Assignment 7: Due Oct 30 (Thus)

1. Let $E = \bigoplus_{i \in I} E_i$ be an A-module. Prove that E is flat if and only if each E_i is flat.

2. Let $(A, \mathfrak{m})$ be a local ring and, M and N f.g. A-modules. Prove that if $M \otimes_A N = 0$, then $M = 0$ or $N = 0$.

3. Let M, N be two submodules of an A-module E such that $M + N$ is flat. Prove that for M and N to be flat, it is necessary and sufficient that $M \cap N$ is flat.

4. A ring A is called absolutely flat if every A-module is flat. Prove that the following are equivalent.

   i. A is absolutely flat;

   ii. every principal ideal is idempotent;

   iii. every f.g. ideal is a direct summand of A.

5. Recall the definition of direct limits of modules. Prove that

   i. Tensor product commutes with direct limits, i.e.

   $$\varinjlim (M_i \otimes_A N) \cong (\varinjlim M_i) \otimes_A N;$$

   ii. If

   $$M = \varinjlim M_i$$

   then M is flat if and only if each M_i is flat.