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PRESSURE WAVE PROPAGATION IN SNOW WITH NONUNIFORM PERMEABILITY

by

Yin-Chao Yen and S. S. T. Fan*

Introduction

The unsteady flow of gases through porous media has been studied by Duwez and Wheeler (1948), Muskat (1937) and Hetherington et al. (1942). More recently, Green and Wilts (1951) obtained a numerical solution for the pressure distribution arising from the unsteady-state, isothermal flow of gas through porous media by means of an electrical analogy. Aronofsky and Jenkins (1951) solved the nonlinear differential equation describing unsteady flow of gases through porous media by the method of finite difference. Roberts (1951) employed a stepwise linearization process to obtain an approximate solution to the equations of unsteady flow through porous media. Bruce et al. (1953) solved the gas flow problem with a digital computer in connection with a gas production study. However, in these previous studies, the physical properties of the porous medium and the imposed boundary conditions have all been assumed or kept constant. The present study extended the investigation to a medium with nonuniform permeability. The results have practical application in describing the propagation of pressure in a deep layer of snow following a massive explosion or detonation above the surface. In order to simulate the surface pressure variation resulting from an explosion, the boundary condition imposed on the snow was made to decay exponentially with time. The conventional case of constant boundary conditions was also studied. To facilitate the mathematical analysis, several assumptions were introduced. The flow was assumed to be isothermal and the gas was assumed to behave ideally. The porous medium was considered as a rigid and incompactible mass.

Theory

Assuming that Darcy's law holds for the case of unsteady state flow of gases through porous media with nonuniform permeability resulting from an impinging pressure front on the surface of the medium, we can write

$$\frac{\partial}{\partial x}\left(\rho \frac{k}{\mu} \frac{\partial p}{\partial x}\right) = \phi \frac{\partial \rho}{\partial t} \tag{1}$$

where

k = permeability of the medium

μ = viscosity of the fluid

p = pressure

x = distance in the direction of flow

ρ = density of fluid

ϕ = porosity

t = time

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Assuming ideal gas behavior, i.e., $\rho = p/rt$, and constant viscosity eq 1 could be written as

$$\frac{1}{2\mu}\left[\frac{\partial k}{\partial x}\frac{\partial p}{\partial x} + k\frac{\partial^2 p^2}{\partial x^2}\right] = \phi\frac{\partial p}{\partial t} \tag{2}$$

with the following initial and boundary conditions

$$p = p_i \text{ for } 0 \leqslant x \leqslant L, \quad t \leqslant 0 \tag{3}$$

$$p = p_{b,t} \text{ at } x = 0, \quad t \geqslant 0 \tag{4}$$

$$\frac{\partial \rho}{\partial x} = 0 \text{ at } x = L, \quad t \geqslant 0 \tag{5}$$

where p_i is the initial pressure throughout the medium and $p_{b,t}$ is the pressure imposed on the boundary. For simplicity in the subsequent analysis, we define a set of dimensionless variables as follows:

$$\psi = \frac{p}{p_{b,o}} \tag{6}$$

$$\xi = \frac{x}{L} \tag{7}$$

$$\alpha = \frac{k}{k_o} \tag{8}$$

$$\tau = \frac{p_{b,o}k_o t}{2\mu L^2} \tag{9}$$

where

$p_{b,o}$ = imposed pressure on the boundary at t = 0

L = depth of the porous medium

k_o = permeability at the surface.

Subsequently, eq 2 can be expressed as

$$\frac{1}{\phi}\left[\alpha\frac{\partial^2\psi^2}{\partial\xi^2} + \frac{\partial\alpha}{\partial\xi}\frac{\partial\psi^2}{\partial\xi}\right] = \frac{\partial\psi}{\partial\tau} \tag{10}$$

and in finite difference form

$$\psi_{\xi,\tau+\Delta\tau} = \psi_{\xi,\tau} + \frac{\Delta\tau}{\Delta\xi^2}\frac{1}{\phi_\xi}\left[\frac{\alpha_{\xi+\Delta\xi} - \alpha_{\xi-\Delta\xi}}{4}\left(\psi^2_{\xi+\Delta\xi,\tau} - \psi^2_{\xi-\Delta\xi,\tau}\right) + \alpha_\xi\left(\psi^2_{\xi+\Delta\xi,\tau} + \psi^2_{\xi-\Delta\xi,\tau} - 2\psi^2_{\xi,\tau}\right)\right]. \tag{11}$$

To insure stability of the solution (Richtmyer, 1957), $\Delta\tau/\Delta\xi^2$ was taken to be 1/4; hence $\Delta\tau = \Delta\xi^2/4$. The air permeability k and porosity ϕ of snow appearing in eq 2 are functions of the depth x. Representative data taken from the Arctic ice cap are expressed as

$$k = 64 - 0.012x + 84 \exp(-0.0013x) \tag{12}$$

and

$$\phi = 0.463 - 0.0000574x + 0.166 \exp(-0.00217x) \tag{13}$$

where x is the depth below snow surface in cm (Bader et al., 1955). It appears from eq 12 that k = 0 at x = 5340 cm. Therefore L was taken to be 5340 cm. From eq 7, 8, 12, and 13, we have

$$\alpha = \frac{64 - 64.08\xi + 84 \exp(-6.942\,\xi)}{148} \tag{14}$$

and

$$\phi = 0.462 - 0.3065\xi + 0.166 \exp(-11.59\xi). \tag{15}$$

Equation 10 was solved numerically according to its finite difference forms as shown in eq 11 and employing eq 14 and 15 for the dimensionless permeability and porosity terms, respectively.

Computation and results

In this study, the solution of eq 10 was obtained for two different types of boundary conditions.

Case I: Boundary pressure constant for $\tau > 0$.

In the following discussion the pressure ψ stands for $\psi_{\xi,\tau}$. It was assumed that at $\tau \leqslant 0$, $\psi_{\xi,o} = p_i/p_{b,o} = \psi_i$ everywhere in the medium. At $\tau = 0$, the pressure on the boundary ($\xi = 0$) was suddenly raised to $\psi_{b,o} = 1$. For computation purposes, $\psi_{o,o} = (\psi_{b,o} + \psi_i)/2$. For $\tau > 0$, $\psi_{b,\tau} = 1$. Therefore, the following initial and boundary conditions were correspondingly established:

$$\psi = \psi_i \text{ for } \tau \leqslant 0, \; 0 \leqslant \xi \leqslant 1 \tag{16}$$

$$\psi = \psi_{b,\tau} = 1 \text{ at } \tau > 0, \; \xi = 0 \tag{17}$$

$$\frac{\partial \psi}{\partial \xi} = 0 \text{ at } \xi = 1, \; \tau \geqslant 0. \tag{18}$$

The computation was performed on IBM 1620 and IBM 7094 digital computers with an increment size of $\Delta\xi = 0.1$ for $p_{b,o} = 5$ and 500 atm. The results are presented in Figures 1a and 1b with p_N plotted versus τ at $\xi = 0.1$, 0.3, 0.5, 0.7, and 0.9 for $p_{b,o} = 5$ and 500 atm, respectively. p_N is defined to be the ratio of $(p - p_i)/(p_{b,o} - p_i)$.

Case II: Boundary pressure decays exponentially with time.

The boundary condition for this case will be represented by

$$\psi_{b,\tau} = \psi_i + (1 - \psi_i) \exp(-B\tau) \tag{19}$$

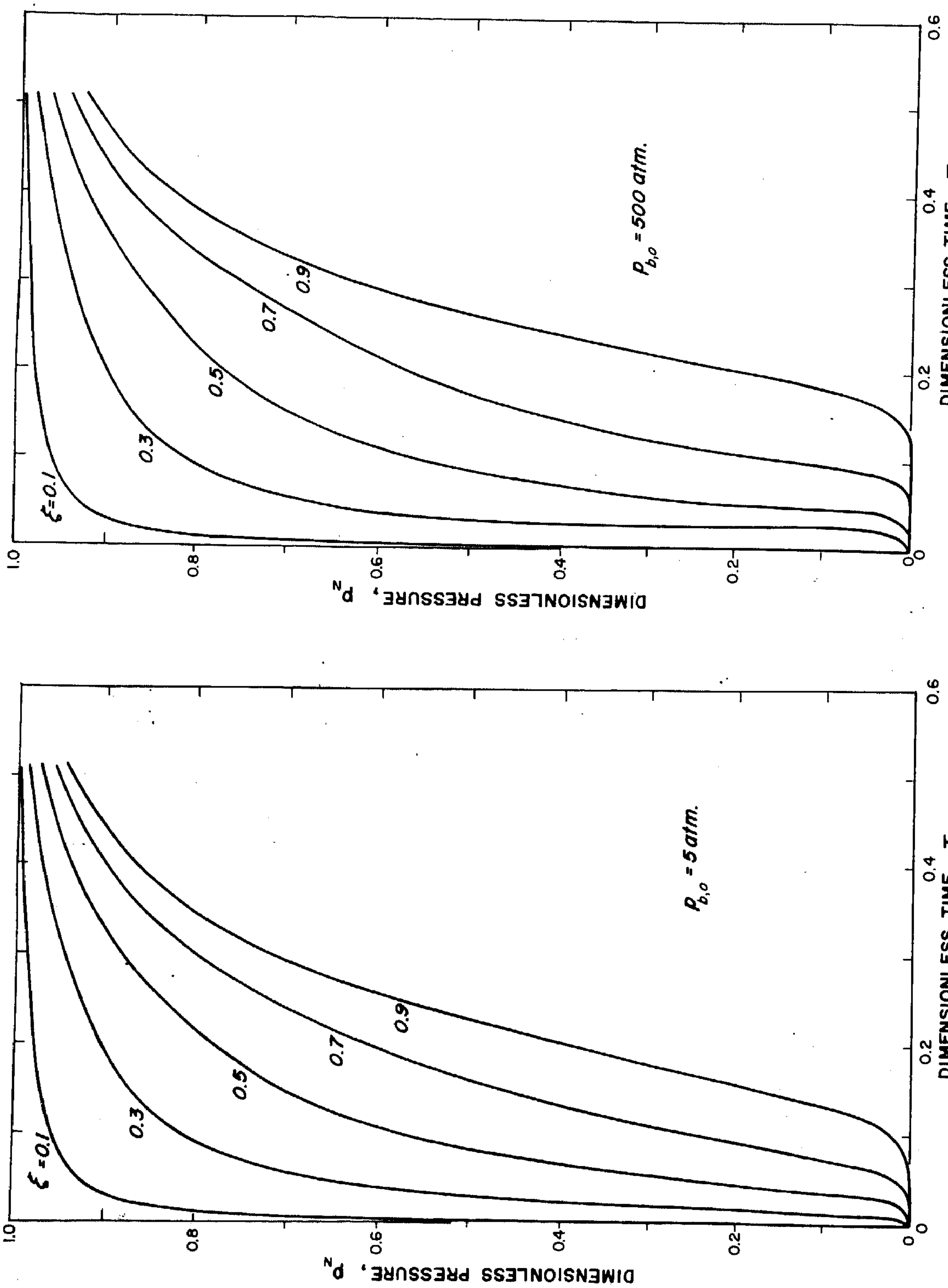


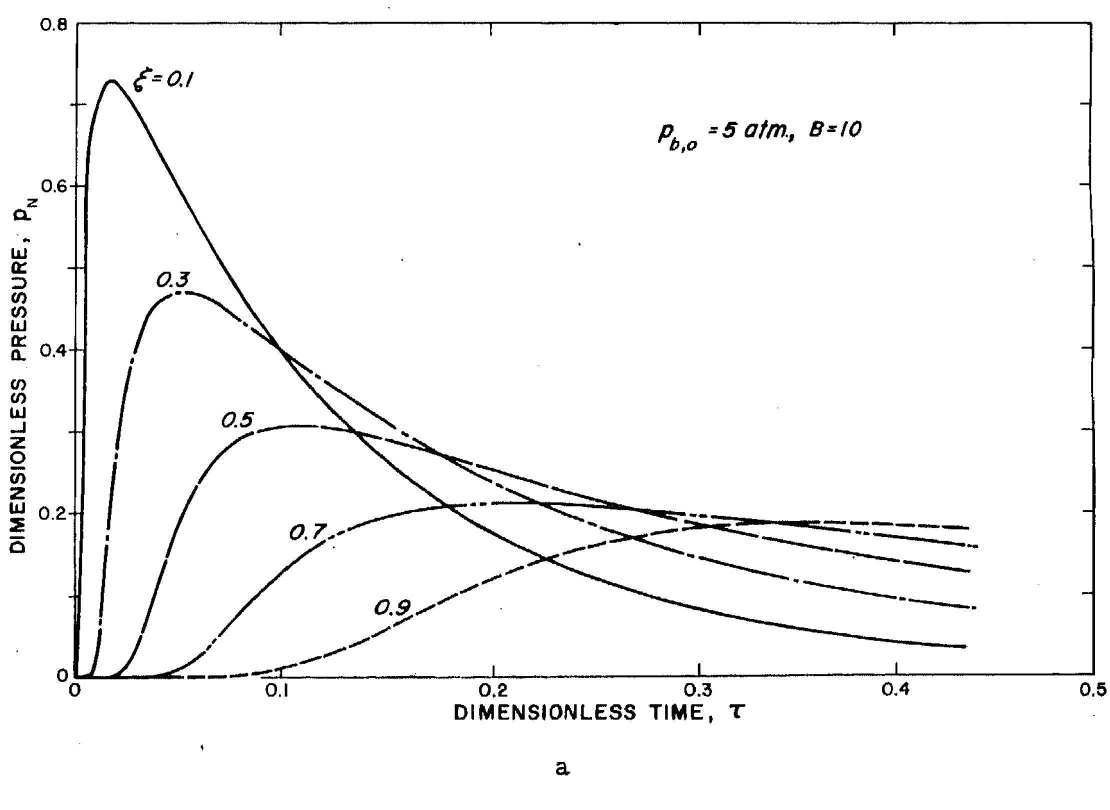
Figure 1. Relationship between P_N and τ, Case I.

in which B is a decay constant. To cover a wide range of decay rates, four values of B, i.e., 10, 20, 30, and 40, were used for all the computations conducted. In Figures 2a and 2b p_N is plotted versus τ at various values of ξ for B = 10 and $p_{b,o}$ = 5 and 500 atm, respectively. A similar plot for B = 40 is shown in Figures 2c and 2d. It should be noted that $\Delta\xi$ = 0.1 was used in all the above computations. To check the adequacy of the grid size, the computation in Case II for B = 10 and $p_{b,o}$ = 5 atm was repeated with $\Delta\xi$ = 0.05; the results are presented in Figure 2e. A comparison between Figures 2a and 2e clearly indicates that, at small values of ξ, the results are almost identical with those obtained by using $\Delta\xi$ = 0.1. At larger values of ξ, the discrepancy becomes more noticeable; however, the deviations amount to only a few per cent at the most. For one set of data, the computation time required for $\Delta\xi$ = 0.05 was eight times longer than with $\Delta\xi$ = 0.1; it was therefore considered justifiable to use the increment $\Delta\xi$ = 0.1 in all the computations.

The relationship between the dimensionless time τ and the real time t is given in eq 9. Substituting numerical values for μ, L, and k_o into eq 9 leads to

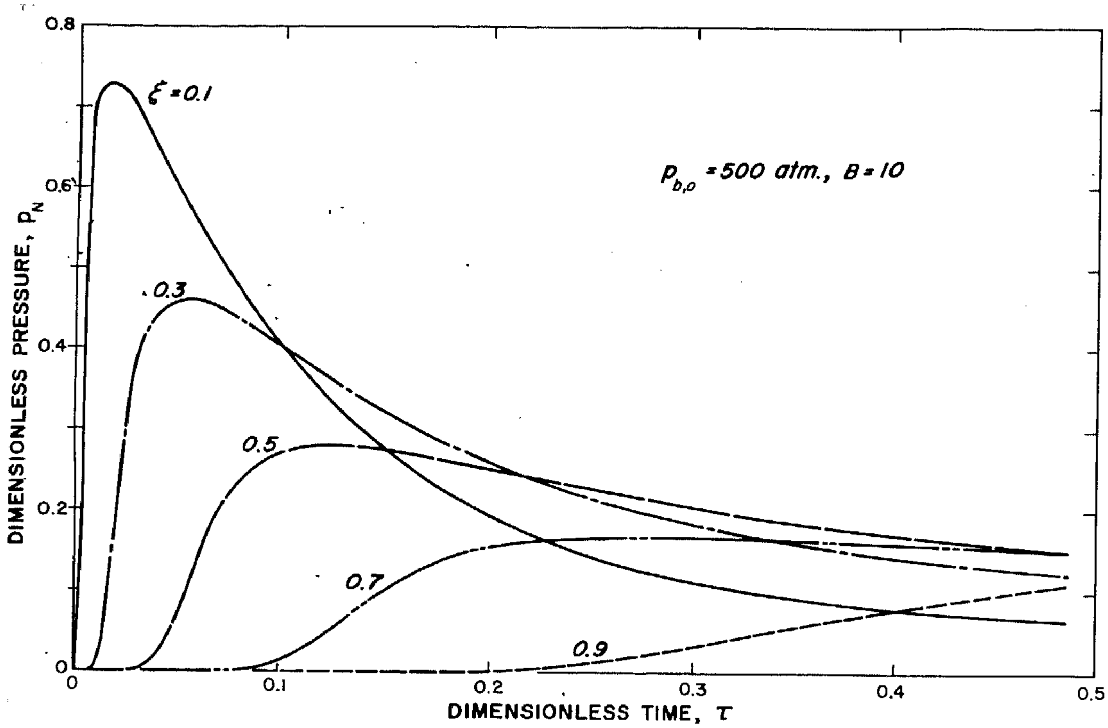
$$t = \frac{373\,\tau}{p_{b,o}} \tag{20}$$

which gives a simple relation between the dimensionless time τ used in the plots and the real time t.



a

Figure 2. Relationship between p_N and τ, Case II.



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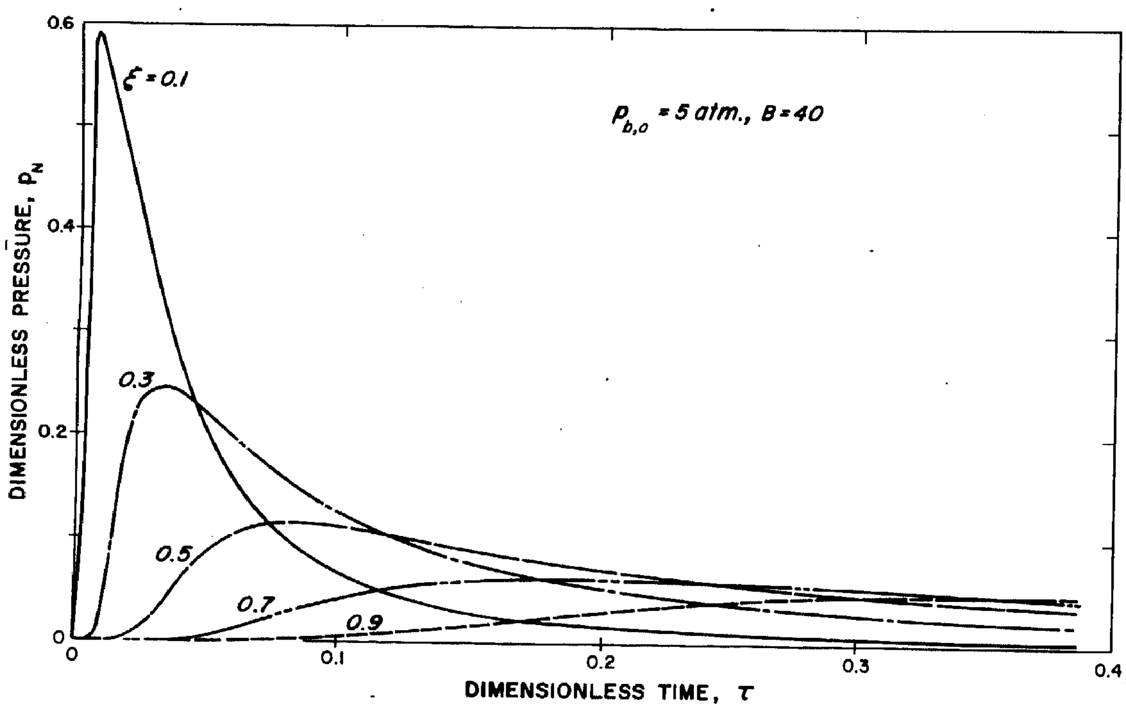

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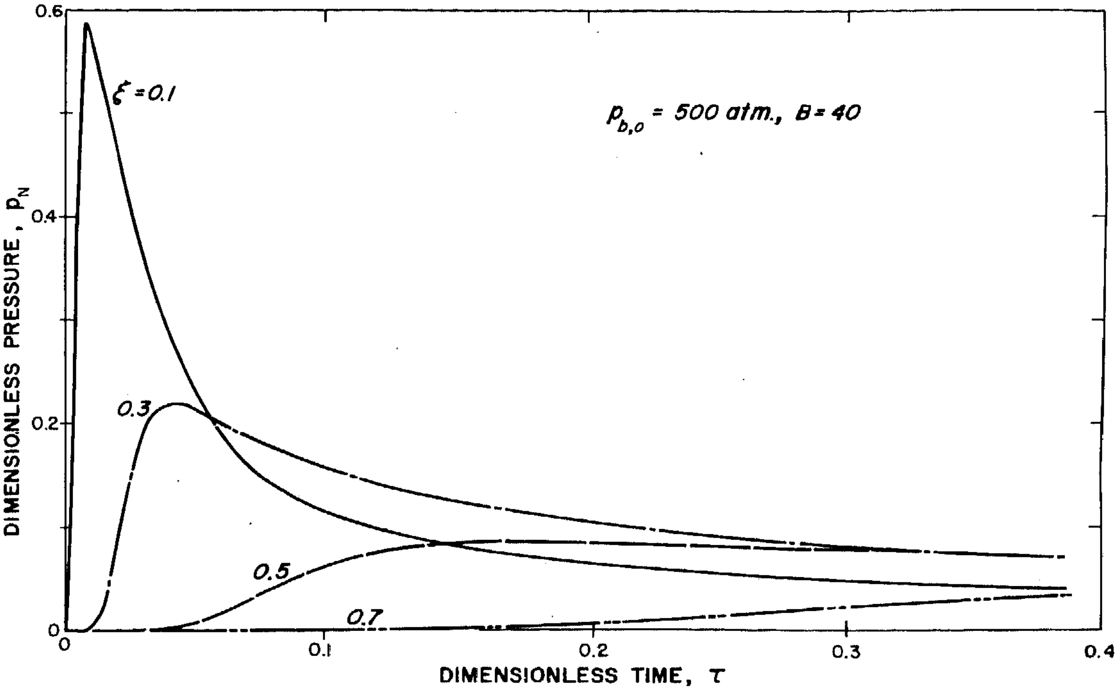
Figure 2 (cont'd). Relationship between p_N and τ, Case II.



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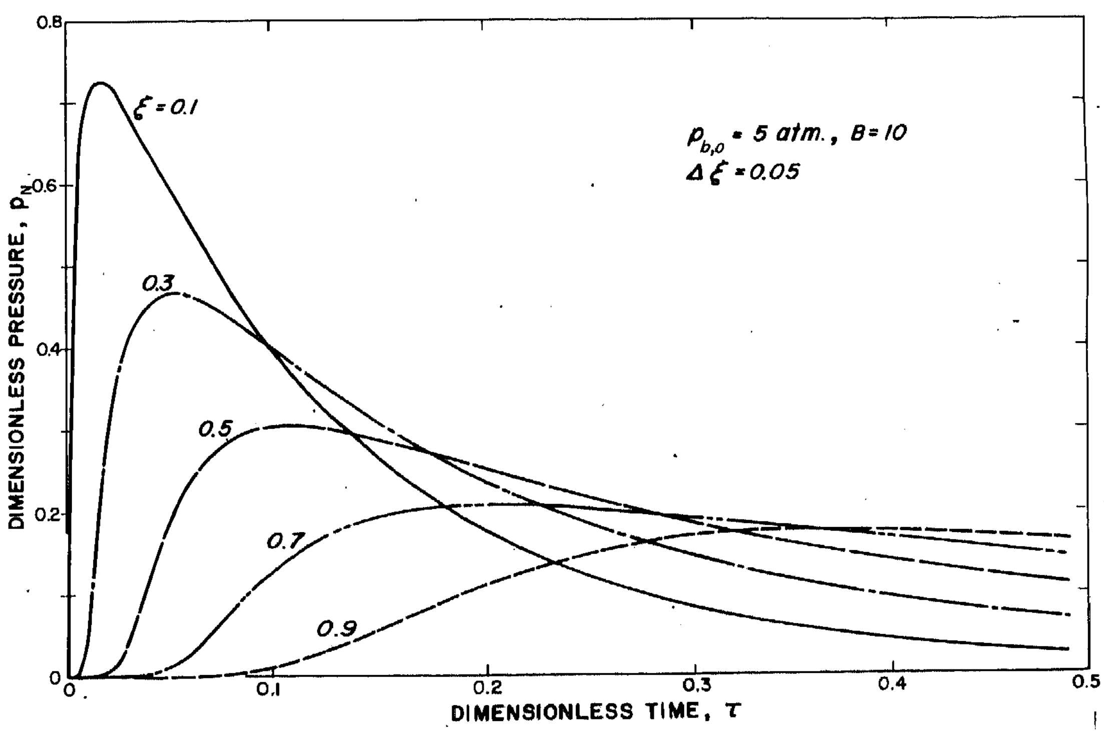

e

Figure 2 (cont'd). Relationship between p_N and τ, Case II.

Conclusion

From the plots in Figures 2a and 2c, it can be noted that the depth and magnitude of the penetration of the pressure front into the snow layer is strongly dependent on the decay constant B. The effect of the maximum pressure $p_{b,o}$ on the propagation of a pressure wave in the medium can be observed by comparing Figures 1a and 1b, Figures 2a and 2b, or Figures 2c and 2d. At high values of $p_{b,o}$ the pressure wave propagates more rapidly and converges more quickly with respect to real time.

One set of the data was plotted as shown in Figure 3 in order to make a comparison with results from the constant permeability study by Aronofsky and Jenkins (1951). However, it should be noted that the dimensionless time $\tau = k_o p_o t/\phi \mu L^2$ employed by Aronofsky and Jenkins differs from the $\tau = k_o p_o t/2\mu L^2$ used in this study by a factor of $2/\phi$.

The porosity considered in this case did not vary drastically with position for $0 < \xi < 0.5$. An average value of $\phi = 0.5$ was considered for the plot $\tau = 0.01$ in Figure 3 to make a comparison. According to the factor $2/\phi$, the plot $\tau = 0.01$ would correspond to the curve of $\tau = 0.04$ in Figure 14 of Aronofsky and Jenkins, which is shown in Figure 3 as the dashed line. It can readily be noted that the pressure front propagated much more slowly in this investigation because of the increase in permeability of the medium with depth.

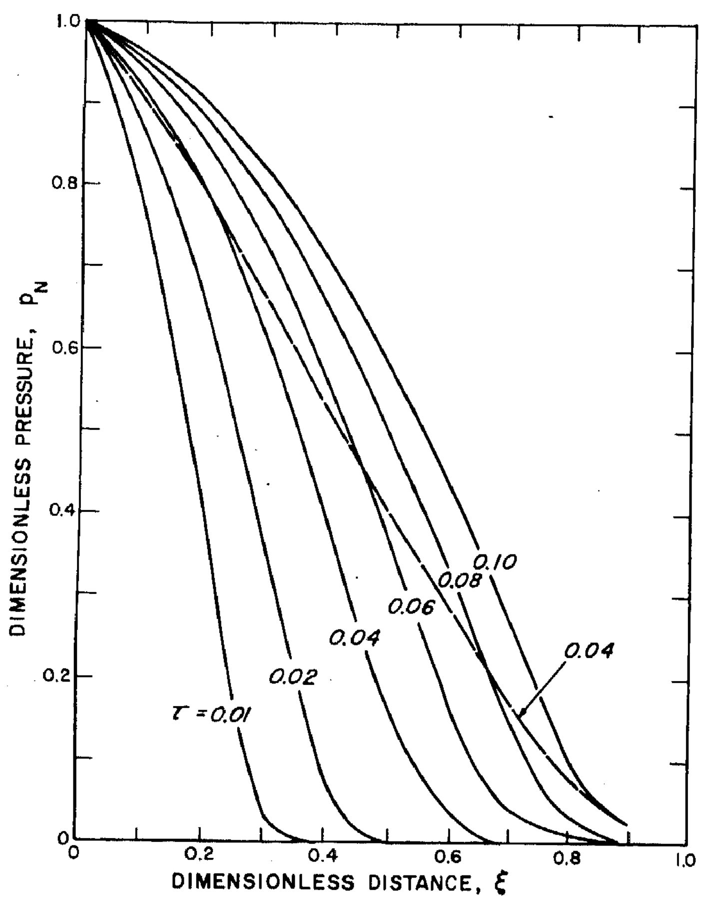

Figure 3. Comparison of the rate of pressure wave propagation through porous media with constant (dashed line) and variable permeabilities.

The assumption of ideal gas flow and a rigid incompactible porous medium will become questionable under high pressure conditions. Therefore, it will be interesting to carry out experimental measurements to check their validity. The analytical study can be continued with other types of boundary conditions, such as the imposition of a damped sinusoidal pressure on the surface.

The introduction of one of the volume-explicit equations of state for real gases will eliminate the assumption of ideal gas behavior; however, the resulting differential equation will become much more complicated.

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