}{K_s} - \frac{1}{K_m} \right), \quad (21a)$$

$$0 = \frac{B}{K_m} + M \left(\frac{1}{K_s} - \frac{1}{K_m} \right). \quad (21b)$$

Note that B and M may also be determined by performing the unjacketed compressibility test,⁵ corresponding to a tensional state of the form

$$\Delta\tau_{11} = \Delta\tau_{22} = \Delta\tau_{33} = -\Delta p = -\Delta p_f, \quad \Delta\hat{p} = 0.$$

Note that, according to (15), $\Delta\phi = 0$. Thus,

$$\frac{\Delta V_b}{\bar{V}_b} = \frac{\Delta V_s}{\bar{V}_s} = \frac{\Delta V_f}{\bar{V}_f}, \quad (22)$$

which in turn implies that

$$e = -\Delta p/K_s. \quad (23)$$

Also, it follows from (5) that

$$\xi = \bar{\phi}(1/K_f - 1/K_s)\Delta p. \quad (24)$$

Now using (23)–(24) in (9) we obtain

$$1 = \frac{K_c}{K_s} + B\bar{\phi} \left(\frac{1}{K_f} - \frac{1}{K_s} \right), \quad (25a)$$

$$1 = \frac{B}{K_s} + M\bar{\phi} \left(\frac{1}{K_f} - \frac{1}{K_s} \right). \quad (25b)$$

Either from (21) or (25) the following expressions for B and M are obtained:

$$B = \frac{K_s K_f (K_s - K_m)}{K_f (K_s - K_m) + K_s \bar{\phi} (K_s - K_f)}, \quad (26a)$$

$$M = \frac{K_s^2 K_f}{K_f (K_s - K_m) + K_s \bar{\phi} (K_s - K_f)}. \quad (26b)$$

II. DETERMINATION OF THE ELASTIC COEFFICIENTS FOR A TWO-PHASE, FLUID-SATURATED POROUS SOLID

In this section we shall consider the problem of determining the elastic coefficients in the stress–strain relations for an elastic, isotropic, homogeneous, porous solid saturated by a mixture of oil and water. The subscripts (or super-

scripts) “ o ” and “ w ” will be used to refer to the oil and water phases. All the results presented here remain valid for any wetting–nonwetting system in which the “ w ” refers to the wetting phase and “ o ” to the nonwetting phase.

Let $S_o = S_o(x)$ and $S_w = S_w(x)$ denote the oil and water saturations. Since the two fluid phases completely saturate the porous part of the bulk material,

$$S_o + S_w = 1.$$

Let u^s , \bar{u}^o , and \bar{u}^w denote the averaged solid, oil, and water displacement vectors. The fluid displacement vectors \bar{u}^o and \bar{u}^w are defined such that for any face F of a cube Q of bulk material, $\int_F \phi S_o \bar{u}^o \cdot \nu \, d\sigma$ represents the amount of oil displaced through F , and a similar hold for \bar{u}^w . Set

$$u^\theta = \phi(\bar{u}^\theta - u^s), \quad \xi^\theta = -\nabla \cdot u^\theta, \quad \theta = o, w.$$

The physical significance of the variables ξ^o and ξ^w is the following. Let us consider a volume \bar{V}_b of homogeneous bulk material containing oil and water volumes \bar{V}_o and \bar{V}_w at pressures \bar{p}_o and \bar{p}_w . Set

$$\bar{V}_f = \bar{V}_o + \bar{V}_w, \quad \bar{S}_\theta = \bar{V}_\theta / \bar{V}_f, \quad \theta = o, w.$$

Note that $\bar{S}_o \xi^o$ and $\bar{S}_w \xi^w$ represent the change in oil and water content per unit volume of bulk material. Thus, if ΔV_θ^c denotes the part of the total change $\Delta V_\theta = V_\theta - \bar{V}_\theta$ in volume due to changes $\Delta p_\theta = p_\theta - \bar{p}_\theta$ in the corresponding fluid pressures, we see that

$$\begin{aligned} \bar{S}_\theta \xi^\theta &= (\Delta V_\theta - \Delta V_\theta^c) / \bar{V}_b \\ &= \bar{\phi} [(\Delta V_\theta - \Delta V_\theta^c) / \bar{V}_f], \quad \theta = o, w, \end{aligned} \quad (27)$$

since in equilibrium $\nabla S_\theta = 0$. Let K_o and K_w denote the oil and water bulk modulus, respectively. Then,

$$\Delta V_\theta^c / \bar{V}_\theta = -\Delta p_\theta / K_\theta, \quad \theta = o, w.$$

Also, neglecting the second-order terms,

$$\Delta V_\theta = \bar{V}_f \Delta S_\theta + \bar{S}_\theta \Delta V_f, \quad \theta = o, w.$$

Thus,

$$\xi^\theta = \bar{\phi} \left(\frac{\Delta S_\theta}{\bar{S}_\theta} + \frac{\Delta V_f}{\bar{V}_f} - \frac{\Delta V_\theta^c}{\bar{V}_\theta} \right), \quad \theta = o, w. \quad (28)$$

Next, set

$$\xi^* = \bar{S}_o \xi^o + \bar{S}_w \xi^w. \quad (29)$$

Then it follows from (27) and (28) that

$$\xi^* = \bar{\phi} [(\Delta V_f - \Delta V_f^c) / \bar{V}_f],$$

where

$$\Delta V_f^c = \Delta V_o^c + \Delta V_w^c.$$

Thus we see that the variable ξ^* in (29) plays the role of the variable ξ of the single-phase case [cf. (5)].

Next, let $\tau_{ij} = \bar{\tau}_{ij} + \Delta\tau_{ij}$ be the total stress tensor of the bulk material, $\Delta\tau_{ij}$ being the change in the total stress with respect to the reference total stress $\bar{\tau}_{ij}$ corresponding to the initial equilibrium state. Also, let $p_o = \bar{p}_o + \Delta p_o$ and $p_w = \bar{p}_w + \Delta p_w$ be the oil and water pressures, Δp_o and Δp_w representing increments in the corresponding pressures with respect to the reference pressures \bar{p}_o and \bar{p}_w associated with the initial equilibrium state. Recall that p_o and p_w are related through the capillary relation,¹⁰

$$p_c = p_c(S_o) = (\bar{p}_o + \Delta p_o) - (\bar{p}_w + \Delta p_w) \\ = p_c(\bar{S}_o) + \Delta p_o - \Delta p_w. \quad (30)$$

Let us normalize by assuming that $\bar{p}_w = 0$, so that $\bar{p}_o = p_c(\bar{S}_o)$. Also set

$$\Delta p_c = \Delta p_o - \Delta p_w, \quad (31)$$

$$\Delta S_o = S_o - \bar{S}_o. \quad (32)$$

Then, ignoring terms of the second order in ΔS_o ,

$$\Delta p_c \approx p'_c(\bar{S}_o) \Delta S_o. \quad (33)$$

Set

$$\beta = p_c(\bar{S}_o)/p'_c(\bar{S}_o).$$

Following Ref. 1, the stress-strain relations can be written in the form

$$\Delta \tau_{ij} = 2N\epsilon_{ij} + \delta_{ij}(\lambda_c^* e - B_1 \xi^o - B_2 \xi^w), \quad (34a)$$

$$(\bar{S}_o + \beta) \Delta p_o - \beta \Delta p_w = -B_1 e + M_1 \xi^o + M_3 \xi^w, \quad (34b)$$

$$\bar{S}_w \Delta p_w = -B_2 e + M_3 \xi^o + M_2 \xi^w. \quad (34c)$$

The inverse relations of (34) that give us the generalized strains ϵ_{ij} , $(\bar{S}_o + \beta) \xi^o$, and $\bar{S}_w \xi^w - \beta \xi^o$ as linear functions of $\Delta \tau_{ij}$, Δp_o , and Δp_w are given¹ by

$$\epsilon_{ij} = (1/2N) \Delta \tau_{ij} + \delta_{ij} (D^* \Delta \tau - F_1 \Delta p_o - F_2 \Delta p_w), \quad (35a)$$

$$(\bar{S}_o + \beta) \xi^o = -F_1 \Delta \tau + H_1 \Delta p_o + H_3 \Delta p_w, \quad (35b)$$

$$\bar{S}_w \xi^w - \beta \xi^o = -F_2 \Delta \tau + H_3 \Delta p_o + H_2 \Delta p_w. \quad (35c)$$

For the analysis that follows, and in analogy with (6), it is convenient to decompose any tensional change $\Delta \tau_{ij}$ into a hydrostatic pressure due to both fluid phases plus an additional residual stress tensor. For that purpose, set

$$\bar{p}_f = S_o p_o + S_w p_w.$$

Ignoring terms of the second order and using (31) and (33) we see that

$$\Delta \bar{p}_f \approx \bar{S}_o \Delta p_o + \bar{S}_w \Delta p_w + \Delta S_o p_c(\bar{S}_o) \\ \approx \bar{S}_o \Delta p_o + \bar{S}_w \Delta p_w + \beta \Delta p_c \\ = (\bar{S}_o + \beta) \Delta p_o + (\bar{S}_w - \beta) \Delta p_w. \quad (36)$$

Thus, we shall use the decomposition

$$\Delta \tau_{ij} = -\Delta \bar{p}_f \delta_{ij} + \Delta \hat{\tau}_{ij} \\ = -[(\bar{S}_o + \beta) \Delta p_o + (\bar{S}_w - \beta) \Delta p_w] \delta_{ij} + \Delta \hat{\tau}_{ij}. \quad (37)$$

Next, we shall obtain an expression for the saturation change ΔS_o that will be useful in the analysis that follows. Let us consider a part Ω of bulk material having bulk volume \bar{V}_b and oil and water volumes \bar{V}_o and \bar{V}_w . Let $\bar{V}_f = \bar{V}_o + \bar{V}_w$ be the total fluid volume. Let us consider another equilibrium state of Ω with corresponding volumes $V_o = \bar{V}_o + \Delta V_o$, $V_w = \bar{V}_w + \Delta V_w$ and $V_f = \bar{V}_f + \Delta V_f$ and saturations $S_o = \bar{S}_o + \Delta S_o$, $S_w = \bar{S}_w + \Delta S_w$. In all the cases to be considered in this work this new equilibrium state will be one attained by the system after the compressibility tests to be described later have been performed. Note that, neglecting terms of the second order,

$$\frac{\Delta V_o}{\bar{V}_o} = \frac{\Delta S_o}{\bar{S}_o} + \frac{\Delta V_f}{\bar{V}_f}$$

and

$$\frac{\Delta V_w}{\bar{V}_w} = -\frac{\Delta S_o}{\bar{S}_w} + \frac{\Delta V_f}{\bar{V}_f}.$$

Hence,

$$\Delta S_o = \bar{S}_o \bar{S}_w \left(\frac{\Delta V_o}{\bar{V}_o} - \frac{\Delta V_w}{\bar{V}_w} \right) = \bar{S}_w \frac{\Delta V_o}{\bar{V}_f} - \bar{S}_o \frac{\Delta V_w}{\bar{V}_f}. \quad (38)$$

Now we proceed to determine the elastic coefficients in (34). First, as in the single-phase case, N is identical to the shear modulus of the porous matrix. To determine the remaining coefficients in (34) it is enough to consider tensional changes $\Delta \tau_{ij}$ of the form

$$\Delta \tau_{11} = \Delta \tau_{22} = \Delta \tau_{33} = \frac{1}{3} \Delta \tau = -\Delta p, \quad \Delta p > 0,$$

$$\Delta \tau_{ij} = 0, \quad i \neq j.$$

Thus, for $\Delta \hat{\tau}$ as in (7), the decomposition (37) becomes

$$-\frac{1}{3} \Delta \tau = \Delta p = \Delta \bar{p}_f + \Delta \hat{p}. \quad (39)$$

Also, (34) and (35) reduce to

$$\frac{1}{3} \Delta \tau = G^* e - B_1 \xi^o - B_2 \xi^w, \quad (40a)$$

$$(\bar{S}_o + \beta) \Delta p_o - \beta \Delta p_w = -B_1 e + M_1 \xi^o + M_3 \xi^w, \quad (40b)$$

$$\bar{S}_w \Delta p_w = -B_2 e + M_3 \xi^o + M_2 \xi^w, \quad (40c)$$

and

$$e = (3D^* + 1/2N) \Delta \tau - 3F_1 \Delta p_o - 3F_2 \Delta p_w, \quad (41a)$$

$$(\bar{S}_o + \beta) \xi^o = -F_1 \Delta \tau + H_1 \Delta p_o + H_3 \Delta p_w, \quad (41b)$$

$$\bar{S}_w \xi^w - \beta \xi^o = -F_2 \Delta \tau + H_3 \Delta p_o + H_2 \Delta p_w, \quad (41c)$$

with

$$G^* = \lambda_c^* + \frac{1}{3} N. \quad (42)$$

Following the ideas given in Sec. I, first we shall determine the bulk modulus K_c^* of the closed system consisting of the saturated rock, where K_c^* is defined by the equation

$$e = -\Delta p / K_c^*. \quad (43)$$

For this purpose, let us consider a "generalized jacketed compressibility test," corresponding to a tensional state such that

$$\Delta p_o = \Delta p_w = 0, \quad e = -\Delta p / K_m = -\Delta \hat{p} / K_m,$$

$$\Delta \tau_{11} = \Delta \tau_{22} = \Delta \tau_{33} = -\Delta \hat{p}. \quad (44)$$

This experiment may be performed as follows. A sample of bulk material is enclosed in an impermeable jacket and then subjected to an external pressure change $\Delta p = \Delta \hat{p}$. A pair of tubes T_o and T_w are connected from the inside of the jacket to corresponding containers filled with oil and water held at the reference pressures \bar{p}_o and \bar{p}_w . The tube T_o is provided with a membrane permeable to oil and impermeable to water, while the tube T_w has a membrane permeable to water and impermeable to oil. This insures that both fluid pressures p_o and p_w inside the sample are held constant; i.e., $\Delta p_o = \Delta p_w = 0$. Note that in this experiment,

$$\Delta S_\theta = 0, \quad \Delta V_\theta^c / \bar{V}_\theta = -\Delta p_\theta / K_\theta = 0, \quad \theta = o, w.$$

Thus, it follows from (15) and (27) that

$$\xi^o = \xi^w = \xi^* = \bar{\phi} \frac{\Delta V_f}{\bar{V}_f} = \left(\frac{1}{K_s} - \frac{1}{K_m} \right) \Delta \hat{p}.$$

Now, applying (44) in (41), we deduce that

$$3D^* + 1/2N = 1/3K_m,$$

$$F_1 = \frac{1}{3}(\bar{S}_o + \beta)(1/K_s - 1/K_m),$$

$$F_2 = \frac{1}{3}(\bar{S}_w - \beta)(1/K_s - 1/K_m).$$

Now we shall use (41a) and the expressions above for $3D^* + 1/2N$, F_1 , and F_2 to determine the bulk modulus K_c^* of the closed system. First note that, for the closed system, there is conservation of mass of both fluid phases; thus, there is no change in oil and water content. Consequently,

$$\frac{\Delta V_\theta}{\bar{V}_\theta} = \frac{\Delta V_\theta^c}{\bar{V}_\theta} = -\frac{\Delta p_\theta}{K_\theta}, \quad \theta = o, w. \quad (45)$$

Then, according to (27),

$$\xi^o = \xi^w = \xi^* = 0. \quad (46)$$

Combining (31), (33), (38), and (45), we obtain

$$\Delta p_o - \Delta p_w = p'_c(\bar{S}_o)\bar{S}_o\bar{S}_w(\Delta p_w/K_w - \Delta p_o/K_o),$$

so that

$$\Delta p_o = \gamma \Delta p_w, \quad (47)$$

with

$$\gamma = \frac{1 + p'_c(\bar{S}_o)\bar{S}_o\bar{S}_w/K_w}{1 + p'_c(\bar{S}_o)\bar{S}_o\bar{S}_w/K_o}.$$

Note that $0 < \gamma < 1$. Let

$$\alpha = (\gamma - 1)(\bar{S}_o + \beta) + 1.$$

Then using (47) we see that for the closed system the decomposition (39) becomes

$$\Delta p = \alpha \Delta p_w + \Delta \hat{p}. \quad (48)$$

Next, from (29) and the fact that the $\xi^* = 0$, we see that

$$\frac{\Delta V_f}{\bar{V}_f} = \bar{S}_o \frac{\Delta V_o^c}{\bar{V}_o} + \bar{S}_w \frac{\Delta V_w^c}{\bar{V}_w} = -\bar{S}_o \frac{\Delta p_o}{K_o} - \bar{S}_w \frac{\Delta p_w}{K_w}. \quad (49)$$

On the other hand, using (15) and (43),

$$\begin{aligned} \frac{\Delta V_f}{\bar{V}_f} &= \frac{\Delta V_b}{\bar{V}_b} + \frac{\Delta \phi}{\bar{\phi}} \\ &= -\frac{\Delta p}{K_c^*} + \frac{1}{\bar{\phi}} \left(\frac{1}{K_s} - (1 - \bar{\phi}) \frac{1}{K_m} \right) \Delta \hat{p}. \end{aligned} \quad (50)$$

Hence, combining (47), (49), and (50) gives

$$-\frac{\Delta p}{K_c^*} + \frac{1}{\bar{\phi}} \left(\frac{1}{K_s} - \frac{(1 - \bar{\phi})}{K_m} \right) \Delta \hat{p} = -\left(\frac{\gamma \bar{S}_o}{K_o} + \frac{\bar{S}_w}{K_w} \right) \Delta p_w. \quad (51)$$

Set

$$\Theta = \frac{1/K_s - 1/K_m + \bar{\phi}(1/K_m - 1/K_c^*)}{\alpha[1/K_s - 1/K_m + \bar{\phi}(1/K_m - 1/K_f^*)]}, \quad (52)$$

where

$$\frac{1}{K_f^*} = \frac{1}{\alpha} \left(\frac{\gamma \bar{S}_o}{K_o} + \frac{\bar{S}_w}{K_w} \right). \quad (53)$$

Then it follows from (48) and (51) that for the closed system the following relation between Δp_w and Δp holds:

$$\Delta p_w = \Theta \Delta p. \quad (54)$$

Next, since for the closed system $\Delta \tau_{11} = \Delta \tau_{22} = \Delta \tau_{33} = -\Delta p$, combining (41a), (43), (47), (52), and (54) leads to the relation

$$\begin{aligned} \frac{1}{K_c^*} &= \frac{1}{K_m} + \left(\frac{1}{K_s} - \frac{1}{K_m} \right) \\ &\times \frac{1/K_s - 1/K_m + \bar{\phi}(1/K_m - 1/K_c^*)}{1/K_s - 1/K_m + \bar{\phi}(1/K_m - 1/K_f^*)}. \end{aligned} \quad (55)$$

Hence,

$$K_c^* = K_s [(K_m + Q^*) / (K_s + Q^*)], \quad (56)$$

where

$$Q^* = K_f^*(K_m - K_s) / \bar{\phi}(K_f^* - K_s).$$

Now we can proceed to determine the elastic coefficients in the right-hand side of (40). First, note that, since for the closed system $\xi^o = \xi^w = 0$, it follows from (43) and (40) that

$$G^* = K_c^*.$$

Also, from (40b), (40c), (43), (46), (47), and (54) we deduce that

$$B_1 = K_c^* \Theta [(\bar{S}_o + \beta)\gamma - \beta] \quad (57)$$

and

$$B_2 = K_c^* \bar{S}_w \Theta. \quad (58)$$

Next, we observe that, for the "generalized jacketed compressibility test" described above, (40), (44), and (45) yield the equations

$$1 = G^*/K_m + (B_1 + B_2)\delta, \quad (59a)$$

$$0 = B_1/K_m + (M_1 + M_3)\delta, \quad (59b)$$

$$0 = B_2/K_m + (M_2 + M_3)\delta, \quad (59c)$$

where

$$\delta = 1/K_s - 1/K_m.$$

A calculation shows that (59a) is compatible with (56)–(58).

Before deriving the additional relations (with some being redundant) needed to determine the elastic coefficients M_1 , M_2 , and M_3 in (40), we observe that combining (28), (31), and (33) produces the following relation between the variables ξ^o and ξ^w :

$$\xi^o - \xi^w = q(\Delta p_o - \gamma \Delta p_w), \quad (60)$$

where

$$q = \bar{\phi} \left(\frac{1}{K_o} + \frac{1}{p'_c(\bar{S}_o)\bar{S}_o\bar{S}_w} \right).$$

Now we consider what will be called a "generalized partially jacketed compressibility test," which may be described as follows. A sample of bulk material is enclosed in an impermeable jacket and immersed in a chamber filled with a fluid

held at the same reference pressure \bar{p}_o as the oil inside the sample. Then an additional pressure change Δp is applied to the fluid in the chamber. To insure that the water pressure in the sample stays at the reference value \bar{p}_w , a tube provided with a membrane permeable to water and impermeable to oil is connected from the inside of the sample to a container filled with water held at the reference pressure \bar{p}_w . Thus, for this experiment,

$$\Delta p_w = 0.$$

Also, since there is a conservation of mass of the oil phase,

$$\xi^o = 0, \quad (61)$$

while (60) implies that

$$\xi^w = -q \Delta p_o. \quad (62)$$

Next note that, according to the decomposition (39),

$$\Delta p = (\bar{S}_o + \beta) \Delta p_o + \Delta \hat{p}. \quad (63)$$

Thus, using the principle of superposition, (14), and (23), we see that

$$e = -(\bar{S}_o + \beta)(\Delta p_o/K_s) - \Delta \hat{p}/K_m. \quad (64)$$

Now, using (61), (62), and (64) in (40a) we see that

$$-\Delta p = [B_2 q - (\bar{S}_o + \beta)(K_c^*/K_s)] \Delta p_o - (K_c^*/K_m) \Delta \hat{p}. \quad (65)$$

Hence, if we add (63) and (65), we conclude that

$$\Delta \hat{p} = \chi \Delta p_o, \quad (66)$$

where

$$\chi = [K_m/(K_c^* - K_m)] [B_2 q + (\bar{S}_o + \beta)(1 - K_c^*/K_s)].$$

Set

$$r = (\bar{S}_o + \beta)/K_s + \chi/K_m.$$

Then it follows from (64) and (66) that

$$e = -r \Delta p_o. \quad (67)$$

Now we can use (61), (62), and (67) in (40b) and (40c) to derive the relations

$$\bar{S}_o + \beta = B_1 r - M_3 q, \quad (68a)$$

$$0 = B_2 r - M_2 q. \quad (68b)$$

From (59b) and (59c), (68a) and (68b) we obtain a system of four linear equations in the three unknowns M_1 , M_2 , and M_3 . Thus,

$$M_3 = -B_2 [1/K_m \delta + r/q] \quad (69a)$$

$$M_2 = B_2 (r/q), \quad (69b)$$

$$M_1 = -B_1/K_m \delta - M_3 \quad (69c)$$

provided the compatibility condition

$$(B_1 + B_2)r + B_2 q/K_m \delta - (\bar{S}_o + \beta) = 0 \quad (70)$$

is satisfied. To show the validity of (70), first we note that after some algebraic manipulations the factor r can be written in the more convenient form

$$r = (K_c^* - K_m)^{-1} [B_2 q - (\bar{S}_o + \beta) K_m \delta].$$

Also, it follows from (57) and (58) that

$$B_1 + B_2 = K_c^* \Theta \alpha.$$

Thus, Eq. (70) can be written in the equivalent form

$$[B_2 q/K_m \delta - (\bar{S}_o + \beta)] \times [1 + K_c^* \Theta \alpha K_m \delta / (K_c^* - K_m)] = 0,$$

which obviously holds since (55) implies that

$$1 + K_c^* \Theta \alpha K_m \delta / (K_c^* - K_m) = 0.$$

Thus, the elastic coefficients in the stress-strain relations (34) are completely determined by the shear modulus N of the solid matrix and the value provided by equations (56), (57), (58), and (69). It should be noted that the same relations for these constants result from interchanging the roles of the two fluids in the generalized partially jacketed compressibility test.

III. PARAMETER STUDY OF THE DIFFERENT TYPES OF WAVES

In this section we compute the elastic coefficients for two different formations saturated by mixtures of oil and water and gas and water, respectively, for different values of the saturation of the nonwetting phase (here taken to be oil or gas). Then the phase velocity and attenuation coefficient for each of the four types of waves will be computed as functions of the saturation of the nonwetting phase in the purely elastic case (i.e., ignoring dissipative effects) and frequency (for given saturations) in the case in which dissipation is included in the model.

The elastic coefficients in the stress-strain relations were computed for mixtures of oil and water and gas and water with the following properties (with ρ_θ , K_θ , and μ_θ denoting the density, bulk modulus, and viscosity of the θ fluid, respectively):

Oil

$$\begin{aligned} \rho_o &= 0.7 \text{ g/cm}^3 \\ K_o &= 0.57 \times 10^{10} \text{ dyn/cm}^2 \\ \mu_o &= 0.1 \text{ P} \end{aligned}$$

Gas

$$\begin{aligned} \rho_g &= 0.1 \text{ g/cm}^3 \\ K_g &= 0.022 \times 10^{10} \text{ dyn/cm}^2 \\ \mu_g &= 0.00015 \text{ P} \end{aligned}$$

Water

$$\begin{aligned} \rho_w &= 1.0 \text{ g/cm}^3 \\ K_w &= 2.25 \times 10^{10} \text{ dyn/cm}^2 \\ \mu_w &= 0.01 \text{ P} \end{aligned}$$

The coefficients were obtained for two different formations from data given in Ref. 11: (1) Berea sandstone, for which the porosity is 0.19, the permeability is 200 millidarcies, the matrix compressional velocity is 3.67 km/s, and the matrix shear velocity is 2.17 km/s; and (2) Fox Hills sandstone, for which the corresponding values are 0.074, 32.5 millidarcies,

4.45 km/s, and 2.515 km/s, respectively. For both formations, the bulk modulus K_s and density ρ_s of the solid grains were taken to be

$$\begin{aligned} K_s &= 37.9 \times 10^{10} \text{ dyn/cm}^2, \\ \rho_s &= 2.65 \text{ g/cm}^3. \end{aligned}$$

A capillary pressure curve $p_c(S_o)$ was obtained by approximating the capillary pressure curve in Ref. 10 on page 50 by an exponential function of the form

$$p_c(S_o) = p_{c_r} e^{-A S_o} (e^{-A S_o} - 1),$$

where

$$A = 6.029\ 158, S_{o_r} = 0.519,$$

$$p_{c_r} = 0.000\ 002\ 650\ 9 \times 10^{10} \text{ dyn/cm}^2.$$

This curve was used in the absence of proper data for the formations being treated and for both the oil and water and gas and water mixtures.

Tables I and II show values of the elastic coefficients for the Berea formation saturated by mixtures of oil and water and gas and water, and for different values of oil (or respectively, gas) saturation. All the coefficients are in units of 10^{10} dynes/cm². The shear modulus of the Berea formation is 10.107 65 (in the same units).

Table III gives values of the coefficients for the Fox Hills formation saturated by a mixture of oil and water. The shear modulus is 15.52147.

Now we proceed to analyze the behavior of the shear and compressional waves as functions of both saturation of the nonwetting phase and temporal frequency. All the results will be presented for the Berea formation saturated by mixtures of oil and water and gas and water, but a similar set of results and conclusions were obtained for the Fox Hills sandstone and other formations.

Using our straightforward generalization of the argument used in the single-phase theory,^{12,13} the mass-coupling parameters \bar{g}_1 , \bar{g}_2 and \bar{g}_3 in Ref. 1 were chosen to be

$$\bar{g}_2 = \bar{S}_w \rho_w F_s / \phi, \quad \bar{g}_3 = 0.1 \sqrt{\bar{g}_1 \bar{g}_2},$$

with $\bar{g}_1 = \bar{S}_o \rho_o F_s / \phi$ for the oil and water mixture and $\bar{g}_1 = \bar{S}_g \rho_g F_s / \phi$ for the gas and water mixture. The parameter F_s is known as the structure factor and was selected here as 2.8 (Refs. 12, 13). The numerical experiments showed that the phase velocities and attenuations for all types of waves increased when lower values of F_s were employed. Changes in the choice of \bar{g}_3 showed almost no change in the values of velocities and attenuation coefficients. To compute the velocities in the dissipative case we needed values for the relative permeabilities k_{r_o} and k_{r_w} . Again in the absence of proper data for the Berea formation, the values were taken from the relative permeability curves in Ref. 10 on page 161. For both the oil and water and gas and water mixtures and a nonwetting phase saturation of 0.6 (i.e., $S_o = 0.6$ or

TABLE II. Elastic coefficients for the Berea sandstone saturated by a gas-water mixture.

S_g	K_c^*	B_1	B_2	M_1	M_2	M_3
0.7	15.4919	0.068 22	0.029 25	0.080 56	0.014 82	0.034 53
0.6	15.5014	0.068 05	0.045 38	0.068 88	0.030 64	0.045 92
0.5	15.5145	0.067 80	0.067 82	0.057 19	0.057 21	0.057 19
0.4	15.5341	0.067 44	0.101 17	0.045 51	0.102 41	0.068 26
0.3	15.5662	0.066 83	0.155 96	0.033 82	0.184 18	0.078 93
0.01	17.3798	0.032 82	3.249 56	0.000 55	5.427 19	0.054 82

$S_g = 0.6$) we chose $k_{r_o} = k_{r_g} = 0.4$ and $k_{r_w} = 0.04$, while for $S_o = 0.01$ or $S_g = 0.01$ we chose $k_{r_o} = k_{r_g} = 0.001$ and $k_{r_w} = 0.99$. Following Ref. 1, for the compressional waves we first computed the eigenvalue $[\alpha^{(j)}]^2, j = \text{I, II, III}$, of the generalized eigenvalue problem

$$\tilde{\mathcal{C}} q^{(a)} = \alpha^2 [\tilde{\mathcal{A}} q^{(a)} - i \tilde{\mathcal{D}} q^{(a)}],$$

with $\alpha = c/a$ and $c = 2\pi f, f$ denoting frequency, and $a = a_r + ia_i$ the wavenumber.

For the shear waves, we computed the wavenumbers $a^{(s)} = a_r^{(s)} + ia_i^{(s)}$ using formula (59) in Ref. 1.

Then the phase velocities for all types of waves were computed by the formula

$$v^{(j)} = c/|a_r^{(j)}|, \quad j = \text{I, II, III, s}. \quad (71)$$

Next, following Ref. 11, instead of the attenuation coefficient $a_i^{(j)}$ we defined and plotted another attenuation coefficient, measured in dB/Hz s, related to the $a_i^{(j)}$'s by the formula

$$b^{(j)} = 20 \log(e) a_i^{(j)} \frac{v^{(j)}}{f} \\ = 2\pi \cdot 8.685\ 889 a_i^{(j)} / |a_r^{(j)}|, \quad j = \text{I, II, III, s}. \quad (72)$$

For each frequency $f, b^{(j)}$ measures the attenuation suffered by the wave after traveling one wavelength.

Figures 1–4 show the phase velocity for each type of wave in the purely elastic case as a function of the saturation of the nonwetting phase. In Figs. 1, 2, and 4 we observe that for low saturations, the phase velocities for compressional waves of types I and II and the shear waves tend to the corresponding values for a single-phase, water-saturated porous solid. In Fig. 1, the different behavior of the wave of type I for each of the mixtures corresponds exactly to a similar behavior for the reference compressional velocity v_c associated with this wave, which is defined to be²

TABLE I. Elastic coefficients for the Berea sandstone saturated by an oil-water mixture.

S_o	K_c^*	B_1	B_2	M_1	M_2	M_3
0.7	16.7389	1.540 83	0.660 36	1.819 57	0.334 23	0.779 81
0.6	16.8718	1.455 20	0.970 15	1.472 96	0.654 67	0.981 97
0.5	17.0348	1.350 17	1.350 17	1.138 87	1.138 88	1.138 86
0.4	17.2395	1.218 26	1.827 40	0.822 08	1.849 70	1.233 12
0.3	17.5042	1.047 67	2.444 57	0.530 23	2.886 80	1.237 20
0.01	19.0357	0.060 76	6.015 14	0.001 02	10.046 1	0.101 48

TABLE III. Elastic coefficients for the Fox Hills sandstone saturated by an oil-water mixture.

S_o	K_c^*	B_1	B_2	M_1	M_2	M_3
0.7	28.5566	1.7468	0.748 64	4.633 36	0.851 08	1.985 71
0.6	28.6231	1.64834	1.098 91	3.747 62	1.665 66	2.498 40
0.5	28.7044	1.52779	1.527 80	2.894 61	2.894 65	2.894 60
0.4	28.8064	1.37676	2.065 14	2.086 76	4.695 24	3.130 13
0.3	28.9378	1.182	2.758 02	1.343 68	7.3156	3.135 25
0.01	29.6899	0.06789	6.721 68	0.002 57	25.2155	0.254 70

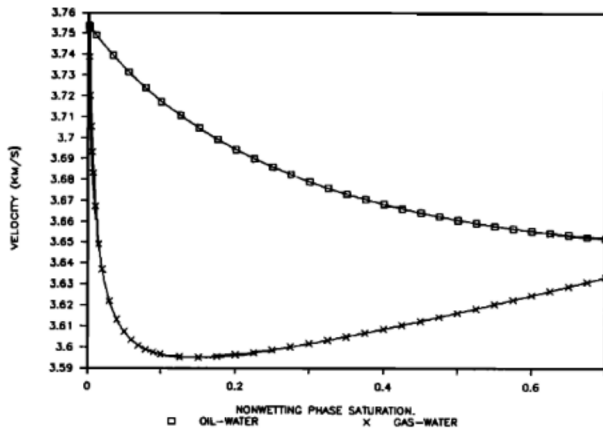


FIG. 1. Type I wave phase velocity.

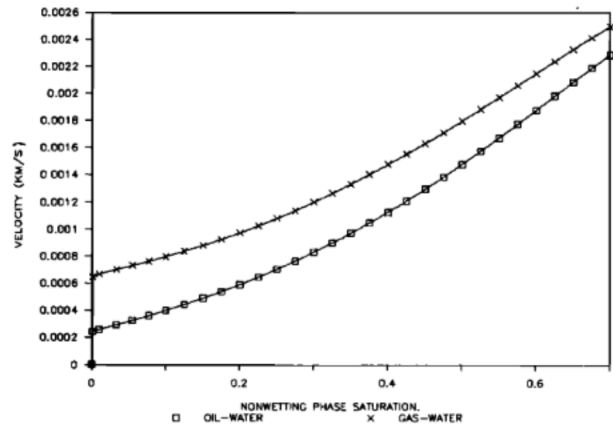


FIG. 3. Type III wave phase velocity.

$$v_c = \sqrt{(K_c + 4/3N)/\rho}$$
,
 with $\rho = (1 - \bar{\phi})\rho_s + \bar{\phi}(\rho_o\bar{S}_o + \rho_w\bar{S}_w)$ being the density of the bulk material.

The wave of type II in Fig. 2 shows a significant change in phase velocity values between the two mixtures. The wave corresponding to the gas and water mixture is slower, since the relative motion, to which this mode is associated, should be smaller for lower density fluids.

In Fig. 3 it can be observed that the phase velocity of the compressional wave of type III is an increasing function of the saturation of the nonwetting phase and tends to zero for low saturations. This is an expected result since the wave is directly associated with the existence of capillary pressure.

Figure 4 shows that the shear wave phase velocity for both mixtures increases as a function of the saturation of the nonwetting phase. The higher values of the phase velocity for the gas-water mixture are explained by the fact that, for a fixed saturation value, the bulk density is smaller for the gas-water than for oil-water mixture.

Figures 5, 6, 7, and 8 show the phase velocity $v^{(j)}$ in (71) for compressional waves of types I, II, and III and shear waves, respectively, in the case in which dissipation is

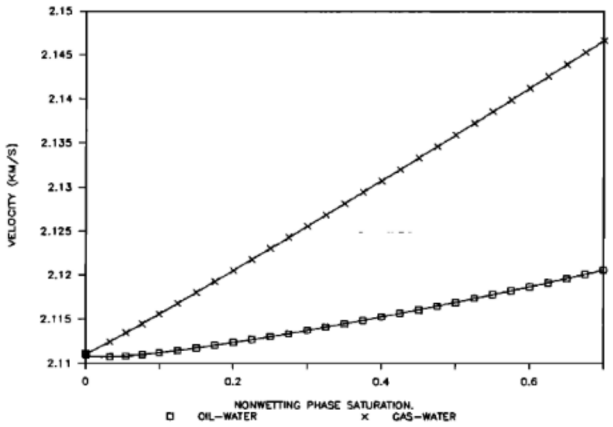


FIG. 4. Shear wave phase velocity.

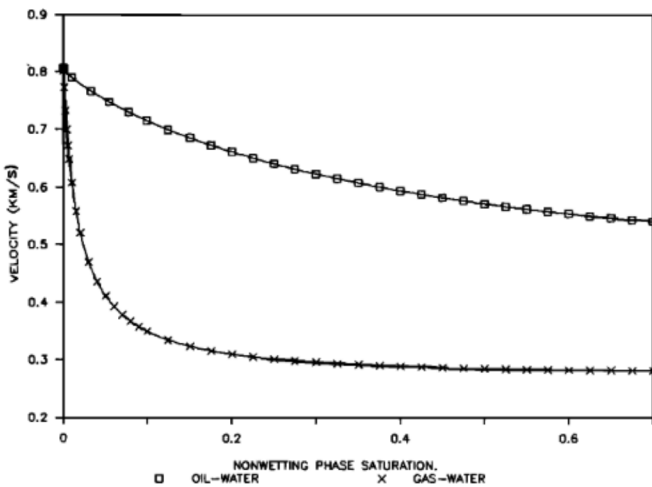


FIG. 2. Type II wave phase velocity.

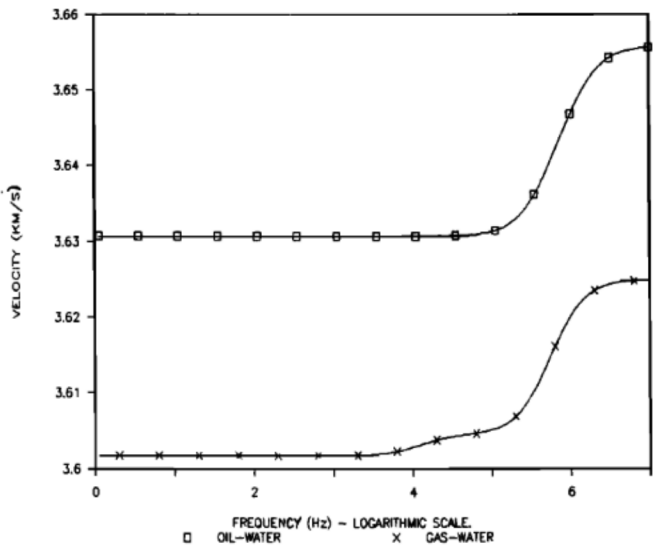


FIG. 5. Type I wave phase velocity.

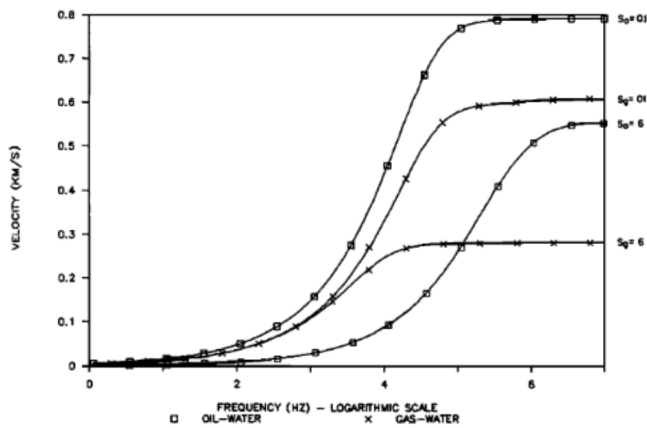


FIG. 6. Type II wave phase velocity.

considered in the model, for frequencies in the range 1 Hz–10 MHz and for fixed values of the nonwetting phase saturation, chosen to be 0.6 for waves of type I and III and shear waves and 0.6 and 0.01 for type II waves. (It is not clear that the model is correct at very high frequencies. It is clear from the derivation of the single-phase Biot model given by Burredge and Keller⁶ that the dissipation related to Darcy's law should be frequency-dependent for high frequencies; Biot² also noted this dependence. Thus, the results in the upper frequencies presented in the figures are more mathematical than physical in nature.)

For waves of type I and shear waves (Figs. 5 and 8) very little dispersion is observed over the whole range of frequencies. For higher frequencies the phase velocities first increase and then stabilize the corresponding values for the purely elastic case (Figs. 1 and 4). At zero frequency the phase velocity of the type I wave tends to the reference value v_c defined above, while the shear wave velocity tends to the value $\sqrt{N/\rho}$.

A completely different behavior is observed for the waves of types II and III. In Fig. 6 we plotted the phase velocities for the type II wave for oil and water and gas and

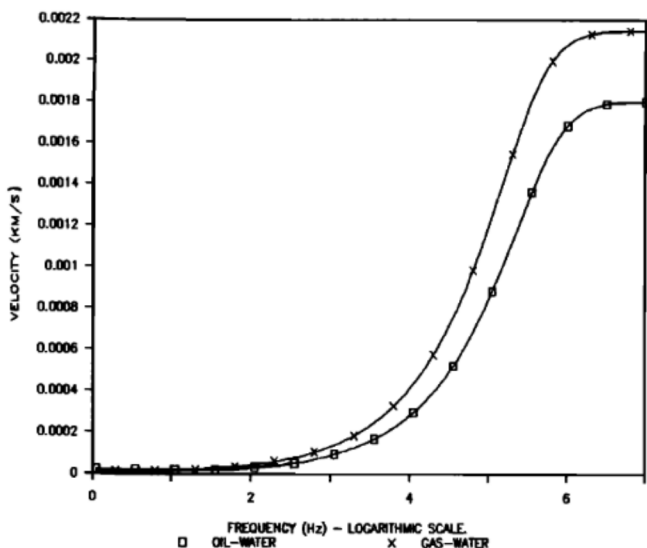


FIG. 7. Type III wave phase velocity.

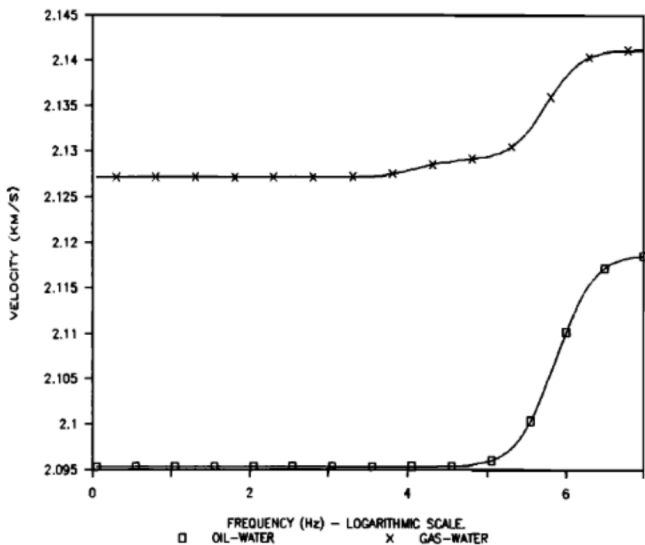


FIG. 8. Shear wave phase velocity.

water mixtures of saturations 0.01 and 0.6. It can be seen that there exists noticeable dispersion even in the low-frequency range, and the phase velocities tend to zero for frequencies tending to zero. Note that for low frequencies and saturation tending to 0.6 the wave associated with the gas and water mixture shows higher phase velocities than those corresponding to the oil and water mixture, due to the lower viscosity of the gas. At high frequencies, the curves stabilize at frequencies where the inertial terms are dominant with respect to the viscous terms associated with the dissipative effects.

The waves of type III (Fig. 7) have almost zero velocity in the low-frequency range. At approximately 100 Hz they start to show some dispersion, increasing with increasing frequencies. As before, for high frequencies, the phase velocities of the waves of types II and III for both mixtures tend to limits which coincide with the values for the purely elastic case for the corresponding saturations. In Figs. 9–12 we have

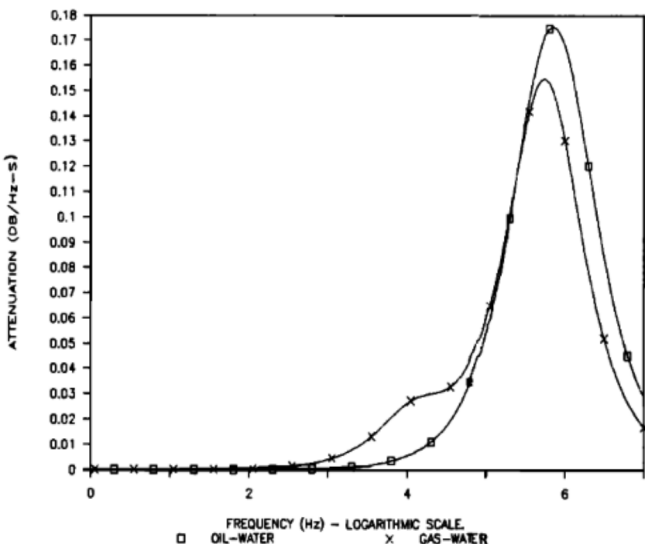


FIG. 9. Type I wave attenuation.

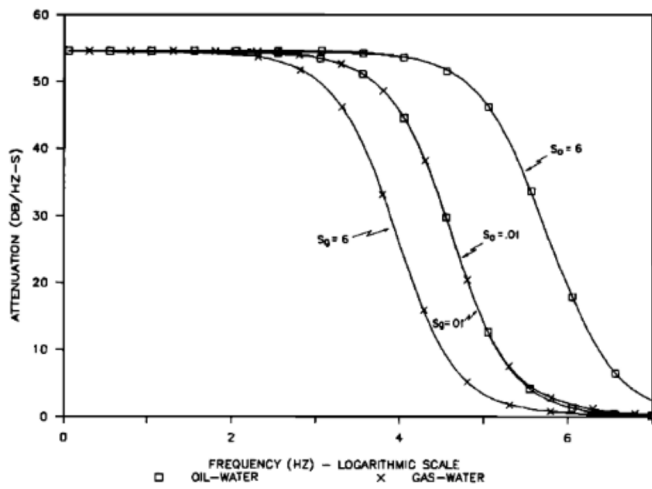


FIG. 10. Type II wave attenuation.

plotted the attenuation coefficients $b^{(j)}$ in (72) for the four types of waves as functions of frequency and for the same saturation values as in Figs. 5–8. For the wave of type I (Fig. 9) the attenuation coefficient $b^{(I)}$ is almost zero in the low-frequency range and very small for high frequencies, with peaks of approximately 0.175 dB/Hz s and 0.155 dB/Hz s for the oil and water and gas and water mixtures, respectively. The attenuation coefficient $b^{(S)}$ for the shear waves shows almost the same behavior (Fig. 12), although for the oil and water mixture the attenuation reaches a peak of about 0.283 dB/Hz s, which is about double that for the corresponding peak for the type I wave. For the gas and water mixture both coefficients $b^{(I)}$ and $b^{(S)}$ are quite similar in magnitude and frequency dependence.

Again, the attenuation coefficients $b^{(II)}$ and $b^{(III)}$ show a completely different behavior. For the wave of type II (Fig. 10) and saturation 0.01, the attenuation coefficient is practically the same for both mixtures, being very high (approximately 54 dB/Hz s) in the range 1–100 Hz. Then it decays rapidly tending to zero at the 10 MHz frequency. For saturation

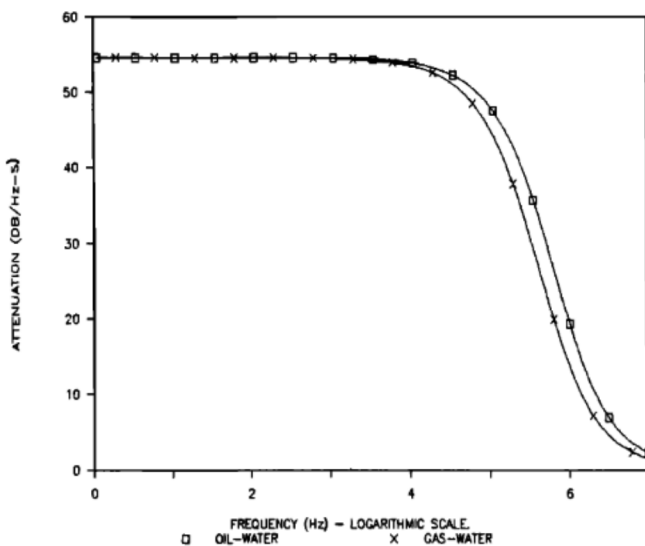


FIG. 11. Type III wave attenuation.

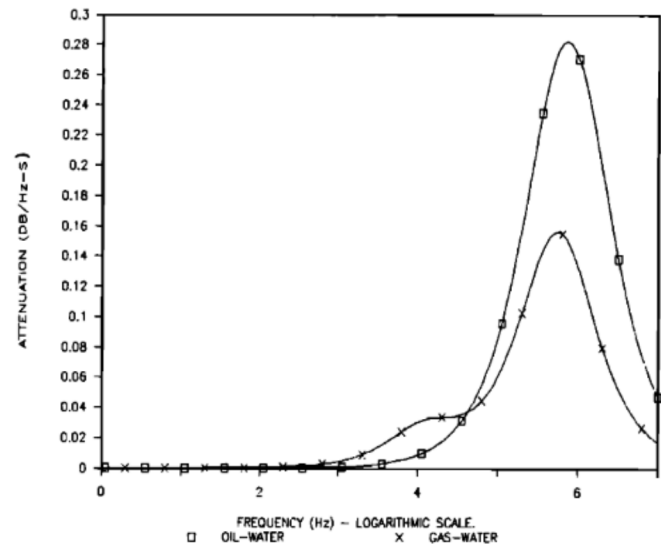


FIG. 12. Shear wave attenuation.

0.6, the attenuation is again approximately 54 dB/Hz s in the ranges 1–100 Hz for the gas–water mixture and 1 Hz–10 kHz for the oil–water mixture, respectively. Above these limits the coefficients start to decay, reaching almost zero in the case of the gas–water mixture. Finally, the wave of type III (Fig. 11) is strongly attenuated in the range 1 Hz–10 kHz. Then the attenuation coefficient $b^{(III)}$ decays to approximately 2 dB/Hz s for both mixtures at a frequency of 10 MHz.

We can conclude that the waves of types I and II and shear waves behave in general as the corresponding waves in the single-phase theory (Ref. 11). Also, the waves of type II and III have similar behavior, being diffusion-type waves. The low attenuation values and the steady behavior of the phase velocity curves for all types of waves in both mixtures at high frequencies indicate that viscous effects are negligible over the corresponding ranges. Finally, it is obvious from the behavior of the type II wave shown in Figs. 6 and 10 that an accurate measurement of the phase velocity in the high-frequency range would allow us to distinguish a single-phase water saturated from an oil and water or gas and water saturated porous medium.

ACKNOWLEDGMENT

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