DIFFUSION LENGTH CALCULATION

(1)

(Ref: Haines Pride, Geophysics, 71, 2006 p. N57–N65)

J. SANTOS Feb 19, 2009

(1) $\rho \ddot{u}^{s} + \rho_f \ddot{u}^{f} = \nabla \cdot \tau$

(2) $\rho_f \ddot{u}^{s} + g \ddot{u}^{f} + \frac{\eta}{k} \dot{u}^{f} = -\nabla p_f$

(3) $\tau_{ij} = \left[\lambda_c \nabla \cdot u^s + \alpha K_{2v} \nabla \cdot u^f\right] \delta_{ij} + 2\mu\, \varepsilon_{ij}(u^s)$

(4) $p_f = -\alpha K_{2v} \nabla \cdot u^s - K_{2v} \nabla \cdot u^f$

Assume small accelerations and $u^s \simeq 0$

$\nabla \cdot u^s \approx 0$ (low frequency)

Then taking time derivative in (4)

(5) $\frac{\partial p_f}{\partial t} = -K_{2v} \nabla \cdot \frac{\partial u^f}{\partial t}$

Also, take divergence in (2) to get

(6) $\frac{\eta}{k} \nabla \cdot \frac{\partial u^f}{\partial t} = -\nabla \cdot \left(\nabla \frac{\partial p_f}{\partial t}\right) = -\Delta p_f$

Then using (6) in (5)

(7) $$\frac{\partial p_f}{\partial t} = -K_{2v}\left(-\frac{K}{\eta}\Delta p_f\right) = \frac{K_{2v}K}{\eta}\Delta p_f$$

Set (8) $$D = \frac{K_{2v}K}{\eta} = \text{diffusion coefficient}$$

The diffusion length is defined as

(9) $$L_D = \sqrt{\frac{D}{\omega}} = \sqrt{\frac{K_{2v}K}{\eta\,\omega}}$$

This is also called the BIOT SKIN DEPTH δ_{BIOT}. Then

(10) $$\delta_{BIOT} \simeq \sqrt{\frac{K_{2v}K}{\eta\,\omega}}$$

ω may be taken the main angular frequency of the pulse —

The fluid pressure equation is

(11) $$\frac{\partial p_f}{\partial t} = D\,\Delta p_f$$

3

The solution of (11) is (in 1D)

$$\rho_f(x,t) = \frac{\rho_{f,0}}{\sqrt{4\pi D}} e^{-\frac{x^2}{4Dt}}$$

$$\tau = \frac{x^2}{4D} = \text{characteristic time}$$

$$f_B = \frac{1}{\tau} = \text{characteristic frequency}$$