

$$\begin{aligned}
 1. \quad p(\lambda) &= \begin{vmatrix} \lambda-4 & -1 & 1 \\ -2 & \lambda-5 & 2 \\ -1 & -1 & \lambda-2 \end{vmatrix} = \begin{vmatrix} \lambda-4 & 0 & 1 \\ -2 & \lambda-3 & 2 \\ -1 & \lambda-3 & \lambda-2 \end{vmatrix} \\
 &= (\lambda-4) \begin{vmatrix} \lambda-3 & 2 \\ \lambda-3 & \lambda-2 \end{vmatrix} + \begin{vmatrix} -2 & \lambda-3 \\ -1 & \lambda-3 \end{vmatrix} \\
 &= (\lambda-3) \left[(\lambda-4)(\lambda-4) - 1 \right] = (\lambda-3)(\lambda^2 - 8\lambda + 15) \\
 &= (\lambda-3)^2 (\lambda-5)
 \end{aligned}$$

$$\lambda_1 = 3 \quad \lambda_2 = 5$$

$$1^\circ \lambda_1 = 3$$

$$[\lambda_1 - A | 0] = \left[\begin{array}{ccc|c} -1 & -1 & 1 & 0 \\ -2 & -2 & 2 & 0 \\ -1 & -1 & 1 & 0 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$V_{11} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad V_{12} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$2^\circ \lambda_2 = 5$$

$$[\lambda_2 - A | 0] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -2 & 0 & 2 & 0 \\ -1 & -1 & 3 & 0 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$V_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

3. A has distinct eigenvalues, thus diagonalizable.

$$A \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} \Rightarrow |A| = 1 \cdot (-1) \cdot 2 \cdot 4 \neq 0 \Rightarrow A \text{ invertible.}$$

All statements are correct.

4. Only D, E can be non-diagonalizable.

$$D. \lambda_1 = \lambda_2 = 1$$

$$[\lambda I - A \mid 0] = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right] \Rightarrow v = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ NOT diagonalizable}$$

$$E. p(\lambda) = \begin{vmatrix} \lambda - 1 & -1 \\ -1 & \lambda - 1 \end{vmatrix} = \lambda^2 - 2\lambda + 1 - 1 = \lambda(\lambda - 2) = 0$$

$$\lambda_1 = 0 \quad \lambda_2 = 2 \quad \text{So diagonalizable.}$$

2. (a) (b)

$$(a) (c\lambda)v = (cA)v$$

$$(b) (cA)v = (c\lambda)v$$

$$(c) A(cv) = \underline{\underline{\lambda}}(cv)$$

(d) $m_i \leq k_i$ might not be equal.