A Numerical Rocks Physics Approach to Model Wave Propagation in Hydrocarbon Reservoirs

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Juan E. Santos,

Instituto del Gas y del Petróleo (IGPUBA), Universidad de Buenos Aires (UBA), Argentina, Department of Mathematics, Purdue University, West Lafayette, Indiana, USA, and Universidad Nacional de La Plata (UNLP), Argentina.

In collaboration with P. M. Gauzellino (UNLP), G. B. Savioli (UBA), A. Sanchez Camus (UNLP) and R. Martinez Corredor, (UNLP).

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Seismic waves in fluid-saturated poroelastic materials. I

- Fast compressional or shear waves travelling through a fluid-saturated porous material (a Biot medium) containing heterogeneities on the order of centimeters (mesoscopic scale) suffer attenuation and dispersion observed in seismic data.
- The mesoscopic loss effect occurs because different regions of the medium may undergo different strains and fluid pressures.
- This in turn induces fluid flow and Biot slow waves causing energy losses and velocity dispersion due to energy transfer between wave modes.

Seismic waves in fluid-saturated poroelastic materials. II

- Since extremely fine meshes are needed to represent these type of mesoscopic-scale heterogeneities, numerical simulations are very expensive or not feasible.
- Alternative: In the context of Numerical Rock Physics, perform compressibility and shear time-harmonic experiments to determine a long-wave equivalent viscoelastic medium to a highly heterogeneous Biot medium.
- This viscoelastic medium has in the average the same attenuation and velocity dispersion than the highly heterogeneous Biot medium.
- Each experiment is associated with a Boundary Value Problem (BVP) that is solved using the Finite Element Method (FEM).

Biot's equations in the diffusive range of frequencies.

Frequency-domain stress-strain relations in a Biot medium

$$\tau_{kl}(\mathbf{u}) = 2G\,\epsilon_{kl}(\mathbf{u}^s) + \delta_{kl}\left(\lambda_u\,\nabla\cdot\mathbf{u}^s + B\nabla\cdot\mathbf{u}^f\right),$$
$$p_f(\mathbf{u}) = -B\nabla\cdot\mathbf{u}^s - M\nabla\cdot\mathbf{u}^f,$$

$\mathbf{u} = (\mathbf{u}^s, \mathbf{u}^f)$, $\mathbf{u}^s = (u_1^s, u_3^s), \mathbf{u}^f = (u_1^f, u_3^f)$.

Biot's equations in the diffusive range:

$$\nabla\cdot\tau(\mathbf{u}) = 0,$$
$$\mathrm{i}\omega\mu\kappa^{-1}\mathbf{u}^f + \nabla p_f(\mathbf{u}) = 0,$$

μ: fluid viscosity, κ:frame permeability.

The complex P-wave modulus of the long-wave equivalent viscoelastic medium. I

Biots' s equations are be solved in the 2-D case on square sample $\Omega = (0, L)^2$ with boundary $\Gamma = \Gamma^L \cup \Gamma^B \cup \Gamma^R \cup \Gamma^T$ in the (x_1, x_3)-plane. The domain Ω is a representative sample of our fluid saturated poroelastic material.

$$\Gamma^L = \{(x_1, x_3) \in \Gamma : x_1 = 0\}, \quad \Gamma^R = \{(x_1, x_3) \in \Gamma : x_1 = L\},$$
$$\Gamma^B = \{(x_1, x_3) \in \Gamma : x_3 = 0\}, \quad \Gamma^T = \{(x_1, x_3) \in \Gamma : x_3 = L\}.$$

For determining the complex plane wave modulus, we solve Biots' s equations with the boundary conditions

$$\tau(\mathbf{u})\nu \cdot \nu = -\Delta P, \quad (x_1, x_3) \in \Gamma^T,$$
$$\tau(\mathbf{u})\nu \cdot \chi = 0, \quad (x_1, x_3) \in \Gamma,$$
$$\mathbf{u}^s \cdot \nu = 0, \quad (x_1, x_3) \in \Gamma^L \cup \Gamma^R \cup \Gamma^B,$$
$$\mathbf{u}^f \cdot \nu = 0, \quad (x_1, x_3) \in \Gamma.$$

The complex P-wave modulus of the long-wave equivalent viscoelastic medium. II

The *equivalent* undrained complex plane-wave modulus $\overline{E_u}(\omega)$ is determined by the relation

$$\frac{\Delta V(\omega)}{V} = -\frac{\Delta P}{\overline{E_u}(\omega)},$$

valid for a viscoelastic homogeneous medium in the quasi-static case. V: original volume of the sample. Then to approximate $\Delta V(\omega)$ use

$$\Delta V(\omega) \approx L u_3^{s,T}(\omega),$$

$u_3^{s,T}(\omega)$: average vertical solid displacements $u_3^s(x_1, L, \omega)$ on Γ^T.

The complex shear modulus of the long-wave equivalent viscoelastic medium. I

Solve Biots' s equations with the boundary conditions

$$-\tau(\mathbf{u})\nu = \mathbf{g}, \quad (x_1, x_3) \in \Gamma^T \cup \Gamma^L \cup \Gamma^R,$$
$$\mathbf{u}^s = 0, \quad (x, y) \in \Gamma^B,$$
$$\mathbf{u}^f \cdot \nu = 0, \quad (x, y) \in \Gamma,$$

$$\mathbf{g} = \begin{cases} (0, \Delta G), & (x_1, x_3) \in \Gamma^L, \\ (0, -\Delta G), & (x_1, x_3) \in \Gamma^R, \\ (-\Delta G, 0), & (x_1, x_3) \in \Gamma^T. \end{cases}$$

The change in shape of the rock sample allows to recover its *equivalent* complex shear modulus $\overline{G}_u(\omega)$ using the relation

$$\mathrm{tg}(\theta(\omega)) = \frac{\Delta T}{\overline{G}_u(\omega)},$$

$\theta(\omega)$: departure angle from the original positions of the lateral boundaries

The complex shear modulus of the long-wave equivalent viscoelastic medium. II

To find an approximation to $\mathrm{tg}(\theta(\omega))$, compute the average horizontal displacement $u_1^{s,T}(\omega)$ of the horizontal displacements $u_1^s(x_1, L, \omega)$ at the top boundary Γ^T. Then use

$$\mathrm{tg}(\theta(\omega)) \approx u_1^{s,T}(\omega)/L,$$

that allows to determine the shear modulus $\overline{G}_u(\omega)$

Phase velocities and Quality factors.

The complex P-wave and shear velocities are

$$v_{sc}(\omega) = \sqrt{\frac{\overline{G_u}(\omega)}{\overline{\rho}}} \quad v_{pc}(\omega) = \sqrt{\frac{\overline{E_u}(\omega)}{\overline{\rho}}},$$

The compressional phase velocities $v_p(\omega), v_s(\omega)$ and quality factor $Q_p(\omega), Q_s(\omega)$ are

$$v_p(\omega) = \left[\mathrm{Re}\left(\frac{1}{v_{pc}(\omega)}\right)\right]^{-1}, \quad \frac{1}{Q_p(\omega)} = \frac{\mathrm{Im}(v_{pc}(\omega)^2)}{\mathrm{Re}(v_{pc}(\omega)^2)}.$$

$$v_s(\omega) = \left[\mathrm{Re}\left(\frac{1}{v_{sc}(\omega)}\right)\right]^{-1}, \quad \frac{1}{Q_s(\omega)} = \frac{\mathrm{Im}(v_{sc}(\omega)^2)}{\mathrm{Re}(v_{sc}(\omega)^2)}.$$

Schematic representation of the experiments to determine the complex P-wave and shear modulus

(a) (b)

Figures (a) show how to determine $\overline{E}_u(\omega)$, (b) show how to determine $\overline{G}_u(\omega)$.