

Analysis of capillary pressure effect on the seismic response of a CO₂-storage site applying multiphase flow and wave propagation simulators

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Abstract

We analyze the influence of capillary pressure on the seismic response of a saline aquifer, where CO₂ has been stored in the Utsira Sand at the Sleipner field. For this purpose, we present a novel methodology integrating numerical simulation of CO₂-brine flow and seismic wave propagation, using a geological model that includes mudstone layers and natural apertures. The simultaneous flow of CO₂ and brine in an aquifer is modeled by the differential equations that describe the two-phase fluid flow in porous media. The multiphase flow functions are determined from well-log data, using the relation between resistivity index, relative permeabilities and capillary pressure. Seismic monitoring is performed with a wave equation that includes attenuation and dispersion effects due to mesoscopic scale heterogeneities in the petrophysical and fluid properties. The fluid simulator properly models the CO₂ injection and upward migration, obtaining accumulations below the mudstone layers as injection proceeds. Moreover, we are able to identify the time-lapse distribution of CO₂ from the synthetic seismograms, which show the typical pushdown effect due

to the spatial distribution of CO₂. Finally, a sensitivity analysis is performed by modifying the capillary pressure threshold in order to evaluate its effect over the CO₂ plume and the corresponding synthetic seismogram.

Keywords: CO₂ Sequestration, Multiphase Flow Functions, Fluid Flow Simulation, Seismic Monitoring

1. Introduction

The first industrial CO₂ injection project began in 1996 at the Sleipner gas field in Norway [1]. In this project, CO₂ separated from natural gas is being injected in the Utsira Sand, a high permeable sandstone, with several mudstone layers that limit the vertical migration of the CO₂ [2]-[3]. Numerical modeling of CO₂ injection and storage is an important tool to analyze the effectiveness of this procedure in reducing the amount of greenhouse gases in the atmosphere [2].

We introduce a methodology to simulate the CO₂ injection and storage in saline aquifers, combined with a petrophysical model of the Utsira Sand in order to compare our results against actual data obtained from an undergoing CO₂ storage project. The petrophysical model is based on initial porosity and clay content and considers the variation of rock properties with pore pressure and saturation [4]. Therefore, these properties are time dependent due to the CO₂ injection, but they change at a much slower rate than pressure and saturations. As a consequence, we have two time scales, and we use a larger time step to update petrophysical properties than to run the flow simulator. The time increment for updating was chosen empirically, as the maximum time step at which the saturation distributions remain unchanged (30 days).

The initial porosity is assumed to have a fractal spatial distribution [5] and the permeability is assumed to be anisotropic, obtained from first principles as a function of porosity and grain sizes.

Several authors have conducted extensive analysis in order to determine the topographic, stratigraphic and petrophysical characterizations of the Utsira

25 Sand based on well-log, 2D and 3D-seismic data, as well as time-lapse seismic
26 imaging of the CO₂ plume[6]-[7]-[8]. Our model for the Utsira Sand takes into
27 account these findings, which makes it closer to the actual properties.

28 This geological model can simulate embedded mudstone layers of very low
29 permeability. Brine and CO₂ relative permeabilities and capillary pressure are
30 determined from well-log data, using the relation between resistivity index, rel-
31 ative permeabilities and capillary pressure [9].

32 The Black-Oil formulation for two-phase flow in porous media [10] is used
33 to simulate the CO₂ - brine flow. The numerical solution is obtained with the
34 public-domain software BOAST [11], which solves the differential equations by
35 finite differences with the IMPES algorithm (IMplicit Pressure Explicit Satu-
36 ration) [10]. The basic idea of IMPES is to obtain a single pressure equation
37 combining the flow equations. Once pressure is implicitly computed for the
38 new time, saturation is then updated explicitly. After applying the fluid flow
39 model to obtain the distribution for CO₂ saturation and pressure within the
40 saline aquifer, we use this information as an input to our own wave propagation
41 simulator in order to obtain the corresponding synthetic seismograms.

42 The wave propagation is based on an isotropic viscoelastic model that con-
43 sider dispersion and attenuation effects. The complex P- and S-wave moduli
44 are determined using the Zener model to represent the viscoelastic behavior of
45 the material in the brine saturated mudstone layers [12]. Outside the mud-
46 stone layers, we use White's theory [13] to model P-wave attenuation based on
47 a model of porous layers alternately saturated with brine and CO₂. S-wave at-
48 tenuation is also taken into account by making the shear modulus complex and
49 frequency dependent using another relaxation mechanism related to the P-wave
50 attenuation model [14].

51 Summarizing, in this paper we present a new approach that combines fluid
52 flow and wave propagation simulation for CO₂ storage inside a saline aquifer.
53 The results of the flow simulator applied to the petrophysical (poro-viscoelastic)
54 model, allows us to calculate the phase velocities and attenuation coefficients
55 of the P- and S-waves. The wave propagation model needs these velocities and

56 coefficients to obtain the corresponding seismic images.

57 Also, we use this approach to evaluate the effect of the capillary pressure on
 58 the propagation of the CO₂ plume and its corresponding synthetic seismograms.

59 2. Model Description

60 In this work we present a new approach that consists in a chained set of
 61 models that produce the storage site seismic images that approximate the real
 62 ones, based on real data. First, we obtain the distribution of CO₂ and brine sat-
 63 urations and pressures in an aquifer by applying a fluid flow simulator. Then, we
 64 feed that information to the petrophysical model which yields updated porosity,
 65 permeabilities, dry-rock bulk and shear moduli and computed values of veloc-
 66 ity and attenuation of wave propagation in this media. Finally, we take those
 67 parameters and run our own wave propagation simulator to obtain the seismic
 68 response.

69 2.1. CO₂ Injection Model

70 Our first step is to model the simultaneous flow of CO₂ and brine in an
 71 aquifer. This is achieved by solving the differential equations that describe
 72 the two-phase fluid flow in porous media [10]. These equations, obtained by
 73 combining the mass conservation equations with Darcy's empirical law, are

$$\begin{aligned} \nabla \cdot \left(\kappa \left(\frac{k_{rg}}{B_g \eta_g} (\nabla p_g - \rho_g g \nabla z) + \frac{R_s k_{rb}}{B_b \eta_b} (\nabla p_b - \rho_b g \nabla z) \right) \right) + \frac{q_g}{\rho_g^{SC}} \quad (1) \\ = \frac{\partial \left[\phi \left(\frac{S_g}{B_g} + \frac{R_s S_b}{B_b} \right) \right]}{\partial t}, \end{aligned}$$

$$\nabla \cdot \left(\kappa \frac{k_{rb}}{B_b \eta_b} (\nabla p_b - \rho_b g \nabla z) \right) + \frac{q_b}{\rho_b^{SC}} = \frac{\partial \left[\phi \frac{S_b}{B_b} \right]}{\partial t}, \quad (2)$$

74 where g , b denote CO₂ and brine phases, respectively and the unknowns are the
 75 fluid pressures p_β and saturations S_β ($\beta = b, g$). Also ρ_β is the density, ρ_β^{SC} is
 76 the density at standard conditions, q_β the injection mass rate per unit volume,

77 $k_{r\beta}$ the relative permeability, η_β the viscosity, ϕ the porosity, $\underline{\kappa}$ the absolute
 78 permeability tensor, g the gravity constant and z the depth. Finally, R_s , B_g
 79 and B_b are the PVT parameters, which are the CO₂ solubility in brine, CO₂
 80 formation volume factor and brine formation factor respectively.

81 These equations were obtained assuming that the CO₂ component may dis-
 82 solve in the brine phase, but the brine is not allowed to vaporize into the CO₂
 83 phase.

84 Two algebraic equations relating the saturations and pressures complete the
 85 system:

$$S_b + S_g = 1, \quad p_g - p_b = P_C(S_b), \quad (3)$$

86 where $P_C(S_b)$ is the capillary pressure function.

87 Appendix A describes how these differential equations are obtained and
 88 solved.

89 2.1.1. Multiphase flow functions

90 Two types of multiphase flow functions appear on the system of equations
 91 (Eq. 1, Eq. 2 and Eq. 3): relative permeability and capillary pressure, where
 92 both are function of saturation. In order to obtain these parameters for CO₂
 93 and brine, we compute the resistivity index from actual well-log data and then
 94 apply a procedure that correlates all three variables [9].

95 The resistivity index ($RI(S_b)$) is computed from the conductivity measure-
 96 ments as

$$RI(S_b) = \frac{\sigma(1)}{\sigma(S_b)}, \quad (4)$$

97 where $\sigma(1)$ is the conductivity at full brine saturated rock obtained from well-
 98 log data and $\sigma(S_b)$ is the conductivity at saturation S_b , which is determined as
 99 follows [15]:

$$\sigma(S_b) = (1 - \phi)\sigma_q^\gamma + \phi(1 - S_b)\sigma_g^\gamma + \phi S_b \sigma_b^\gamma, \quad (5)$$

100 where σ_b , σ_g and σ_q are the brine, CO₂ and grain conductivities, respectively.
 101 Moreover, γ is a free parameter assumed equal to 0.5 according to the CRIM
 102 model [16].

103 Once the resistivity index is calculated, the relative permeability curves can
 104 be obtained as functions of brine saturations as

$$k_{rb}(S_b) = \frac{\Lambda(S_b)}{RI(S_b)}, \quad (6)$$

105

$$k_{rg}(S_b) = (1 - \Lambda(S_b))^2 [1 - \kappa_{rb}(S_b)], \quad (7)$$

106 with

$$\Lambda(S_b) = \frac{S_b - S_{rb}}{1 - S_{rb}}, \quad (8)$$

107 being S_{rb} the residual brine saturation.

108 To obtain the capillary pressure function ($P_c(S_b)$) it is necessary to adjust
 109 the brine relative permeability curve with a free parameter n ,

$$\kappa_{rb}(S_b) = \frac{\Lambda(S_b)}{RI(S_b)} = \Lambda(S_b) \left(\frac{2+n}{n} \right). \quad (9)$$

110 Then, $P_c(S_b)$ can be represented using a potential model as follows.

$$P_{cD}(S_b) = \Lambda(S_b)^{(-1/n)}, \quad (10)$$

111 with

$$P_{cD}(S_b) = \frac{P_c(S_b)}{P_{ce}}, \quad (11)$$

112 where P_{cD} is the dimensionless capillary pressure and P_{ce} is the entry or thresh-
 113 old pressure, that is to say, the pressure at which the CO₂ phase is sufficiently
 114 connected to allow flow. This parameter was used to perform the capillary
 115 pressure sensitivity analysis presented in this work.

116 2.2. Petrophysical model

117 This section describes the procedure used to determine the petrophysical
 118 and fluid flow parameters needed in the CO₂ injection and seismic simulations.
 119 Firstly, it is used to calculate the aquifer properties update during CO₂ injection.
 120 Secondly, it allows us to calculate the phase velocities and quality factors for the
 121 P- and S-waves, which are necessary to run the wave propagation simulation.

122 The porosity pressure dependence is based on the following relationship be-
 123 tween porosity and average pore pressure $p(t) = S_b p_b(t) + S_g p_g(t)$,

$$\frac{(1 - \phi_c)}{K_{solid}}(p(t) - p_H) = \phi_0 - \phi(t) + \phi_c \ln \frac{\phi(t)}{\phi_0}, \quad (12)$$

124 where ϕ_c is a critical porosity, ϕ_0 is the initial porosity at hydrostatic pore
 125 pressure p_H and K_{solid} is the bulk modulus of the solid grains composing the dry
 126 porous matrix [4]. The rock is assumed to have two components: quartz (bulk
 127 modulus of 40 GPa) and clay (bulk modulus of 15 GPa). K_{solid} is computed as
 128 the arithmetic average of the Hashin Shtrikman upper and lower bounds.

129 The relationship among horizontal permeability κ_x , porosity and clay con-
 130 tent C is [4],

$$\frac{1}{\kappa_x(t)} = \frac{45(1 - \phi(t))^2}{\phi(t)^3} \left(\frac{(1 - C)^2}{R_q^2} + \frac{C^2}{R_c^2} \right), \quad (13)$$

131 where R_q and R_c are the average radii of the sand and clay grains. As absolute
 132 permeability is anisotropic, we assume the following relationship between the
 133 horizontal and vertical permeability κ_z [4],

$$\frac{\kappa_x(t)}{\kappa_z(t)} = \frac{1 - (1 - 0.3a) \sin \pi S_b}{a(1 - 0.5 \sin \pi S_b)}, \quad (14)$$

134 being a the permeability-anisotropy parameter.

135 The dry-rock bulk modulus, K_{dry} , is computed using the Krief equation [17]
 136 as follows.

$$K_{dry}(t) = K_{solid}(1 - \phi(t))^{A/(1 - \phi(t))}, \quad (15)$$

137 where A is a dimensionless parameter which depends on the pore shape and
 138 Poisson ratio of the matrix. This parameter is a pore compliance coefficient and
 139 takes a value of about 2 for spherical pores. Assuming a relation $K_{dry}/G_{dry} =$
 140 K_{solid}/G_{solid} , we obtain the dry-rock shear modulus G_{dry} as

$$G_{dry}(t) = G_{solid}(1 - \phi(t))^{A/(1 - \phi(t))}, \quad (16)$$

141 where G_{solid} is the solid shear modulus. Using the moduli K_{solid} , K_{dry} , G_{dry} ,
 142 the porosity ϕ and permeabilities κ_x , κ_z , as well as the fluids bulk moduli and

143 viscosities (computed using the Peng-Robinson model [18] for CO₂), we deter-
 144 mine the complex and frequency (ω) dependent Lamé coefficients $\lambda_G(\omega), \mu(\omega)$
 145 as follows. In the brine saturated mudstone layers, these coefficients are ob-
 146 tained by using a Zener model [12]. Outside the mudstone layers, we con-
 147 sider P-wave attenuation due to wave induced fluid flow at mesoscopic scale
 148 using a model of porous layers alternately saturated with brine and CO₂ [13].
 149 This approach yields a complex and frequency dependent P-wave modulus
 150 $E(\omega) = \lambda_G(\omega) + 2\mu(\omega)$ for the formation.

151 Both, Zener and White models, require the knowledge of the bulk modulus
 152 K_{solid} and density ρ_{solid} of the grains, the bulk and shear modulus K_{dry} and
 153 G_{dry} as well as the porosity ϕ and permeabilities κ_x, κ_z of the matrix. They
 154 also need the fluid bulk modulus and viscosity.

155 The phase velocities $v(\omega)$ and quality factors $Q(\omega)$ are defined by the rela-
 156 tions

$$v_t(\omega) = \left[\operatorname{Re} \left(\frac{1}{v_{ct}(\omega)} \right) \right]^{-1}, \quad Q_t(\omega) = \frac{\operatorname{Re}(v_{ct}(\omega)^2)}{\operatorname{Im}(v_{ct}(\omega)^2)}, \quad t = P, S, \quad (17)$$

157 where $v_{ct}(\omega)$ are the complex and frequency dependent compressional velocities
 158 defined as

$$v_{cP}(\omega) = \sqrt{\frac{E(\omega)}{\rho}}, \quad v_{cS}(\omega) = \sqrt{\frac{\mu(\omega)}{\rho}}, \quad (18)$$

159 being ρ the bulk density defined as $\rho = (1 - \phi)\rho_{solid} + \phi\rho_{fluid}$, with ρ_{solid} and
 160 ρ_{fluid} the solid and fluid densities respectively, $\rho_{fluid} = S_b\rho_b + S_g\rho_g$.

161 2.3. Wave propagation model

162 Wave propagation simulation is performed with our own model, which con-
 163 siders the effects of both wave dispersion and velocity attenuation. One of the
 164 main phenomena occurring in rocks, in particular partially saturated with CO₂,
 165 is the mesoscopic-loss effect [13]-[12]. This caused by heterogeneities in the fluid
 166 and solid phases greater than the pore size but much smaller than the dom-
 167 inant wavelengths. The mesoscopic loss causes wave attenuation and velocity
 168 dispersion of the compressional and shear waves. Since the mesoscopic scale

169 is typically on the order of centimeters, any finite-element or finite-difference
 170 numerical procedure based on Biot’s equations is impractical. To overcome this
 171 difficulty, we use an alternative approach, based on an equivalent viscoelastic
 172 medium that includes these mesoscopic-scale attenuation and dispersion effects.

173 The equation of motion in a 2D isotropic viscoelastic domain Ω with bound-
 174 ary $\Gamma = \partial\Omega$ can be stated in the space-frequency (x, ω) domain as

$$-\omega^2 \rho u - \nabla \cdot \sigma(u) = f(x, \omega), \quad \Omega \quad (19)$$

$$-\sigma(u)\nu = i\omega \mathcal{D}u, \quad \Gamma, \quad (20)$$

175 where $u = (u_x, u_z)$ is the displacement vector and $\sigma(u)$ is the stress tensor.

176 Eq. (20) is a first-order absorbing boundary condition [19], where

$$\mathcal{D} = \rho \begin{bmatrix} \nu_1 & \nu_2 \\ -\nu_2 & \nu_1 \end{bmatrix} \begin{bmatrix} v_{cP}(\omega) & 0 \\ 0 & v_{cS}(\omega) \end{bmatrix} \begin{bmatrix} \nu_1 & -\nu_2 \\ \nu_2 & \nu_1 \end{bmatrix},$$

177 with $\nu = (\nu_1, \nu_2)$ the unit outward normal on Γ .

178 The stress tensor is defined in the space-frequency domain by

$$\sigma_{jk}(u) = \lambda_G(\omega) \nabla \cdot u \delta_{jk} + 2\mu(\omega) \varepsilon_{jk}(u), \quad \Omega, \quad (21)$$

179 where $\varepsilon_{jk}(u)$ denotes the strain tensor, δ_{jk} is the Kroenecker delta, and $\lambda_G(\omega)$
 180 and $\mu(\omega)$ the complex and frequency dependent Lamé coefficients defined in
 181 Section 2.2.

182 Appendix B describes the method used to solve the wave propagation model.

183 3. Numerical examples

184 This section presents a series of numerical examples performed with the
 185 model described in Section 2. First, we build the aquifer model based on the
 186 Utsira Sand. Then, we run the CO₂ injection simulation for several time spans to
 187 obtain the resulting CO₂ spatial distribution. Finally, we apply our wave prop-
 188 agation model to simulate seismic monitoring and to obtain the corresponding
 189 seismic images. We also run this series of analysis for different values of entry
 190 capillary pressure in order to evaluate its effect over the CO₂ storage procedure.

191 *3.1. Aquifer model*

192 We consider a 3D model of the Utsira Sand ranging 1,200 m on the x -
193 direction, 10,000 m on the y -direction, and 400 m on the vertical z -direction.
194 The aquifer top is considered to be at a depth of 770 m (b.s.l.) [20]. The sim-
195 ulation is performed using a mesh with equally spaced blocks in each direction,
196 considering 300 cells on the x -direction, 11 on the y -direction and 400 on the
197 z -direction. Since the properties on the y -direction are constant, it is fair to say
198 that the model is actually 2.5D.

199 We consider the aquifer at hydrostatic pressure $p_H = \rho_b g z$, with $\rho_b = 1040$
200 kg/m^3 the density of brine, g the gravity constant and z the depth (in m b.s.l.).
201 For the temperature profile we assume $T = 31.7z + 3.4$, with T in $^\circ\text{C}$ [21].

202 The initial porosity ϕ_0 (at hydrostatic pore pressure) for the Utsira sandstone
203 is assumed to have a fractal spatial distribution, based on the so-called von
204 Karman self-similar correlation functions [5], using a mean value of 36.7% [3]-
205 [2]. Horizontal and vertical permeabilities are determined by using Eq. 13 and
206 Eq. 14, considering an anisotropy parameter $a = 0.1$ and a fixed clay content
207 $C = 6\%$ [22]. The minimum, average and maximum initial porosities obtained
208 are 35.5%, 36.7% and 38.3%. The associated vertical permeabilities are 0.1
209 D, 0.12 D and 0.145 D. The spatial vertical permeability distribution at initial
210 conditions is shown in Figure 1. It can be observed that, within the formation,
211 there are several mudstone layers (low permeability and porosity) which act as
212 barriers to the vertical motion of the CO_2 . The mudstone layers topography
213 is considered in order to represent the actual internal structure of Utsira Sand
214 [6]. In this regard, we have taken a picture of seismic data [8] and have picked
215 the main reflections to produce a line drawing. We later have generated a
216 geological model and produced synthetic data with plain-wave simulation. We
217 have adjusted the model taking into account that the first internal mudstone
218 layer is 6.5m thick and the rest are 1m thick [7]. Since the model is generated
219 from real data, we do not claim that it is perfect, but representative of the real
220 topography. Nevertheless, these layers are not completely sealed, having low and
221 constant porosity and vertical permeability values of 24% and 0.033 D [6]-[8].

222 Besides, each of them has an opening allowing the upward migration of CO₂.
 223 These openings are not strictly aligned, and distributed in order to help match
 224 observations from the seismic images [8]. The overburden and underburden of
 225 the Utsira Sand are assigned constant porosity and permeability values of 32%
 226 and 0.02 D.

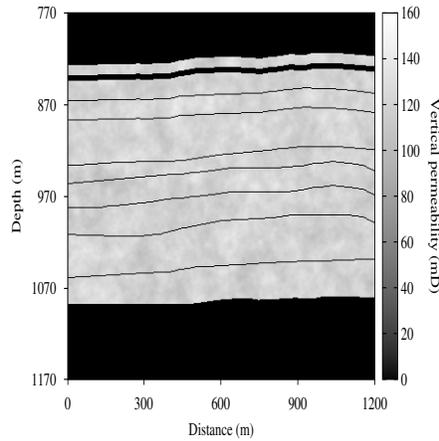


Figure 1: Initial vertical permeability map

227 Figure 2b shows the relative permeability and dimensionless capillary pres-
 228 sure curves that have been obtained from the conductivity log from well 15/9-13
 229 of the Utsira Sand shown in Figure 2a, by taking into account the relationships
 230 with the resistivity index, as described in section 2.1.1. In particular, this cap-
 231 illary pressure curve has been obtained using $P_{ce} = 10$ kPa based on previous
 232 studies of CO₂-brine systems [23].

233 3.2. CO₂ Injection

234 We apply the BOAST simulator to model 7 years of CO₂ injection in the
 235 Utsira Sand. The simulation is performed at a constant flow rate of one million
 236 tons per year at the injection point which is located at the bottom of the for-
 237 mation: $x = 600$ m, $y = 5,000$ m, $z = 1,082$ m. To satisfy the CFL stability
 238 condition due to IMPES formulation [24], a constant time step of 0.08 days is

(a) Conductivity log (b) Multiphase flow functions

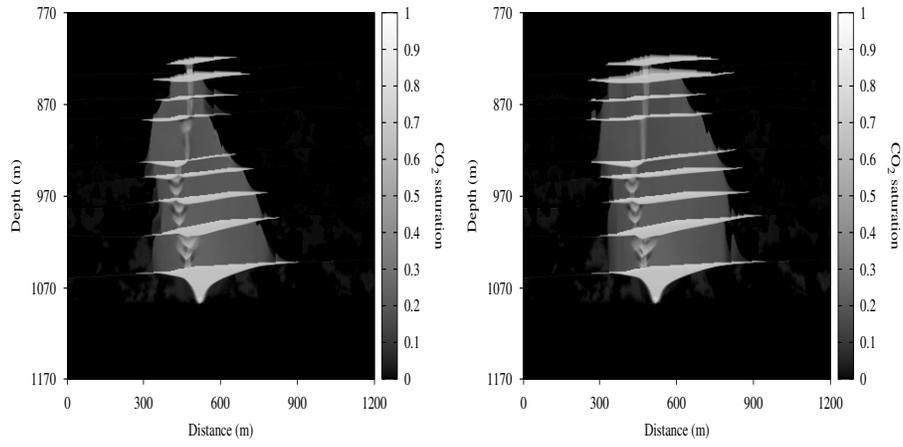
Figure 2: Relative permeabilities and dimensionless capillary pressure from Utsira conductivity log (well 15/9-13)

239 used. Recall that the petrophysical properties of the formation are time depen-
240 dent due to the CO₂ injection and consequent increase in pore pressure, but
241 they change at a much slower rate than pressure and saturation. As a conse-
242 quence, we have two time scales, and we use a much larger time step to update
243 petrophysical properties than to run the flow simulator. A thorough analysis
244 has been conducted to validate that the optimum time to update the petrophys-
245 ical properties would be 30 days. In this analysis, we run several simulations by
246 gradually reducing the time step of properties update. Big differences can be
247 observed among time steps of 1 year, 6 months and 1 month. But running the
248 model with an update time step shorter than 30 days yield negligible differences.

249 Therefore a time step of 30 days is considered adequate.

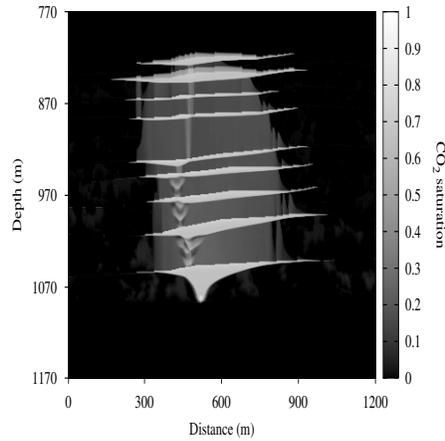
250 Figure 3 shows the CO₂ saturation maps after 3, 5 and 7 years of injection.
251 These maps are vertical cross sections at the injection point. CO₂ accumulations
252 below the mudstone layers that spread laterally can be observed. As injection
253 proceeds, the injected fluid migrates upwards through the aquifer, since its den-
254 sity is lower than that of the brine. Due to the presence of mudstone layers
255 apertures, this migration generates "chimneys" because the CO₂ follows a pre-
256 ferred path through these openings. However, a smaller portion of it moves
257 through the mudstone layers themselves. The openings distribution can be in-
258 ferred from the shape of the chimney, which strongly depends on the position
259 of those openings. As stated before, this distribution has been determined from
260 seismic images [8]. The vertical fluid flow observed is controlled by the vertical
261 permeability. CO₂ plume reaches greater heights within the aquifer and starts
262 to spread laterally when it reaches the top. As can be seen in Figure 3a, after
263 3 years of injection the plume has already reached the top of the formation.
264 Similar results were also achieved by other authors applying different models
265 [25]-[26].

266 We also evaluate the behavior of the different properties that affect the
267 fluid flow within the aquifer. Both the porosity and the horizontal permeability
268 do not suffer significant variations, because they depend only on pore pressure
269 (Eq.12 and Eq.13). However, the vertical permeability is assumed as saturation-
270 dependent according to the petrophysical model (Eq.14), based on qualitative
271 observations in rocks saturated with water and gas. In this model at full brine
272 saturation, $k_x > k_z$. As water saturation is reduced, and the larger pores drained
273 first, k_x becomes less than k_z . In fact, with CO₂ saturation around 0.5, the
274 vertical permeability can become 1.7 times the horizontal permeability. At the
275 other end, i.e. full CO₂ saturation, $k_x > k_z$ again. In the equations of two-phase
276 flow in porous media (Eq.1 and Eq.2), two different permeabilities are used: the
277 absolute permeability tensor, that contemplates the solid matrix permeabilities
278 in each direction, and the relative permeabilities, that take into account the
279 interactions between rock and fluids and depend only on fluid saturation. Eq.



(a) After 3 years of injection

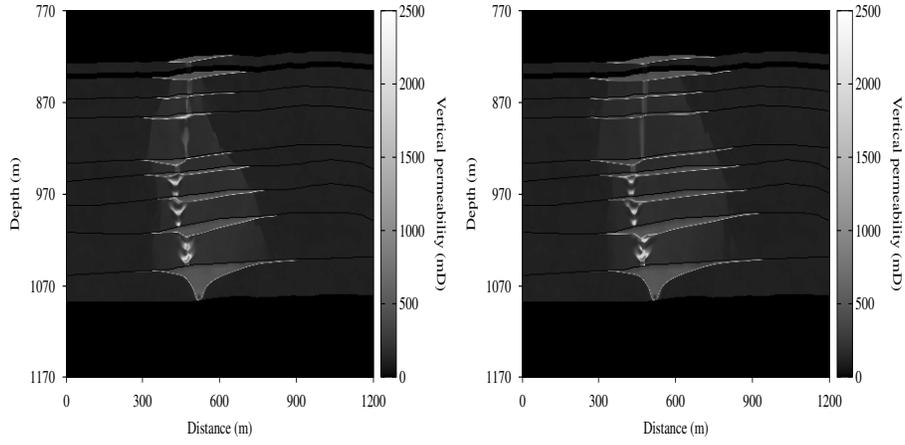
(b) After 5 years of injection



(c) After 7 years of injection

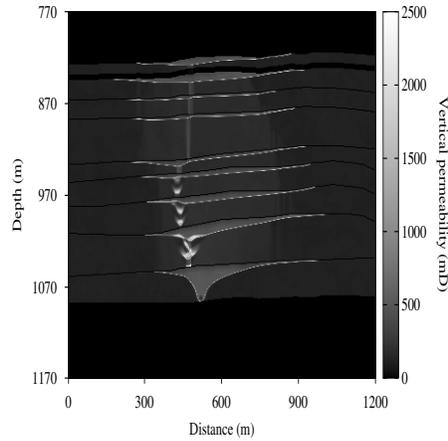
Figure 3: CO₂ saturation maps

280 14 allows for relative permeability anisotropy, because the relationship between
 281 vertical and horizontal absolute permeabilities is function of saturation. This is
 282 why vertical permeability, besides saturation, is the most affected property as
 283 CO₂ injection evolves, as we can see in Figure 4.



(a) After 3 years of injection

(b) After 5 years of injection



(c) After 7 years of injection

Figure 4: Vertical permeability maps

284 *3.3. Seismic monitoring*

285 In this section, we analyze the capability of seismic monitoring to identify
 286 zones of CO₂ accumulation and migration. With this purpose, saturation and
 287 pressure maps obtained by the fluid model and the corresponding updated rock
 288 properties (as described in section 2.2) are used as input data to compute syn-
 289 thetic seismograms. The iterative procedure described in Eq. (B.1) and Eq.

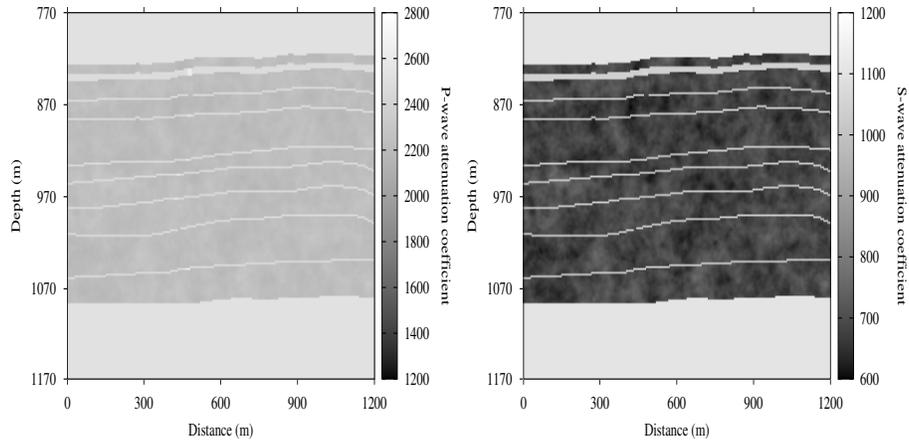
290 (B.2) (Appendix B) is used to compute the time Fourier transforms of the dis-
291 placement vector for 200 equally spaced temporal frequencies in the interval
292 (0, 200) Hz. The seismic source used is a spatially localized plane wave of dom-
293 inant frequency 60 Hz at $z = 772$ m. A line of receivers is located at the same
294 depth to record the Fourier transforms of the vertical displacements. Then, a
295 discrete inverse Fourier transform is performed to compute the synthetic seis-
296 mograms [19].

297 Figure 5 displays the spatial distribution of the P- and S-wave phase veloc-
298 ities (v_p and v_S maps) and the corresponding migrated synthetic seismogram
299 before injection, where the mudstone layers are clearly identified. On the other
300 hand, Figure 6 shows the v_p map, v_S map and the seismogram associated with
301 the CO₂ saturation distribution obtained after 3 years of injection (Figure 3a).
302 The reduction in the P-wave velocities due to CO₂ accumulation is clearly ob-
303 served. This synthetic seismic section (as well as all others) may be migrated
304 to eliminate the diffractions.

305 The waves generated by the source are reflected and transmitted due to
306 the CO₂ accumulations. The reflected waves suffer a delay when traveling in
307 zones of CO₂ accumulation between mudstone layers. Therefore the reflections
308 observed in the seismograms show the CO₂ distribution and accumulation.

309 Figure 7 presents a comparison between the seismograms obtained after 3, 5
310 and 7 years of simulation. It is also clear that for different simulation times we
311 get distinct saturation distributions (3), which result in different seismograms.
312 As CO₂ injection develops, it spreads both laterally and vertically. Even though
313 the absolute horizontal permeability is in general greater than the vertical one,
314 the difference in densities for the brine and CO₂ gives the latter a much greater
315 upward mobility. Seismic analysis can clearly detect the evolution of the CO₂
316 plume along time.

317 Finally, Figure 8 compares the migrated synthetic seismogram obtained after
318 5 years of injection (Figure 8a) with a real seismic image (Figure 8b) obtained
319 from [3]. It can be observed that the pushdown effect that appears in real
320 seismograms due to CO₂ accumulations is also observed on the synthetic one.



(a) V_p map before injection

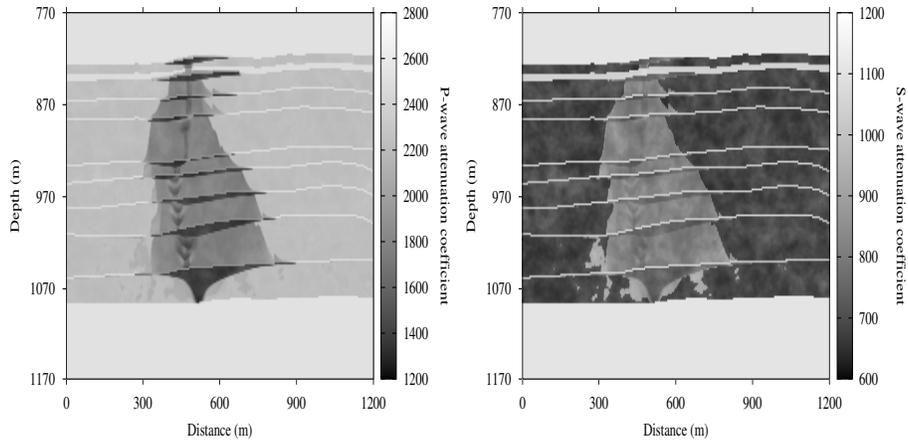
(b) V_s map before injection

(c) Migrated synthetic seismogram before injection

Figure 5: Seismic monitoring before injection

321 Also, the delay between the real and synthetic one is comparable. This allows
 322 us to conclude that it is possible to correlate the real saturation distribution
 323 and that obtained through the injection model.

324 Although the saturation map around the chimney section allows to accu-
 325 rately reproduce the pushdown effect, it is also observable in Figures 9 and 10



(a) V_p map after 3 years

(b) V_s map after 3 years

(c) Migrated synthetic seismogram after 3 years

Figure 6: Seismic monitoring after 3 years of injection

326 that the lateral spread does not match the actual seismic images exactly. This
 327 is likely due to the discretization size in the y -direction, which has been selected
 328 to control the computational cost. Recall that the main objective of this work
 329 is not to study the plume behavior in 3D.

(a) After 3 years of injection

(b) After 5 years of injection

(c) After 7 years of injection

Figure 7: Time evolution of migrated synthetic seismograms

330 *3.4. Capillary pressure sensitivity analysis*

331 The results shown in previous sections consider an unique capillary pressure
332 curve, characterized by an entry capillary pressure value of $P_{ce} = 10$ KPa. In
333 this section, a sensitivity analysis is conducted for this parameter to determine
334 its influence over the CO₂ plume evolution and the corresponding seismograms.

335 Several entry capillary pressure values are considered, ranging from 0.01 kPa

(a) Migrated synthetic seismogram after 5 years (b) Real cross-correlogram after 5 years [3]

Figure 8: Comparison between numerical results and Utsira actual data

336 to 200 kPa. To evaluate the plume evolution, we analyze the maximum height
337 that CO₂ reaches within the aquifer. In order to do this, we keep track of the
338 maximum plume length through all the time steps. This is easily computed by
339 sweeping all the horizontal x -cells for every height z and checking whether the
340 CO₂ saturation is higher than 0.01 (indicating CO₂ presence). This progression
341 is shown in Figure 11 for entry capillary pressure values of 10, 50, 100 and
342 200 kPa. Simulated cases with P_{ce} values lower than 10 kPa show slightly
343 differences with the $P_{ce} = 10$ kPa case. It can be observed that, as capillary
344 pressure increases, the plume upward migration velocity decreases. This CO₂
345 plume velocity decrease is clearly demonstrated by the reduction of the slopes
346 of the different curves in Figure 11 as higher values of entry capillary pressure
347 are used.

348 We also seek to evaluate the capability of the seismic analysis to identify the
349 capillary pressure effect over the CO₂ distribution within the aquifer. For this
350 matter we use the saturation maps obtained from the CO₂-brine flow model
351 after 3 years of injection for each capillary pressure curve (e.g. Figure 3a for
352 $P_{ce} = 10$ kPa). With this information we obtain the seismic images shown in

(a) Migrated synthetic seismogram

(b) Real seismic image [27]

Figure 9: Comparison between numerical results and Utsira actual data after 5 years of injection

353 Figure 12. From these images, it is clearly observed that the seismic analysis
354 allows to identify the P_{ce} influence over the plume velocity. The presence of CO₂
355 is identified due to the velocity attenuation and the corresponding push down
356 effect. As we can see in Figure 12, the seismic images show the presence of CO₂
357 in different zones depending on the entry capillary pressure value. As capillary
358 pressure increases, the plume reaches lower heights in the aquifer. That is to
359 say, the variation of capillary pressure has a great impact on the seismic images.

(a) Migrated synthetic seismogram

(b) Actual time-lapse seismic image [27]

Figure 10: Comparison between numerical results and Utsira actual data after 14 years of injection

360 **4. Conclusions**

361 We present a methodology integrating numerical simulation of CO₂-brine
362 flow and seismic wave propagation, combined with a petrophysical model of
363 the formation, to study and monitor CO₂ storage inside a saline aquifer. We
364 apply this methodology to analyze the influence of capillary pressure during this
365 process.

Figure 11: CO₂ Plume height evolution for different entry capillary pressures

366 The flow simulator considers the CO₂ solubility in brine through a simplified
367 thermodynamic model, with CO₂ properties determined by the Peng-Robinson
368 equations. The petrophysical model is based on fractal porosity and considers
369 the variation of properties with pore pressure and fluid saturation. The wave
370 equation takes into account wave velocity changes and attenuation effects due
371 to the presence of mesoscopic scale heterogeneities caused by patches of CO₂.

372 The proposed methodology has been applied to the Utsira Sand, which con-
373 tains several thin low-permeability mudstone layers, which are not completely
374 impermeable, allowing the upward migration of CO₂. The mechanism used
375 to obtain the multiphase flow functions from well logs allows us to derive the
376 properties that represent the fluid behavior through the Utsira Sand. The fluid-
377 flow simulator, considering the petrophysical properties updating, yields realistic
378 CO₂ accumulations below the mudstone layers. Besides the reflections observed
379 in the seismograms show the progressive increase in CO₂ accumulations. In
380 particular, the pushdown effect is clearly observed.

(a) $P_{Ce} = 10$ KPa

(b) $P_{Ce} = 50$ KPa

(c) $P_{Ce} = 100$ KPa

(d) $P_{Ce} = 200$ KPa

Figure 12: Entry capillary pressure effect over the migrated synthetic seismograms

381 As can be deduced from numerical examples, capillary forces affect the mi-
382 gration and diffusion of the CO₂ plume; higher values of these forces cause
383 a slower CO₂ upward migration and lateral spreading of accumulation zones
384 below mudstone layers. Higher entry capillary pressure values somehow slow
385 down the advance of the CO₂ plume within the aquifer. This makes it very
386 important to have a correct estimation of this parameter before running the

387 simulations, because an improper value could generate wrong estimations of the
388 CO₂ distribution within the aquifer.

389 Summarizing, the described methodology constitutes an important tool to
390 monitor the migration of the CO₂ plume, to analyze storage integrity and to
391 make long term predictions based on storage site actual data.

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397 Appendix A. Black-Oil formulation of two phase flow in porous media

398 The simultaneous flow of brine and CO₂ is described by the well-known
399 Black-Oil formulation [10]. In this approach, brine is identified with oil and
400 CO₂ with gas assuming supercritical properties. Therefore, the CO₂ component
401 may dissolve in the brine phase but the brine is not allowed to vaporize into
402 the CO₂ phase. The Black-Oil formulation uses, as a simplified thermodynamic
403 model, the PVT data defined as

- 404 • R_s : CO₂ solubility in brine
- 405 • B_g : CO₂ formation volume factor
- 406 • B_b : brine formation volume factor

407 The conversion of compositional data from equations of state into the Black-
408 Oil PVT data is based on an algorithm developed by [28],

- 409 • $R_s = \frac{\tilde{\rho}_b^{SC} \chi_g}{\tilde{\rho}_g^{SC} (1 - \chi_g)}$
- 410 • $B_b = \frac{\rho_b^{SC}}{\rho_b (1 - \omega_g)}$,

411 where $\tilde{\rho}_b^{SC}$ and $\tilde{\rho}_g^{SC}$ are the brine and CO₂ molar densities at standard condi-
 412 tions, respectively and χ_g and ω_g are the CO₂ mole and mass fraction in the
 413 brine phase.

414 Then, the mass conservation equation for each component can be expressed
 415 as

$$-\nabla \cdot \left(\frac{1}{B_g} \underline{v}_g + \frac{R_s}{B_b} \underline{v}_b \right) + q_g = \frac{\partial \left[\phi \left(\frac{S_g}{B_g} + \frac{R_s S_b}{B_b} \right) \right]}{\partial t} \quad \text{for } CO_2 \text{ component, (A.1)}$$

416

$$-\nabla \cdot \left(\frac{1}{B_b} \underline{v}_b \right) + q_b = \frac{\partial \left[\phi \frac{S_b}{B_b} \right]}{\partial t} \quad \text{for brine component, (A.2)}$$

417 where g, b denote CO₂ and brine phases respectively, ϕ is the porosity, \underline{v}_β the
 418 phase velocity, S_β the saturation and q_β the injection mass rate per unit volume,
 419 with $\beta = b, g$.

420 The phase velocities can be expressed by Darcy's empirical law as

$$\underline{v}_g = -\frac{\underline{\kappa} k_{rg}}{\eta_g} (\nabla p_g - \rho_g g \nabla z), \quad (A.3)$$

421

$$\underline{v}_b = -\frac{\underline{\kappa} k_{rb}}{\eta_b} (\nabla p_b - \rho_b g \nabla z), \quad (A.4)$$

422 where p_β are the fluid pressures, η_β the viscosity, $k_{r\beta}$ the relative permeability,
 423 $\underline{\kappa}$ the absolute permeability tensor, g the gravity constant and z the depth.

424 Finally, combining the mass conservation equations (Eq. A.1) and Eq. (A.2)
 425 with Darcy's law (Eq. A.3) and Eq. (A.4) we obtain the following two differen-
 426 tial equations

$$\begin{aligned} \nabla \cdot \left(\frac{\underline{\kappa} k_{rg}}{B_g \eta_g} (\nabla p_g - \rho_g g \nabla z) + \frac{R_s k_{rb}}{B_b \eta_b} (\nabla p_b - \rho_b g \nabla z) \right) + \frac{q_g}{\rho_g^{SC}} \\ = \frac{\partial \left[\phi \left(\frac{S_g}{B_g} + \frac{R_s S_b}{B_b} \right) \right]}{\partial t}, \end{aligned} \quad (A.5)$$

427

$$\nabla \cdot \left(\frac{\underline{\kappa} k_{rb}}{B_b \eta_b} (\nabla p_b - \rho_b g \nabla z) \right) + \frac{q_b}{\rho_b^{SC}} = \frac{\partial \left[\phi \frac{S_b}{B_b} \right]}{\partial t}, \quad (A.6)$$

428 The unknowns for the Black-Oil model are the fluid pressures p_β and saturations
 429 S_β .

430 Two algebraic equations relating the saturations and pressures complete the
 431 system:

$$S_b + S_g = 1, \quad p_g - p_b = P_C(S_b), \quad (\text{A.7})$$

432 where $P_C(S_b)$ is the capillary pressure function.

433 The numerical solution of the system is obtained with the public-domain
 434 software BOAST [11]. BOAST solves the differential equations using IMPES
 435 (Implicit Pressure Explicit Saturation), a finite-difference technique [10]. Finite
 436 differences is the standard in commercial reservoir simulators, and the improved
 437 versions use both structured and unstructured grids with local refinements to
 438 accurately represent reservoir geometry. The basic idea of IMPES is to obtain a
 439 single pressure equation by a combination of the flow equations. Once pressure
 440 is implicitly computed, saturation is updated explicitly. We briefly describe
 441 IMPES for these particular system (Eq. A.5, Eq. A.6 and Eq. A.7). The
 442 first step is to obtain the pressure equation, combining flow equations: Eq. A.5
 443 multiplied by B_g and Eq. A.6 multiplied by $(B_b - R_s B_g)$ are added. In this
 444 way, the right side of the resulting equation is:

$$B_g \frac{\partial \left[\phi \left(\frac{S_g}{B_g} + \frac{R_s S_b}{B_b} \right) \right]}{\partial t} + (B_b - R_s B_g) \frac{\partial \left[\phi \frac{S_b}{B_b} \right]}{\partial t}.$$

445 Using the chain rule to expand the time derivatives and after some compu-
 446 tations and rearrangements,

$$\phi \left[\frac{1}{\phi} \frac{d\phi}{dp_b} + S_g \left(-\frac{1}{B_g} \frac{dB_g}{dp_b} \right) + S_b \left(-\frac{1}{B_b} \frac{dB_b}{dp_b} + \frac{B_g}{B_b} \frac{dR_s}{dp_b} \right) \right] \frac{\partial p_b}{\partial t},$$

447 where all the time derivatives with respect to the saturation disappear.

448 Defining the following,

- 449 • Formation compressibility: $c_f = \frac{1}{\phi} \frac{d\phi}{dp_b}$

- 450 • CO₂ compressibility: $c_g = -\frac{1}{B_g} \frac{dB_g}{dp_b}$,
- 451 • Brine compressibility: $c_b = -\frac{1}{B_b} \frac{dB_b}{dp_b} + \frac{B_g}{B_b} \frac{dR_s}{dp_b}$,
- 452 • Total compressibility: $c_t = c_f + S_g c_g + S_b c_b$,

453 the right side of the resulting pressure equation is simply expressed as,

$$\phi c_t \frac{\partial p_b}{\partial t}.$$

454 Finally, replacing p_g by $p_b + P_C(S_b)$ in the left side of the combined equation,
 455 the following pressure equation in p_b is obtained,

$$\begin{aligned} & B_g \left[\nabla \cdot \left(\kappa \left(\frac{k_{rg}}{B_g \eta_g} (\nabla p_b - \rho_g g \nabla D) + \frac{R_s k_{rb}}{B_b \eta_b} (\nabla p_b - \rho_b g \nabla D) + \frac{k_{rg}}{B_g \eta_g} \nabla P_C \right) \right) \right] \\ & + (B_b - R_s B_g) \left[\nabla \cdot \left(\kappa \frac{k_{rb}}{B_b \eta_b} (\nabla p_b - \rho_b g \nabla D) \right) \right] \\ & + B_g \frac{q_g}{\rho_g^{SC}} + (B_b - R_s B_g) \frac{q_b}{\rho_b^{SC}} = \phi c_t \frac{\partial p_b}{\partial t}. \end{aligned} \quad (\text{A.8})$$

456 In the BOAST simulator, Eq. (A.6) and Eq. (A.8) are discretized using a
 457 block centered grid. The system is linearized by evaluating the pressure and
 458 saturation dependent functions (PVT parameters, viscosities, relative perme-
 459 abilities and capillary pressure) in the pressure and saturation values of the
 460 previous time step. The pressure equation is solved implicitly, applying a Block
 461 Successive Over Relaxation method (BSOR) to compute the linear system solu-
 462 tion. The saturation equation is solved explicitly, therefore stability restrictions
 463 are considered to select the time step [24].

464 Appendix B. The iterative domain decomposition algorithm

465 In order to solve the differential system (Eq. 19 and Eq. 20) we apply
 466 an iterative finite-element domain decomposition procedure, formulated in the
 467 space-frequency domain. To define a global finite-element method we use the
 468 nonconforming finite-element space described below, based on rectangular ele-
 469 ments first presented in [29]. For $h > 0$, let \mathcal{T}_h be a quasiregular partition of

470 $\bar{\Omega}$ such that $\bar{\Omega} = \cup_{j=1}^J \bar{\Omega}_j$ with Ω_j being rectangles of diameter bounded by h .
 471 Set $\Gamma_j = \partial\Omega \cap \partial\Omega_j$ and $\Gamma_{jk} = \Gamma_{kj} = \partial\Omega_j \cap \partial\Omega_k$, we denote by ξ_j and ξ_{jk} the
 472 centroids of Γ_j and Γ_{jk} , respectively.

473 We consider a nonconforming finite element space constructed using the
 474 following reference rectangular element:

$$\hat{R} = [-1, 1]^2 \quad S_2(\hat{R}) = \text{Span} \left\{ \frac{1}{4} \pm \frac{1}{2}x - \frac{3}{8} \left((x^2 - \frac{5}{3}x^4) - (y^2 - \frac{5}{3}y^4) \right), \right. \\ \left. \frac{1}{4} \pm \frac{1}{2}y + \frac{3}{8} \left((x^2 - \frac{5}{3}x^4) - (y^2 - \frac{5}{3}y^4) \right) \right\}.$$

475 The four degrees of freedom associated with $S_2(\hat{R})$ are the values at the mid
 476 points of the faces of \hat{R} , i.e., the values at the nodal points $a_1 = (-1, 0)$, $a_2 =$
 477 $(0, -1)$, $a_3 = (1, 0)$ and $a_4 = (0, 1)$. For example the basis function $\psi_1(x, y) =$
 478 $\frac{1}{4} - \frac{1}{2}x - \frac{3}{8} \left((x^2 - \frac{5}{3}x^4) - (y^2 - \frac{5}{3}y^4) \right)$ is such that $\psi_1(a_1) = 1$ and $\psi_1(a_j) =$
 479 $0, j = 2, 3, 4$.

480 A useful property of applying nonconforming elements for wave propagation
 481 phenomena is that it almost halves the number of points per wavelength neces-
 482 sary to reach a given accuracy as compared with the standard bilinear elements
 483 [30].

484 Set $NC_j^h = S_2(\Omega_j)$ and define a nonconforming finite-element space in the
 485 following manner

$$NC^h = \left\{ v \mid v_j := v|_{\Omega_j} \in NC_j^h, j = 1, \dots, J; \quad v_j(\xi_{jk}) = v_k(\xi_{jk}), \forall \{j, k\} \right\}.$$

486 The global nonconforming Galerkin procedure is defined as follows: find
 487 $\hat{u}^h \in [NC^h]^2$ such that

$$-(\rho\omega^2 \hat{u}^h, \varphi) + \sum_{pq} (\sigma_{pq}(\hat{u}^h), \varepsilon_{pq}(\varphi)) + i\omega \langle \langle \mathcal{D}\hat{u}^h, \varphi \rangle \rangle_{\Gamma} = (\hat{f}, \varphi), \quad \varphi \in [NC^h]^2,$$

488 Here $(f, g) = \int_{\Omega} f \bar{g} d\Omega$ and $\langle f, \bar{g} \rangle = \int_{\Gamma} f \bar{g} d\Gamma$ denote the complex $[L^2(\Omega)]^N$ and
 489 $[L^2(\Gamma)]^N$ inner products. Also, $\langle \langle \cdot, \cdot \rangle \rangle$ denotes the approximation of $\langle \cdot, \cdot \rangle$ on the
 490 boundary faces by the midpoint quadrature rule.

491 Instead of solving the global problem, we will use the parallelizable domain
 492 decomposition iterative hybridized procedure defined in [19]. This approach is
 493 a requirement when dealing with large 2D (or 3D) problems.

494 One of the main advantages of using nonconforming elements to solve wave
 495 propagation phenomena in parallel architectures is that the amount of informa-
 496 tion exchanged among processors in a domain decomposition iterative procedure
 497 is considerable reduced as compared to the case when conforming elements are
 498 used. Besides, it is possible to obtain an estimate on the speed of convergence
 499 of the iterative domain decomposition procedure as a function of the mesh size
 500 h .

501 To define the iterative procedure, we introduce a set $\tilde{\Lambda}^h$ of Lagrange multi-
 502 pliers λ_{jk}^h associated with the stress values $-\tau(\hat{u}_j)\nu_{jk}(\xi_{jk})$:

$$\tilde{\Lambda}^h = \{\lambda^h : \lambda^h|_{\Gamma_{jk}} = \lambda_{jk}^h \in [P_0(\Gamma_{jk})]^2 = [\Lambda_{jk}^h]^2\}.$$

503 Here $P_0(\Gamma_{jk})$ are constant functions on Γ_{jk} . Note that Λ_{jk}^h and Λ_{kj}^h are consid-
 504 ered to be distinct.

505 Then, given an initial guess $(\hat{u}_j^{h,0}, \lambda_{jk}^{h,0}, \lambda_{kj}^{h,0}) \in [NC_j^h]^2 \times [\Lambda_{jk}^h]^2 \times [\Lambda_{kj}^h]^2$,
 506 compute $(\hat{u}_j^{h,n}, \lambda_{jk}^{h,n}) \in [NC_j^h]^2 \times [\Lambda_{jk}^h]^2$ as the solution of the equations

$$\begin{aligned} & -(\rho\omega^2\hat{u}_j^{h,n}, \varphi)_j + \sum_{pq} (\tau_{pq}(\hat{u}^{h,n}), \varepsilon_{pq}(\varphi))_j + i\omega \left\langle \left\langle \mathcal{D}\hat{u}_j^{h,n}, \varphi \right\rangle \right\rangle_{\Gamma_j} \\ & + \sum_k \left\langle \left\langle \lambda_{jk}^{h,n}, \varphi \right\rangle \right\rangle_{\Gamma_{jk}} = (\hat{f}, \varphi)_j, \quad \varphi \in [NC_j^h]^2, \end{aligned} \quad (\text{B.1})$$

$$\lambda_{jk}^{h,n} = -\lambda_{kj}^{h,n-1} + i\beta_{jk}[\hat{u}_j^{h,n}(\xi_{jk}) - \hat{u}_k^{h,n-1}(\xi_{jk})], \quad \text{on } \Gamma_{jk}. \quad (\text{B.2})$$

507 It can be shown that

$$[\hat{u}^{h,n} - \hat{u}^h]^2 \rightarrow 0 \text{ in } [L^2(\Omega)]^2 \quad \text{when } n \rightarrow \infty,$$

508 so that in the limit the global nonconforming Galerkin approximation is obtained
 509 [19].

510 Appendix C. List of Symbols

- 511 a : permeability-anisotropy parameter
 512 A : dimensionless parameter
 513 B_b : brine formation volume factor

514 B_g : CO₂ formation volume factor
515 c_b : brine compressibility
516 c_f : formation compressibility
517 c_g : CO₂ compressibility
518 c_t : total compressibility
519 C : clay content
520 $E(\omega)$: frequency dependent P-wave modulus
521 g : gravity constant
522 $k_{r\beta}$: β phase relative permeability
523 K_{dry} : dry-rock bulk modulus
524 K_{solid} : solid grains bulk modulus
525 n : fitting parameter
526 p : average pore pressure
527 p_β : β phase pressure
528 p_H : hydrostatic pressure
529 $P_C(S_b)$: capillary pressure function
530 P_{cD} : dimensionless capillary pressure
531 P_{ce} : entry capillary pressure
532 q_β : β phase injection mass rate per unit volume
533 $Q(\omega)$: quality factor
534 R_c : average radii of the clay grains
535 R_q : average radii of the sand grains
536 R_s : CO₂ solubility in brine
537 $RI(S_b)$: resistivity index
538 S_β : β phase saturation
539 S_{rb} : residual brine saturation
540 T : temperature
541 u : displacement vector
542 $v(\omega)$: Darcy phase velocity
543 $v_{ct}(\omega)$: compressional velocity
544 γ : free parameter

545 δ_{jk} : Kroenecker delta
 546 $\varepsilon_{jk}(u)$: strain tensor
 547 η_β : β phase viscosity
 548 η_f : fluid viscosity
 549 $\underline{\kappa}$: absolute permeability
 550 κ_x : horizontal permeability
 551 κ_z : vertical permeability
 552 $\lambda(\omega)$: frequency dependent Lamé coefficient
 553 G_{dry} : dry-rock shear modulus
 554 G_{solid} : solid grains shear modulus
 555 $\mu(\omega)$: frequency dependent Lamé coefficient
 556 ρ_β : β phase density
 557 ρ_{solid} : solid density
 558 ρ_{fluid} : fluid density
 559 ρ : bulk density
 560 $\sigma(1)$: conductivity at full brine saturated rock
 561 $\sigma(S_b)$: conductivity at saturation S_b
 562 σ_b : brine conductivity
 563 σ_g : CO₂ conductivity
 564 σ_q : grain conductivity
 565 ϕ : porosity
 566 ϕ_c : critical porosity
 567 ϕ_0 : initial porosity

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