Assignment 8: Due Nov 18 (Tues)

1. Let $A \subset B$ be integral domains and suppose that A and B have the same field of fractions; if B is faithfully flat over A then $A = B$.

2. If B is a faithfully flat A-algebra and B is Noetherian, then A is Noetherian. 3. If A is a Noetherian ring, I and J ideals of A, and A is complete for both $I=$-adic and J-adic topologies, then A is also complete for the $(I+J)$-adic topology.

4. Let A be a Noetherian ring that is I-adically complete. Then A is also J-adically complete for any ideal $J \subset I$.

5. Let A be a Noetherian ring and $\mathfrak{p} \in \mathrm{Ass}(A)$. Then there is an integer $c > 0$ such that $\mathfrak{p} \in \mathrm{Ass}(A/\mathfrak{a})$ for every ideal $\mathfrak{a} \subset P^c$.

6. Complete the proof of Hensel's lemma as stated in class.