1502 B6
Practice Mid Term I
Time: 1hrs 20 min.

Write your name in the top left corner. Attempt all questions. You must show all work in order to obtain credit

1. Find the Taylor polynomial P_4 for the function $f(x) = \sqrt{1+x}$. (4pts)

2. Write down a non-constant function f such that $f(x) = P_{100}(x)$, where P_{100} is the Taylor polynomial of f of degree 100. (2pts)

3. Express $g(x) = x^n$ in powers of $(x-1)$. (4pts)

4. Evaluate the following limits (or prove that the limits do not exist). (6pts)

(a)

$$\lim_{x \to 0} \frac{x}{\sin x}$$

(b)

$$\lim_{x \to 0} x \sin 1/x$$

(c)

$$\lim_{x \to 0} \frac{1}{x} \sin 1/x$$

5. Evaluate the improper integral (4pts)

$$\int_0^1 \frac{dx}{\sqrt{x}}.$$

6. Consider the infinite series,

$$\sum_{k=0}^{\infty} \frac{1}{(2k+1)(2k+3)}.$$

Express the partial sums s_n directly as a function of n and evaluate the limit $\lim_{n\to\infty} s_n$. (3pts)

7. Prove the convergence or divergence of the following series. (3pts)

$$\sum_{k=0}^{\infty} \frac{k^9 + k^5 + 17k + 1}{k^{10} + 113}.$$

Justify your carefully in order to obtain any credit.

8. State carefully the ratio test for convergence of a series with positive terms. (4pts)

9. What can you deduce from the ratio test regarding convergence or divergence of the harmonic series, (2pts)

$$\sum_{k=1}^{\infty} \frac{1}{k}?$$

10. Let $\{a_k\}$ be a decreasing sequence of positive numbers. Consider the alternating series

$$\sum_{k=0}^{\infty} (-1)^k a_k.$$

Prove that the odd partial sums s_{2m+1} are all positive and form an increasing sequence. (3pts)

11. Test the following series for absolute as well as conditional convergence. (3pts)

$$\sum_{k=0}^{\infty}(-1)^k \frac{k}{k^2+1}$$

12. Find the interval of convergence of the following power series. (2pts)

$$\sum_{k=0}^{\infty} \frac{x^k}{k^k}.$$